A MODEL FOR SMALL-SCALE INTERACTION IN MERGING TURBULENT REGIONS

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ABSTRACT

The dynamics of two turbulent regions initially separated by a non-turbulent background comprise three distinct phases. First, each turbulent layer grows independently; then, the two streams start interacting and coalesce; and finally the flow homogenises. The present study focuses on the second, and arguably most intriguing phase: the merging region. Therein, several small-scale aspects of turbulent/non-turbulent interfaces (TNTIs) are observed to deviate from those of classical TNTIs due to the mutual interaction. Specifically, this results in a more homogeneous flow relative to classical TNTIs, i.e. a greater balance between enstrophy and strain. We interpret this result with a model based on two approaching Burgers vortices (Kimura & Moffatt, 2014), located within the turbulent regions and adjacent to the interacting interfaces, which qualitatively reproduces our observations from direct numerical simulations.

INTRODUCTION

Two parallel turbulent streams developing within an irrotational environment spread by entraining non-turbulent fluid. As soon as both have grown by an amount comparable to their initial separation, their interaction becomes unavoidable; the streams eventually merge and the flow homogenises into a final uniform state. In between these initial and final states of simplicity, i.e. from the approximately isolated spreading of turbulence to the canonical situation of homogenous isotropic decay, the flow is dictated by an intriguing dynamics influenced by both large and small-scales (Branco *et al.*, 2025). At the core of interaction, there is the defining question of how the outermost turbulent region is influenced by the proximate approaching stream.

For an isolated turbulent region in an irrotational background, it is well-established that the thin turbulent/non-turbulent interface (TNTI), separating turbulent from irrotational fluid, is populated by intense vortical structures. These vortices, located immediately adjacent to the TNTI, are typically aligned tangentially to the interface and persist even within a few Kolmogorov micro-scales from the irrotational boundary (IB) (da Silva *et al.*, 2011). A simple conceptual model of the TNTI, therefore, places these vortical structures directly at the turbulence boundary, where their geometry deforms the interface. Building on this fact, it is tempting to analyse the interfacial dynamics, namely the fluxes of mass and entrainment of irrotational fluid, with regard to these structures. Indeed, such has been envisaged in isolated interfaces (e.g. Watanabe *et al.* (2017)), but no phenomenological interpretation is available whenever two TNTIs become liable to interact. Here, we study a situation of merging turbulent regions using direct numerical simulations (DNS), focusing on how their mutual interaction influences the smallest scales of motion near the TNTIs. To interpret these effects, a model based on two approaching Burgers vortices, strained by an axisymmetric background field in the spirit of Kimura & Moffatt (2014), is proposed.

DIRECT NUMERICAL SIMULATIONS

The DNS dataset to be analysed in the present work has been purposely generated for the study of merging turbulent fronts in the absence of mean shear (Branco *et al.*, 2025). The simulations were performed using a code that combines pseudo-spectral methods (in cross-stream, y, and spanwise, z, directions) with a 6th-order compact finite differences scheme along the streamwise direction, x. An explicit 3rd-order Runge-Kutta scheme is used for time advancement, while pressure-velocity coupling is treated with a fractional step method. The spatially developing flow is made possible by the inflow condition, where velocity planes from precursor simulations of homogenous isotropic turbulence are injected in the spatial domain after adding a uniform convection velocity, U_∞. The central non-turbulent region is created by convolving these precursor velocity planes with a sharp filter that effectively removes velocity fluctuations at the inner region of the domain. Both simulations are synchronised with the same time step and the velocity fluctuations are retrieved from a fixed plane, ensuring that the temporal correlations from the precursor simulation are made equivalent in the spatial domain. The result are two turbulent fronts, with characteristics controlled

