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TIMESCALE-BASED UNDERSTANDING TURBULENCE MODULATION BY POLYMER ADDITIVES

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ABSTRACT

We numerically investigate the physics of isotropic homogeneous turbulence modulated by polymer additives. We employ an Eulerian–Lagrangian approach that couples direct numerical simulation (DNS) with Brownian dynamics simulation (BDS). We model a polymer as a finitely extensible nonlinear elastic (FENE) dumbbell. In the present study, we focus on how polymers affect the statistical properties of turbulence, specifically the power-law scaling of the energy spectrum. We show that for large enough Weissenberg numbers, the energy spectrum exhibits the -3 power-law scaling with the wavenumber. This implies that the characteristic timescale of the modulated turbulence becomes constant regardless of the wavenumber, where the timescale is comparable with the integral timescale of turbulence. We characterize the unified timescale by focusing on the steady state of polymers in solutions. In this state, the polymer length is constant on average, indicating a balance between stretching and relaxation. By evaluating this microscopic balance, we define an effective relaxation time. We find that this effective relaxation time is comparable with the unified turbulent timescale. We thus conclude that the balance of timescales between turbulence and the polymers is the origin of the -3 power-law scaling.

INTRODUCTION

Even a small amount of polymers can drastically alter the structures and statistics of turbulence compared to those in a Newtonian fluid. The phenomenon is closely related to the Toms effect, a well-known frictional drag reduction in wall-bounded turbulence (Toms, 1948). It is therefore important to understand how polymers modulate turbulence. This modulation occurs through mechanical interactions between turbulence and polymers. Specifically, kinetic energy is transferred from the turbulence to the polymers as they are stretched. However, the underlying physics is not fully understood.

Many previous studies have examined the statistical properties of polymer-laden turbulence, such as the velocity structure functions and energy spectrum, as their power-law scaling reflects the fundamental nature of turbulence. However, there is a lack of consensus. Although recent numerical simulations (Rosti *et al.*, 2023) showed the -2.3 power-law scaling for the energy spectrum, other power-laws were also reported by some experimental and theoretical studies (Vonlanthen & Monkewitz, 2013; Fouxon & Lebedev, 2003). In the present study, we aim to understand the physics of turbulence modulation by providing a new timescale-based physical picture governing the power-law spectra.

GOVERNING EQUATIONS

We conduct hybrid Eulerian–Lagrangian numerical simulations of turbulence in dilute polymer solutions similar to Watanabe & Gotoh (2013). This approach consists of two parts: direct numerical simulation (DNS) for the turbulence and Brownian dynamics simulation (BDS) for the polymers. The governing equations for the DNS are the continuity equation,

$$\boldsymbol{\nabla}\cdot\boldsymbol{u}=\boldsymbol{0}, \tag{1}$$

and the Navier–Stokes equation,

$$\frac{\partial \boldsymbol{u}}{\partial t}+\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u}=-\frac{1}{\rho_{\mathrm{st}}}\boldsymbol{\nabla}p+\nu_{\mathrm{sv}}\nabla^2\boldsymbol{u}+\boldsymbol{f}_{\mathrm{ex}}+\boldsymbol{\nabla}\cdot\boldsymbol{T}_{\mathrm{p}}, \tag{2}$$

where ρ is the solution density, ν_{sv} is the solvent kinematic viscosity, $\boldsymbol{f}_{\mathrm{ex}}$ is the external force, and $\boldsymbol{T}_{\mathrm{p}}$ is the polymer elastic stress. The external force $\boldsymbol{f}_{\mathrm{ex}}$ drives the turbulence to maintain a constant energy injection rate (Lamorgese *et al.*, 2005). Before adding the polymers, the initial Taylor microscale-based Reynolds number is fixed at $Re_\lambda \approx 160$. As a polymer model, we use finitely extensible nonlinear elastic (FENE)

