

THE $-5/3$ SCALING FOR ENERGY SPECTRA IN A CIRCULAR CYLINDER WAKE

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ABSTRACT

We examine why an apparent $-5/3$ band can occur in one-dimensional energy spectra of a circular-cylinder wake at finite Reynolds number, using three-component hot-wire measurements acquired with a four-X-wire probe in the mean-shear plane at $x/d = 10, 20$, and 40 for $Re_d = 2500$ – 15000 , based on the free-stream velocity U_∞ and the cylinder diameter d . At $Re_d = 15000$ and $x/d = 20$, the lateral spectra ϕ_v and spanwise spectra ϕ_w exhibit a broader $-5/3$ range over $y/d \simeq 0$ – 1.2 than the streamwise spectrum ϕ_u , and the scaling progressively weakens as y/d increases. To interpret this behaviour, we adopt a physical-space scale-by-scale (SbS) budget for the second-order increment energy $(\Delta q)^2$, which quantifies the contribution of large-scale forcing in the separation range associated (via Taylor's hypothesis) with the observed spectral band. The SbS analysis shows that mean-shear production and cross-stream transport/diffusion remain dynamically significant across that range, so the $-5/3$ band should not be interpreted as a classical inertial subrange with negligible forcing. Instead, the results support an overlap-scaling interpretation in which persistent large-scale forcing sustains outer similarity while small-scale Kolmogorov scaling is already meaningful.

Introduction

Kolmogorov (1941a,b) proposed two hypotheses to predict small-scale turbulent motions at sufficiently high Reynolds numbers. The first hypothesis asserts that, in locally homogeneous and isotropic turbulence, a universal equilibrium range of small scales exists, in which the turbulent statistics are uniquely determined by the mean energy dissipation rate $\bar{\epsilon}$ (the overbar denotes temporal or spatial averaging) and the kinematic viscosity ν . Kolmogorov used dimensional analysis to derive the velocity scale $u_K = (\bar{\epsilon}\nu)^{1/4}$ and the length scale $\eta = (\nu^3/\bar{\epsilon})^{1/4}$. The second hypothesis states that the existence of an inertial subrange at scales much larger than η yet much smaller than the integral scale. Within this range, the statistics depend solely on $\bar{\epsilon}$, with viscous effects becoming

negligible. As a consequence, dimensional analysis yields the well-known $-5/3$ power-law scaling of the energy spectrum. At moderate Reynolds numbers, where classical inertial subrange is not well established, a number of studies (Parnaudeau *et al.*, 2008; Wissink & Rodi, 2008; Gomes *et al.*, 2014, 2015; Laizet *et al.*, 2015; Paul *et al.*, 2017, 2018; Portela *et al.*, 2017, 2020; Yasuda *et al.*, 2020; Ortiz *et al.*, 2021; Zhou *et al.*, 2018, 2019, 2020, 2023) have nevertheless reported the presence of an approximate $-5/3$ scaling over a restricted range of scales. For example, Wissink & Rodi (2008) identified a weak $-5/3$ -like scaling in the streamwise velocity spectrum $\phi_u(f)$ in the near wake ($x/d = 6$) of a circular cylinder at $Re_d = 3300$, where f denotes frequency. In this paper, the one-dimensional streamwise, lateral, and spanwise energy spectra are denoted by ϕ_u , ϕ_v , and ϕ_w , respectively. u_i is the fluctuating velocity component in the x_i direction. u_1 , u_2 , and u_3 will be used interchangeably with u , v , w ; similarly for x_1 , x_2 , x_3 and x , y , z . The Reynolds number is defined as $Re_d = U_\infty d/\nu$, based on the free-stream velocity U_∞ and the cylinder diameter d (or the side width of a square cylinder). Similar observations have been reported in square-cylinder wakes. In the near-wake region ($x/d = 1$ – 8) at $Re_d = 3900$, Portela *et al.* (2017, 2020) showed that the centreline spectra $\phi_u(f)$, $\phi_v(f)$, and $\phi_w(f)$ exhibit an approximate $-5/3$ scaling despite pronounced inhomogeneity and anisotropy. At a lower Reynolds number ($Re_d = 2500$), Zhou *et al.* (2023) further reported comparable behaviour in the wavenumber spectrum $\phi_u(\kappa_1)$ at $x/d = 6$ and 26 , where κ_1 denotes the streamwise wavenumber. More complex configurations also display similar features. For two side-by-side square cylinders, Zhou *et al.* (2019, 2020) observed an approximate $-5/3$ scaling in $\phi_u(f)$ at $x/d = 6$, even though the turbulence remains in a developing and highly intermittent state.