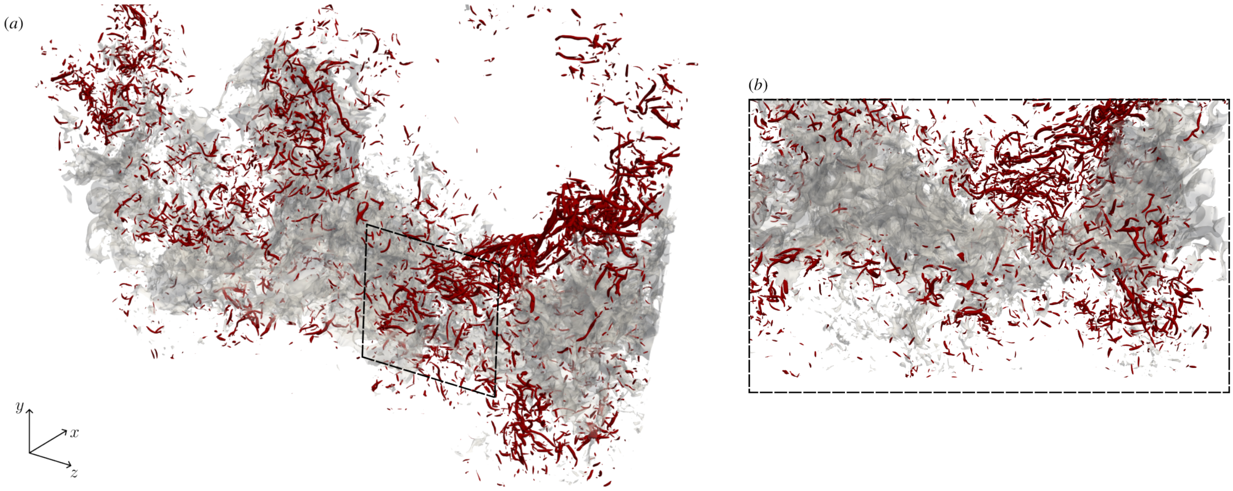

Figure 1. A slice of the spatial domain, in the merging region. The irrotational boundaries are shown in grey, while the small-scale intense vortices ($Q \gg 0$) that populate the turbulent/non-turbulent interfaces are presented in red. (*a*) Three-dimensional view of the slice; (*b*) inset of the (z, y)-plane where merging is eminent.

by the precursor simulation, separated by an irrotational region with initial slot-width H, that evolve without mean shear. As such, this configuration provides a clean set-up to study the small-scale aspects of merging without the additional effects of large-scale shear. More details are given in Branco *et al.* (2025).

CONDITIONAL STATISTICS

The effects of the interaction between both turbulent regions are analysed recurring to conditional statistics of enstrophy and strain in relation to the interface location. To that end, the irrotational boundary (IB), often idealized as a surface that separates turbulent and irrotational flow, is detected as the collection of iso-points of low enstrophy. The threshold value to detect the IB is obtained by the standard procedure of analysing the histogram of turbulent volume fraction (Zecchetto & da Silva (2021), and references therein). At each point of the IB, a local reference frame is created, aligned with the interface normal direction and pointing towards positive enstrophy gradients, i.e. turbulent flow. In this local frame, flow statistics are collected by trilinear interpolation of the domain points and accumulated as function of their distance to the IB. This procedure allows to compute the evolution of any scalar field across the TNTI, without having to reconstruct the actual IB surface, since it is conceptualized as a collection of points. This is especially important in the context of merging turbulent fronts, as the IB surface geometry can be very complex. The conditional average of any variable performed as described above is denoted as $\langle \cdot \rangle_I$.

SMALL-SCALE INTERACTIONS

Figure 1 presents the geometry of the flow to be investigated. In this slice of the domain, the two irrotational boundaries (surfaces in grey) are interacting, and the intense vortical structures that reside within them (in red) start to approach. The details of this interaction are evident in figure 2*a*, where the enstrophy field conditioned to the distance from the IB is presented for different streamwise locations, after normalisation by the local rate-of-strain. The colour gradient in figure 2 encodes the streamwise location where the conditional profiles were obtained, with darker colours corresponding to locations more downstream, i.e. closer to the merging region. Initially, the conditional profiles are similar to those typical of a classical isolated TNTI, with the peak located at $\approx 12\langle \eta \rangle_I$ from the irrotational boundary and its magnitude $\langle \omega_i \omega_i \rangle / \langle 2S_{ij}S_{ij} \rangle \approx 1.4$ evidencing the characteristic inhomogeneity of this layer, which were shown to be universal (Zecchetto & da Silva, 2021). However, when the merging region is approached two main differences are noticeable: (i) the intensity of the normalised peak decreases, and (ii) its location is displaced towards the irrotational boundary. The former indeed shows that during merging the inherently inhomogeneous region is homogenised by the action of the proximate turbulent stream.

If one recovers the simplified cartoon of a TNTI layer has a vortex tube (supported by the fact that $\langle \omega_i \omega_i \rangle > \langle 2S_{ij}S_{ij} \rangle$), one can advance a model to describe this situation, where the intense vortices within the interface layer are modelled by Burgers-type vortices. Consider an axisymmetric background strain field $\mathbf{U} = (-\alpha x, -\alpha y, 2\alpha z)$, where incompressibility is respected and α controls the magnitude of both stretching, along z direction, and compression, equal in x and y directions. It is evident that the Burgers vortex, a steady solution of the linearised vorticity equation under axisymmetric strain (Hunt & Carruthers, 1990) and an accurate model of small-scale intense vorticity structures (Jiménez *et al.*, 1993), will maintain its characteristics if subjected to the proposed background strain. At any point (x, y, z) of the domain, the strain provided by the background is exactly the one needed by the Burgers vortex to maintain its integrity: it will be stretched along its axial direction, z, while being radially compressed in the other two directions. The axisymmetry dictates that the tubular shape and the circular cross section remain unchanged. The vorticity distribution of this single vortex is given by

$$\omega(x, y) = \omega_0 e^{-x^2 - y^2}, \qquad (1)$$

where $\omega_0 = (\alpha \Gamma_0 / 2\pi\nu)$, Γ_0 being the vortex circulation, ν the kinematic viscosity of the fluid and x, y the spatial coordinates normalised by the vortex radius $R_B = \sqrt{2\nu/\alpha}$ obtained by an equilibrium between enstrophy diffusion and production (Hunt & Carruthers, 1990).