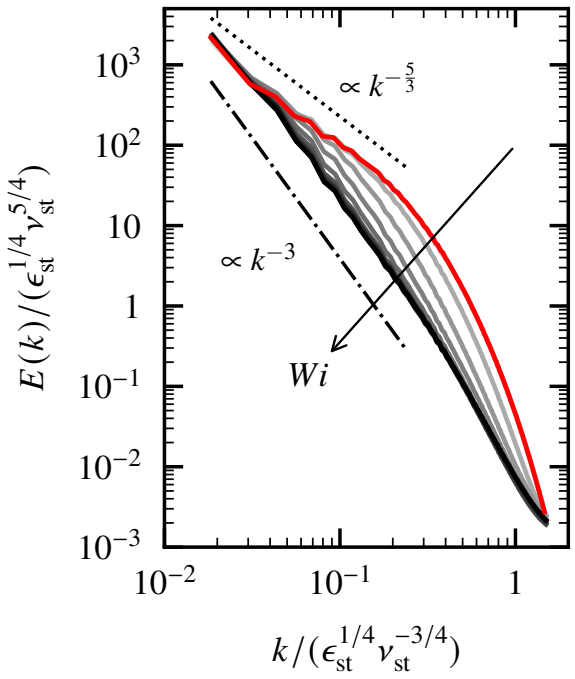

Figure 1. Normalized energy spectra for a Newtonian fluid (red) and polymer solutions (grayscale). Darker colors show the larger Weissenberg numbers. The dot line and dash-dotted line represent the $-3/5$ and -3 power-law scalings, respectively.

dumbbells. The motion of each FENE dumbbell is described by its center of mass $\boldsymbol{r}_\mathrm{g}$ and its end-to-end vector $\boldsymbol{R}$. Their governing equations are the following Langevin equations:

$$\begin{cases} \dfrac{\mathrm{d}\boldsymbol{r}_\mathrm{g}}{\mathrm{d}t} = \boldsymbol{u} + \dfrac{R_\mathrm{eq}}{\sqrt{8\tau_\mathrm{p}}}(\boldsymbol{w}_1 + \boldsymbol{w}_2), & \text{(3a)} \\ \dfrac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}t} = \boldsymbol{\nabla u}\cdot\boldsymbol{R} - \dfrac{1}{2\tau_\mathrm{p}}\dfrac{\boldsymbol{R}}{1-|\boldsymbol{R}|^2/R_\mathrm{max}^2} + \dfrac{R_\mathrm{eq}}{\sqrt{2\tau_\mathrm{p}}}(\boldsymbol{w}_1 - \boldsymbol{w}_2), & \text{(3b)} \end{cases}$$

where $\boldsymbol{u}$ and $\boldsymbol{\nabla u}$ are the fluid velocity and velocity gradient at the position of the dumbbell $\boldsymbol{r}_\mathrm{g}$, R_eq and R_max are the equilibrium and maximum lengths of polymers, τ_p is the relaxation time of the polymer, and $\boldsymbol{w}_i$ $(i = 1, 2)$ are standard normal random variables. A key dimensionless parameter in the present simulations is the Weissenberg number Wi. We define Wi as the ratio of the polymer relaxation time to the initial Kolmogorov time τ_η of the turbulence before adding polymers. By varying τ_p, we investigate the wide range of $Wi = 0.06$–1024.

RESULTS

To quantify turbulence modulation by polymers, we investigate the energy spectra. Figure 1 shows the normalized energy spectra $E(k)/(\varepsilon_\mathrm{st}^{1/4}\nu_\mathrm{st}^{5/4})$ for different Wi as functions of the wavenumber k. In this figure, ν_st and ε_st denote the kinematic viscosity and energy dissipation rate of the solution, respectively. For the polymer solutions, the spectra are attenuated at high wavenumbers. For larger Weissenberg numbers, the spectra tend to follow the -3 power-law scaling with the wavenumber. To elucidate the physical meaning of the scaling exponent -3, we consider the characteristic timescale

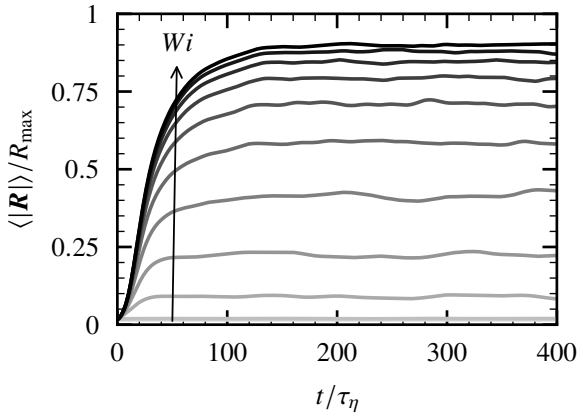

Figure 2. Timeseries of averaged polymer length $\langle|\boldsymbol{R}|\rangle$. Darker colors indicate larger Weissenberg numbers.

$\tau_\mathrm{f}(k) = \left\{k^3 E(k)\right\}^{-1/2}$ of turbulent structures at scale k^{-1}, which is derived from the dimensional analysis. When the spectra follow the -3 power-law scaling (i.e. $E \propto k^{-3}$), $\tau_\mathrm{f}(k)$ becomes constant. This indicates that the turbulence is modulated such that the characteristic time of turbulent structures becomes independent of the wavenumber.