The above studies have provided valuable insights into the occurrence of $-5/3$ scaling in wake flows; however, a comprehensive understanding of this scaling in bluff-body wakes remains still lacking. In particular, its dependence on velocity components, spatial location, and Reynolds number has not yet been systematically clarified. To address this issue, the present study examines the $-5/3$ scaling behaviour in the wake of a

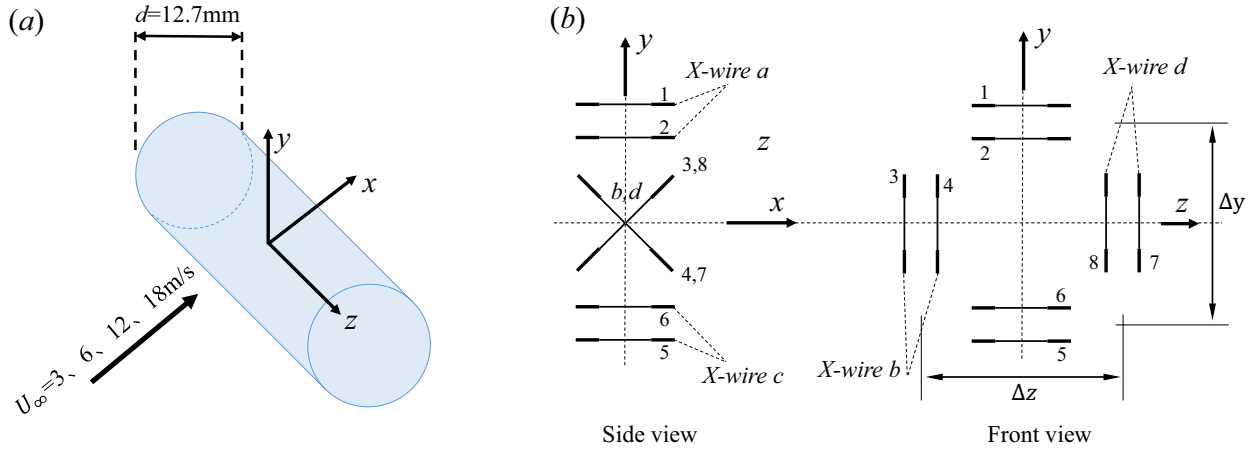


Figure 1. (a) Definition sketch and coordinate system; (b) Sketch of the multi-wire probe.

circular cylinder. Furthermore, we consider the dual scaling of the energy spectrum, first introduced by Tennekes & Lumley (1972) for shear flows. Specifically, the high-wavenumber range is expected to scale with inner (Kolmogorov) variables (u_K, η) , whereas the low-wavenumber range scales with outer variables (u', L_u) associated with large-scale motions. Here, $u' (= \overline{u'^2})^{1/2}$ denotes the root-mean-square streamwise velocity fluctuation, and $L_u = \int_0^\infty R_{uu}(r) dr$ is the longitudinal integral length scale, where $R_{uu} = \overline{u(x+r)u(x)}/u'^2$ is the longitudinal velocity autocorrelation coefficient and r is the spatial separation. Tennekes & Lumley (1972) suggested that the overlap of these two scaling ranges may give rise to an apparent $-5/3$ scaling when the Taylor-microscale Reynolds number Re_λ is sufficiently large. This interpretation has been supported by subsequent studies in grid turbulence (Gamard & George, 2000; Tang *et al.*, 2024), where dual scaling has been identified in both energy spectra and structure functions. Motivated by these findings, we investigate whether the $-5/3$ scaling observed in the present cylinder wake can be interpreted within a similar dual-scaling framework.