From rapid distortion theory, after linearising the equations, the resulting vorticity field of two equal-strength Burgers

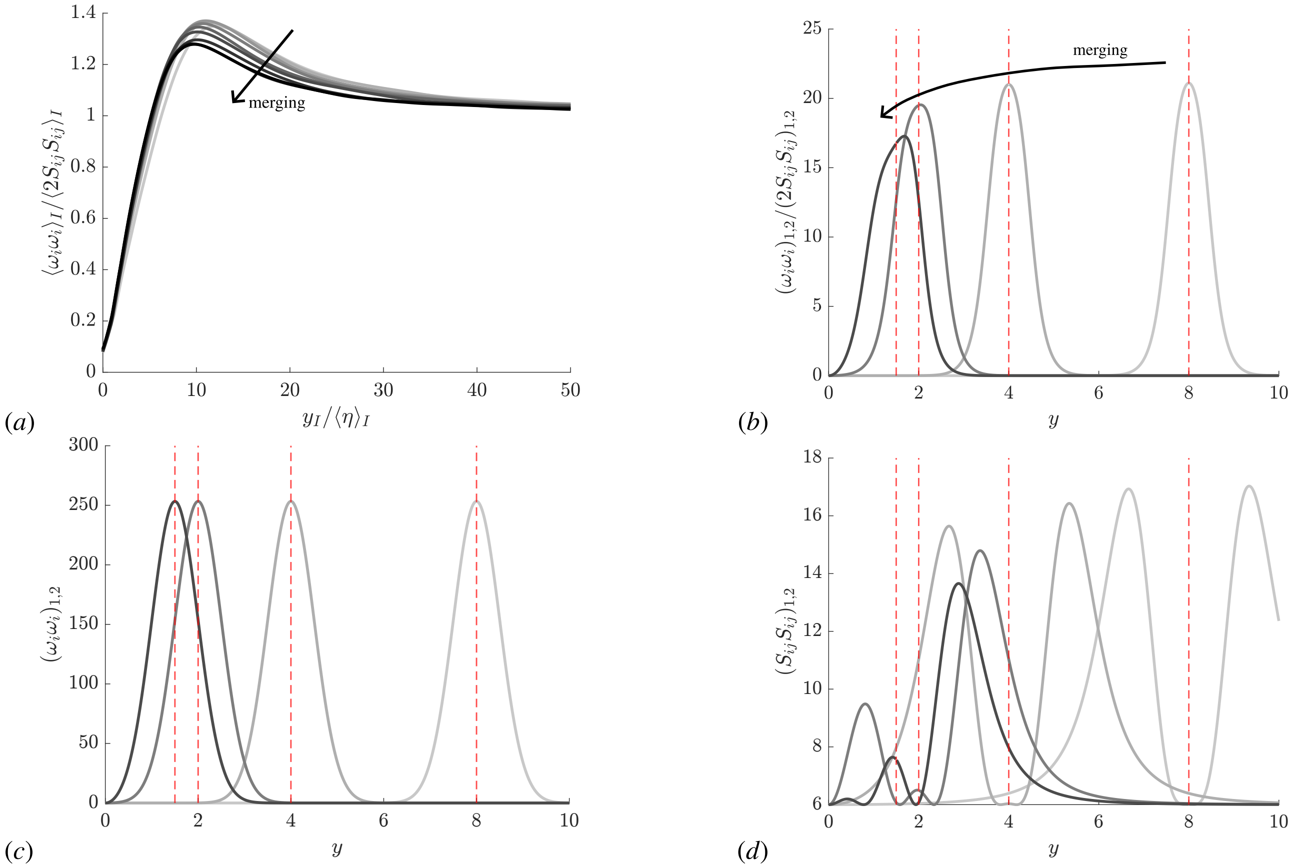

Figure 2. (*a*) Turbulent/non-turbulent interface conditional enstrophy profiles normalised by the local strain rate $\langle \omega_i \omega_i \rangle_I / \langle 2S_{ij}S_{ij} \rangle_I$, for different streamwise sections within the merging region. (*b*) Profiles of $(\omega_i \omega_i)_{1,2}/(2S_{ij}S_{ij})_{1,2}$ for the linearised model of two merging Burgers vortices, for successively decreasing distances between the vortex cores ($a = 8, 4, 2, 1.5$) (units of Burgers radius). We have chosen $\Gamma_0 = 50$ and $\alpha = 1$, but the behaviour is independent of the specific values as long as $\Gamma_0 \gg \alpha$. The red dashed vertical lines mark the position of the vortex core at each instant of time. (*c*) Profiles of enstrophy $(\omega_i \omega_i)_{1,2}$ and (*d*) rate-of-strain $(S_{ij}S_{ij})_{1,2}$ during merging along the positive semi-axis $x = 0$. Darker colours denote later instants of time.