Next, we investigate the relationship between this unified timescale of turbulence and the polymer relaxation timescale. Since τ_p is a model parameter and can be arbitrarily large, it may not represent a physically relevant timescale for sufficiently large Wi. Therefore, we introduce an effective polymer relaxation time τ_p^*, which depends on the averaged length $\langle|\boldsymbol{R}|\rangle$ of polymers. Figure 2 shows the timeseries of $\langle|\boldsymbol{R}|\rangle$. At the statistical steady state ($t/\tau_\eta > 100$), $\langle|\boldsymbol{R}|\rangle$ remains constant on average. This suggests that polymers are in a state with statistical balance between stretching and relaxation. From (3b), this balance can be expressed as

$$\begin{aligned} 2\langle(\boldsymbol{\nabla u}\cdot\boldsymbol{R})\cdot\boldsymbol{R}\rangle &= \frac{1}{\tau_\mathrm{p}}\left\langle\frac{|\boldsymbol{R}|^2}{1-|\boldsymbol{R}|^2/R_\mathrm{max}^2}\right\rangle \\ \Leftrightarrow\ \Gamma_\mathrm{fluid} &= \Gamma_\mathrm{elast}. \end{aligned} \tag{4}$$

The left- and right-hand sides of (4) represent the contribution of stretching by the turbulence and relaxation by the polymers, respectively. This balance can be rewritten in terms of timescale balance:

$$\frac{\langle|\boldsymbol{R}|^2\rangle}{\Gamma_\mathrm{fluid}} = \frac{\langle|\boldsymbol{R}|^2\rangle}{\Gamma_\mathrm{elast}}. \tag{5}$$

The right-hand side of (5) represents a relaxation timescale expressed solely by the polymer's variables. Therefore, we define the effective relaxation time τ_p^* of polymers as

$$\tau_\mathrm{p}^* = \tau_\mathrm{p}\frac{\langle|\boldsymbol{R}|^2\rangle}{\left\langle|\boldsymbol{R}|^2/(1-|\boldsymbol{R}|^2/R_\mathrm{max}^2)\right\rangle}. \tag{6}$$

In figure 3, we compare the effective relaxation time τ_p^* with the unified turbulent timescale $\tau_\mathrm{f}(k)$. We find that when the Weissenberg number is sufficiently large (i.e., when the spectra exhibit the -3 power-law scaling), the ratio $\tau_\mathrm{p}^*/\tau_\mathrm{f}(k)$

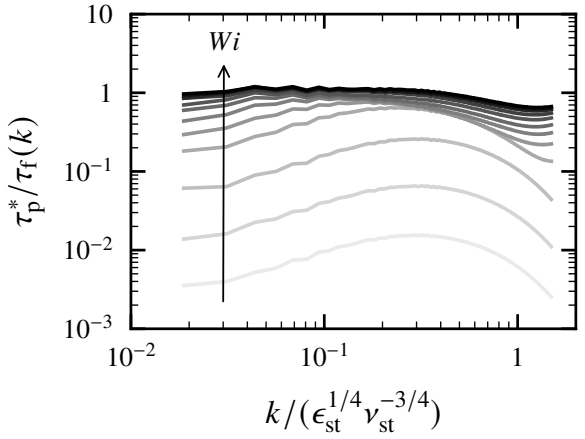

Figure 3. The ratio of the polymer effective relaxation time τ_p^* to modulated turbulent timescale $\tau_f(k)$. Darker colors indicate larger Weissenberg numbers.

becomes unity. This result indicates that the timescale of the modulated turbulence is characterized by the effective relaxation timescale of the polymers.

CONCLUSION

To understand the underlying physics of turbulence modulated by polymer additives, we have conducted numerical simulations of isotropic homogeneous turbulence in dilute polymer solutions with varying polymer relaxation times. Remarkably, we observe that for large enough Weissenberg numbers, the energy spectra tend to follow the -3 power-law scaling with the wavenumber. We interpret this scaling exponent -3 as being consistent with the unification of the turbulent timescale. Furthermore, by introducing the effective relaxation time τ_p^*, derived from the microscopic balance between stretching and relaxation of polymer, we demonstrate that this unified turbulent characteristic timescale is equal to τ_p^*. In other words, the timescales of the turbulence and the polymers are balanced. These results suggest that turbulence modulation by polymer additives can be consistently understood from a timescale perspective.

The -3 power-law scaling of the spectra is also observed in two-dimensional turbulence. The key difference between two- and three-dimensional turbulence is the absence of vortex stretching in the former. This analogy (Bos *et al.*, 2025) leads us to expect that polymers may suppress vortex stretching in three-dimensional turbulence. In our presentation, we will discuss how polymers affect vortex stretching events.

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