Scale-by-scale framework: increment-energy budget and interpretation

To interpret an apparent $-5/3$ spectral band in a spatially developing wake, we use a physical-space, scale-by-scale (SbS) budget for the second-order increment energy

$$\overline{(\Delta q)^2} = \overline{\Delta u \Delta u} + \overline{\Delta v \Delta v} + \overline{\Delta w \Delta w}, \quad (1)$$

where $\Delta u_i \equiv u_i(x+r) - u_i(x)$, which quantifies how energy contained in increments at separation r is affected by interscale transfer, viscous effects, and large-scale forcing/inhomogeneity (Tang *et al.*, 2016). For a statistically stationary wake, the SbS balance may be viewed schematically as

$$\mathcal{A}(r) + \mathcal{T}(r) = \mathcal{P}(r) + \mathcal{D}(r) - \mathcal{E}(r), \quad (2)$$

where $\mathcal{A}$ denotes a large-scale advection contribution associated with streamwise decay, $\mathcal{T}$ denotes nonlinear interscale transfer (third-order increments), $\mathcal{P}$ is the mean-shear production, $\mathcal{D}$ is the cross-stream diffusion, and $\mathcal{E}$ is the viscous destruction term. In a classical inertial subrange, $\mathcal{P}$, $\mathcal{D}$ and $\mathcal{A}$

are negligible over $\eta \ll r \ll L$, so that $\mathcal{T}$ is approximately scale-local and the flux is close to constant. In a near wake, however, mean-shear production and inhomogeneous transport need not be small over the same separations where a $-5/3$ spectral band is observed.

Following Tang *et al.* (2016), two large-scale contributions that can be robustly assessed are the integrated production and cross-stream transport/diffusion terms

$$\begin{aligned} \mathcal{P}(r) &= \frac{2}{r^2} \int_0^r s^2 \overline{\Delta u \Delta v} \frac{\partial U}{\partial y} ds, \\ \mathcal{D}(r) &= \frac{2}{r^2} \int_0^r s^2 \frac{\partial}{\partial y} \overline{v(\Delta q)^2} ds, \end{aligned} \quad (3)$$

which represent, respectively, shear-driven injection of increment energy and inhomogeneous redistribution across the wake. Here, s is a dummy variable. The integral form avoids differentiating noisy data with respect to r and provides the cumulative contribution up to separation r .

With single-point hot-wire records, increments are obtained from time lags τ and mapped to separations $r \simeq U_c \tau$ using Taylor's hypothesis; lateral traverses provide the cross-stream derivative in $\mathcal{D}(r)$. Estimating $\mathcal{A}(r) \sim U \partial(\overline{\Delta q})^2 / \partial x$ would require closely spaced streamwise stations, which motivates our focus on $\mathcal{P}$ and $\mathcal{D}$ as direct indicators of large-scale forcing at $x/d = 20$. In cylinder wakes, this forcing is effectively twofold: mean shear provides production through $\partial U / \partial y$, while vortex shedding (von Kármán street) imposes a coherent large-scale organisation that can modulate $\overline{\Delta u \Delta v}$ and enhance the effective forcing in the near field. We emphasise that we use the SbS balance as a dynamical interpretation consistent with the measured one-dimensional spectra, but we do not attempt a coherent spectral-space budget.

Experiments

Experiments were conducted in an open-circuit, low-turbulence wind tunnel with a working section of 0.35 m in height, 0.35 m in width, and 2.4 m in length. A circular cylinder of diameter $d = 12.7 \text{ mm}$ was mounted horizontally to generate a turbulent wake (figure 1). The free-stream velocities U_∞ were 3, 6, 12, and 18 m/s, corresponding to $Re_d = 2500, 5000, 10000$, and 15000, respectively. Streamwise measurements were performed at $x/d = 10, 20$, and 40 using a traversing four-X-wire probe capable of simultaneously resolving all