vortices initially placed at $(x,y) = (0, \pm a)$ will be the addition of the individual solutions (Kimura & Moffatt, 2014),

$$\omega_{1,2}(x,y) = \omega_0 \left[F_\omega(-a) + F_\omega(a) \right], \tag{2}$$

where $F_\omega(a) = e^{-x^2-(y+a)^2}$ from equation (1). The total velocity field (u,v,w) will also be given by the superposition of the velocities induced by both vortices and their common background $\mathbf{U}$, yielding

$$\begin{aligned} u(x,y) &= -\tilde{\alpha} x + F_u(-a) + F_u(a), \\ v(x,y) &= -\tilde{\alpha} y + F_v(-a) + F_v(a) \\ w(z) &= 2\tilde{\alpha} z, \end{aligned} \tag{3}$$

where

$$\begin{aligned} F_u(a) &= -\frac{y+a}{x^2+(y+a)^2}\left(1 - e^{-x^2-(y+a)^2}\right), \\ F_v(a) &= -\frac{x}{x^2+(y+a)^2}\left(1 - e^{-x^2-(y+a)^2}\right), \end{aligned} \tag{4}$$

and $\tilde{\alpha} = \Gamma_0/(4\pi\nu)$ is a non-dimensional parameter, after the velocities have been normalised by $\Gamma_0/(2\pi R_B)$.

Dynamically, the ambient axisymmetric strain field will drive the vortices together towards the origin $(x,y) = (0,0)$, with the distance between their centres decreasing exponentially fast (Kimura & Moffatt, 2014). Note that the velocity field in (3) closes our problem since we can now evaluate any quantity for this configuration as a function of both space and time. The resulting enstrophy and rate-of-strain distributions along the line $x = 0$ that crosses both vortex centres are displayed in figures 2*c* and *d*, for decreasing distances between the vortex centres. Along their trajectories, the strain field, initially concentrated on a ring around each vortex centre will begin to superimpose with the enstrophy field of the other vortex, with their ratio $(\omega_i \omega_i)_{1,2}/(2S_{ij}S_{ij})_{1,2}$ evolving according to figure 2(*b*). This evolution is in qualitative very good agreement with the actual conditional ratio throughout merging obtained in our DNS (figure 2*a*).

In spite of being a very simple model, and even though it may not capture the complex details of the expected vortex reconnection during merging, it remains appealing because it still retains the minimal ingredients of the process: the interaction leads to an increased rate-of-strain inside the cores of both vortices and the resulting flow homogenisation within the interface. Preliminary results point that the specific mechanism responsible for this enhanced strain is the non-local pressure-strain interactions. Numerical simulations of the full non-linear dynamics of merging Burgers vortices have shown that there is a significant deformation of the vortex cores (Buntine & Pullin, 1989), hinting the reason why the normalised peaks move slightly towards the irrotational boundary (see figure 2*a*).

CONCLUSIONS

The model proposed, based on the linearised interaction of two Burgers vortices, is found to describe well the qualitative behaviour of small-scales in merging turbulent regions. Of course, extensions of this model to account for the effects of non-axisymmetric strain are desirable and possible (Kimura & Moffatt, 2014). Of especial interest is the case where the vortices are subjected to the characteristic strain inside the TNTI, where the average ratio of principal strain rates is dependent on the distance to the interface, reaching $3.5 : 1 : -4.5$ inside the rotational region, while being essentially bi-dimensional in the irrotational part (Elsinga & da Silva, 2019). This should present additional difficulties since not all initial configurations will lead to interaction, but only those where the vortices are displaced exactly along the compressive direction. Moreover, it is also not granted that the cores maintain their circular cross-section along the trajectories (Moffatt *et al.*, 1994).

Because this model has originally been developed to interpret experiments on vortex reconnection, our results suggest that reconnection may play a significant role during the merging, potentially serving as the underlying mechanism by which turbulence is locally generated. Indeed, this is foreseeable given that vortex reconnection is known to produce the celebrated $-5/3$ inertial range of the energy spectrum (Yao & Hussain, 2022). The evaluation of such hypothesis would require, for example, the analysis of interface-conditioned two-point velocity correlations (Mollicone *et al.*, 2025) for merging turbulent/non-turbulent interfaces.

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