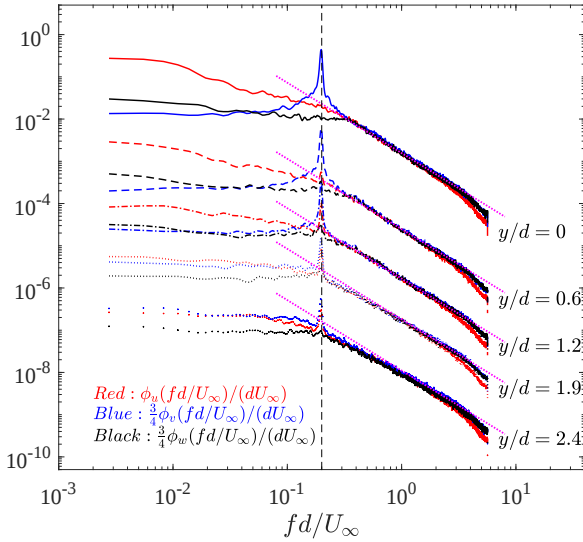


Figure 2. Distributions of one-dimensional energy spectra at $Re_d = 15000$ and $x/d = 20$ for $y/d = 0 - 2.4$, normalized by d and U_∞ . Red: $\phi_u(f)$; Blue: $\phi_v(f)$; Black: $\phi_w(f)$. Magenta dotted lines represent the $-5/3$ slope. Vertical dashed line indicates $f_c d/U_\infty \approx 0.2$, where f_c is the Kármán vortex frequency. The $\phi_v(f)$ and $\phi_w(f)$ distributions are multiplied by a factor of $3/4$, and, for clarity, the distributions for $y/d \geq 0.6$ are shifted downward by $\log 15$, $\log 200$, $\log 1500$, and $\log 20000$, respectively.

three velocity components. Lateral measurements were taken on one side of the centreline over the range $y/d = -0.2$ to 5.1 , with the extent depending on Re_d . The lateral spacing was $0.2d$, except near the free stream at $x/d = 40$, where it was increased to $0.4d$. The probe consisted of four X-wires. Two probes (denoted b and d) were positioned in the (x, y) plane with a spanwise separation of $\Delta z = 1.94$ mm, while the other two (denoted a and c) were arranged in the (x, z) plane with a lateral spacing of $\Delta y = 1.9$ mm. Each X-wire employed a pair of $2.5 \mu\text{m}$ etched Wollaston (Pt-10%Rh) wires and was operated in constant-temperature mode at an overheat ratio of 1.5 . The analog signals were conditioned using bucking and amplification circuits, low-pass filtered at a cutoff frequency f_c , and digitised at a sampling frequency $f_s = 2f_c$.

Results

We examine the one-dimensional energy spectra across lateral locations $0 \leq y/d \leq 2.4$. Figure 2 shows the spectral distributions of $\phi_u(f)$, $\phi_v(f)$, and $\phi_w(f)$ at $x/d = 20$ and $Re_d = 15000$, normalised by d and U_∞ . To facilitate comparison with $\phi_u(f)$, the spectra of $\phi_v(f)$ and $\phi_w(f)$ are multiplied by a factor of $3/4$ (Saddoughi & Veeravalli, 1994). For visual separation, the spectra corresponding to $y/d \geq 0.6$ are shifted vertically by $\log 15$, $\log 200$, $\log 1500$, and $\log 20000$, respectively. In figure 2, the spectra of all three components exhibit an approximate $-5/3$ scaling over a substantial portion of the lateral domain. At the centreline ($y/d = 0$), $\phi_u(f)$ exhibits an approximate $-5/3$ scaling over $0.25 \lesssim fd/U_\infty \lesssim 1.2$, whereas the corresponding ranges for $\phi_v(f)$ and $\phi_w(f)$ extend further, approximately over $0.3 \lesssim fd/U_\infty \lesssim 3$. At $y/d = 0.6$ and 1.2 , the scaling range associated with $\phi_u(f)$ broadens slightly, reaching $0.22 \lesssim fd/U_\infty \lesssim 1.5$, while the range for $\phi_v(f)$ and $\phi_w(f)$ remains nearly identical to those observed at $y/d = 0$. In contrast, at more distant lateral positions ($y/d = 1.9$ and

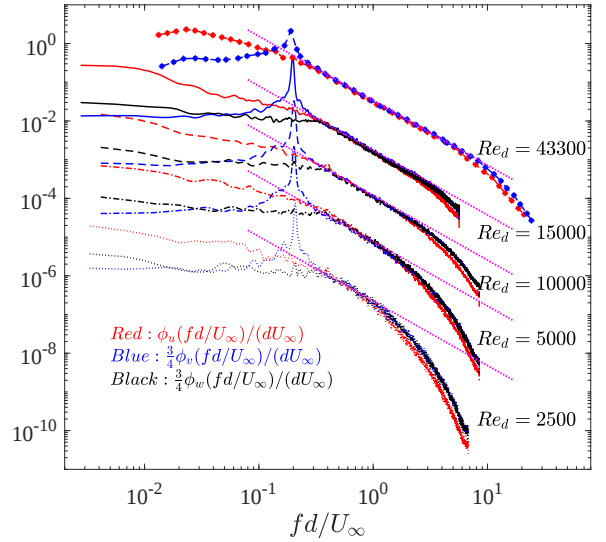


Figure 3. Distributions of one-dimensional energy spectra for $Re_d = 2500 - 15000$ at $x/d = 20$ and $y/d = 0$, normalized by d and U_∞ . Red: $\phi_u(f)$; Blue: $\phi_v(f)$; Black: $\phi_w(f)$. Magenta dotted lines represent the $-5/3$ slope. For clarity, the distributions for $Re_d \leq 10000$ are shifted downward by $\log 15$, $\log 200$, $\log 8000$, respectively. For reference, also shown in this figure are the data for $Re_d = 43300$ at $x/d = 25$ and $y/d = 0$ in a circular cylinder (Kang & Meneveau, 2001, 2002). The $\phi_v(f)$ and $\phi_w(f)$ distributions are also multiplied by $3/4$.

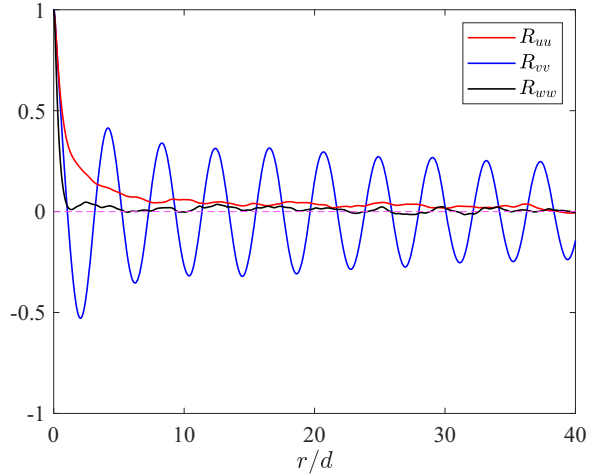


Figure 4. Distributions of two-point auto-correlation coefficients R_{uu} , R_{vv} , and R_{ww} at $x/d = 20$ and $y/d = 0$ for $Re_d = 2500$. The horizontal dashed line denotes the value of 0. The spatial separation r is estimated using Taylor's hypothesis.

2.4), all spectra deviate from the $-5/3$ scaling, with exponents ranging from approximately $-4.5/3$ to $-4.3/3$ (not shown). Overall, these observations suggest that the appearance and extent of the approximately $-5/3$ scaling are influenced by both lateral positions and velocity components. In particular, $\phi_v(f)$ and $\phi_w(f)$ show similar behaviour to each other and extend over a broader range than $\phi_u(f)$.

We next examine the Reynolds-number dependence of the one-dimensional energy spectra along the wake centreline. Figure 3 shows $\phi_u(f)$, $\phi_v(f)$, and $\phi_w(f)$ at $x/d = 20$ for Re_d ranging from 2500 to 15000 , normalized by d and U_∞ .

For comparison, data from a circular cylinder wake for $Re_d = 43300$, measured at $x/d = 25$ and $y/d = 0$ (Kang & Meneveau, 2001, 2002), are also included. The $\phi_v(f)$ and $\phi_w(f)$ spectra are multiplied by a factor of $3/4$ to facilitate comparison with $\phi_u(f)$. To improve visual clarity, the spectra corresponding to $Re_d \leq 10000$ are vertically shifted by $\log 15$, $\log 200$, and $\log 8000$. In figure 3, the extent of the $-5/3$ scaling in all three spectra increases with Re_d . Specifically, at $Re_d = 2500$, only a very narrow range of the approximately $-5/3$ scaling is observed. In contrast, for the higher- Re_d dataset ($Re_d = 43300$), the scaling range becomes much more pronounced, extending up to approximately $0.26 \lesssim fd/U_\infty \lesssim 8.2$ for $\phi_u(f)$ and $0.26 \lesssim fd/U_\infty \lesssim 5$ for $\phi_v(f)$. These observations suggest that the extent of the $-5/3$ scaling in $\phi_u(f)$, $\phi_v(f)$, and $\phi_w(f)$ strongly depends on Re_d in the cylinder wake.

To further examine the dual-scaling behaviour related to the $-5/3$ scaling in the wake, figure 4 shows the two-point autocorrelation coefficients R_{uu} , R_{vv} , and R_{ww} at $x/d = 20$ and $y/d = 0$ for $Re_d = 2500$. Here, R_{vv} and R_{ww} are defined in a manner similar to R_{uu} . We observed that a clear periodic pattern appears only in R_{vv} , reflecting the effect of large-scale vortex shedding. In contrast, such oscillations are absent in R_{uu} and R_{ww} , indicating that coherent motions make only a minor contribution to $\overline{u^2}$ and $\overline{w^2}$ (see figure 2 in Thiesse *et al.* (2014)). This implies that the relevant scales in the energy-containing range should be the lateral root-mean-square velocity v' and the lateral integral length scale L_v . These two quantities are therefore adopted to normalize the large-scale part of the energy spectra in the following analysis of dual scaling.

Figure 5 shows the one-dimensional spectra $\phi_v(\kappa_1)$ at $x/d = 20$ and $y/d = 0$ and 0.6 for $Re_d = 2500 - 15000$, normalised using both outer (large-scale) variables (v' , L_v) and inner (Kolmogorov) scales (u_K , η). These flow parameters are listed in table 1. For reference, we include the data from Lee & Moser (2015) in homogeneous and isotropic turbulence at $Re_\lambda = 433$. In figure 5(a), when scaled with outer variables, the spectra collapse in the low-wavenumber region (i.e. small $\kappa_1 L_v$, corresponding to large scales). As Re_d increases, this collapse progressively extends towards higher $\kappa_1 L_v$, indicating that the range of outer scaling broadens towards smaller scales. Conversely, in figure 5(b), when scaled with Kolmogorov scales, the spectra collapse in the high-wavenumber region (i.e. large $\kappa_1 \eta$, corresponding to small scales). With increasing Re_d , the collapse expands towards lower $\kappa_1 \eta$, implying an increasing influence of inner scaling on larger scales. These results indicate the emergence of dual scaling in the circular cylinder wake. Consistent with the dual-scaling picture of Tennekes & Lumley (1972), we observe an outer collapse of the low-wavenumber range when normalised by (v' , L_v) and an inner collapse of the dissipative range when normalised by (u_K , η) (figure 5). With increasing Re_d , these similarity ranges extend and can overlap, within which an apparent $-5/3$ behaviour is observed. In the present TSFP contribution we use this viewpoint primarily as context; the emphasis is instead on the SbS budget evidence that large-scale forcing remains dynamically significant over the separations associated with the observed spectral band.

To quantify the contribution of large-scale forcing within the $-5/3$ scaling range, we examine the large-scale production and diffusion terms in Eq. 3. Figure 6 shows the production term $\mathcal{P}(r)$ and the diffusion term $\mathcal{D}(r)$ at $x/d = 20$ and $y/d = 0.6$. Both terms are normalized by $\bar{\epsilon}r$. Note that, $\bar{\epsilon}$ is estimated using the spectral chart method, which relies on the observation that all Kolmogorov-normalized spectra collapse in the dissipative range (Djenidi & Antonia, 2012). We

can observe that both the production $\mathcal{P}(r)$ and the diffusion $\mathcal{D}(r)$ make substantial contributions to the transport equations of $(\Delta q)^2$ across scales, with the former being positive and the latter negative. These results indicate that, over the scale range where the $-5/3$ behaviour is observed, large-scale forcing remains dynamically important to scale-by-scale energy transport at this location. Therefore, in the present wake, the $-5/3$ range should not be regarded as evidence of a classical inertial cascade in which production and transport are negligible. Instead, the scale-by-scale budget in figure 6 indicates that production remains important over the same range of scales where the spectra approach $-5/3$. The near-field behaviour is therefore better interpreted as an overlap scaling: large-scale forcing maintains the outer-scale similarity and its associated energy level over a wide range of scales, while the smallest motions already satisfy inner Kolmogorov-type scaling. The co-existence of these two similarity constraints broadens the overlap region and makes the asymptotic $-5/3$ behaviour more evident with increasing Re_d .

Conclusions

We examined the $-5/3$ power-law scaling in the wake of a circular cylinder. At $Re_d = 15000$ and $x/d = 20$, $\phi_v(f)$ and $\phi_w(f)$ exhibit an approximate $-5/3$ scaling over a substantial frequency range for $y/d \approx 0 - 1.2$, whereas $\phi_u(f)$ shows a noticeably narrower $-5/3$ scaling range in the same lateral region. As y/d increases to 1.9 and 2.4, all spectra progressively deviate from the $-5/3$ scaling and become slightly shallower. These results indicate that the occurrence and extent of the $-5/3$ scaling depend strongly on the velocity component and on the lateral positions. Among the three components, $\phi_v(f)$ and $\phi_w(f)$ exhibit a nearly identical scaling behaviour and more pronounced $-5/3$ range than $\phi_u(f)$. The Re_d dependence of energy spectra is examined. At $x/d = 20$, the $-5/3$ scaling range for all spectra progressively broadens as Re_d increases from 2500 to 15000, demonstrating a clear finite Reynolds number effect.

A dual scaling analysis, based on the large-scale parameters (v' , L_v) and Kolmogorov scales (u_K , η), was systematically conducted for $\phi_u(\kappa_1)$, $\phi_v(\kappa_1)$, and $\phi_w(\kappa_1)$ in the wake of a circular cylinder. At $x/d = 20$ and for $y/d \approx 0 - 1.2$, all three spectra exhibit a clear dual-scaling behavior. With increasing Re_d , the scaling based on (v' , L_v) extends to increasingly larger values of $\kappa_1 L_v$, whereas the scaling based on (u_K , η) extends increasingly smaller values of $\kappa_1 \eta$. The two scalings overlap at high Re_d , which leads to a $-5/3$ power-law behavior within the overlap region. In the separation ranges that correspond (via Taylor's hypothesis) to the frequencies over which the spectra exhibit an extended $-5/3$ behaviour, the large-scale production and diffusion terms contribute significantly to the physical-space transport equations for $(\Delta q)^2$. These results support the view that the prominent near-field $-5/3$ range is best interpreted as an overlap-scaling behaviour sustained by persistent large-scale forcing, rather than as evidence of a classical inertial cascade in which production is negligible.

The observed differences in the extent of the apparent $-5/3$ range between components and lateral positions can be rationalised within the SbS viewpoint. In the present mean-shear plane, the dominant mean-gradient is $\partial U / \partial y$, so the principal production pathway is $P = -\overline{uv} \partial U / \partial y$; correspondingly, the SbS production contribution involves $\overline{\Delta u \Delta v} \partial U / \partial y$ and therefore couples most directly to the cross-stream component. This provides a natural interpretation for why the $-5/3$

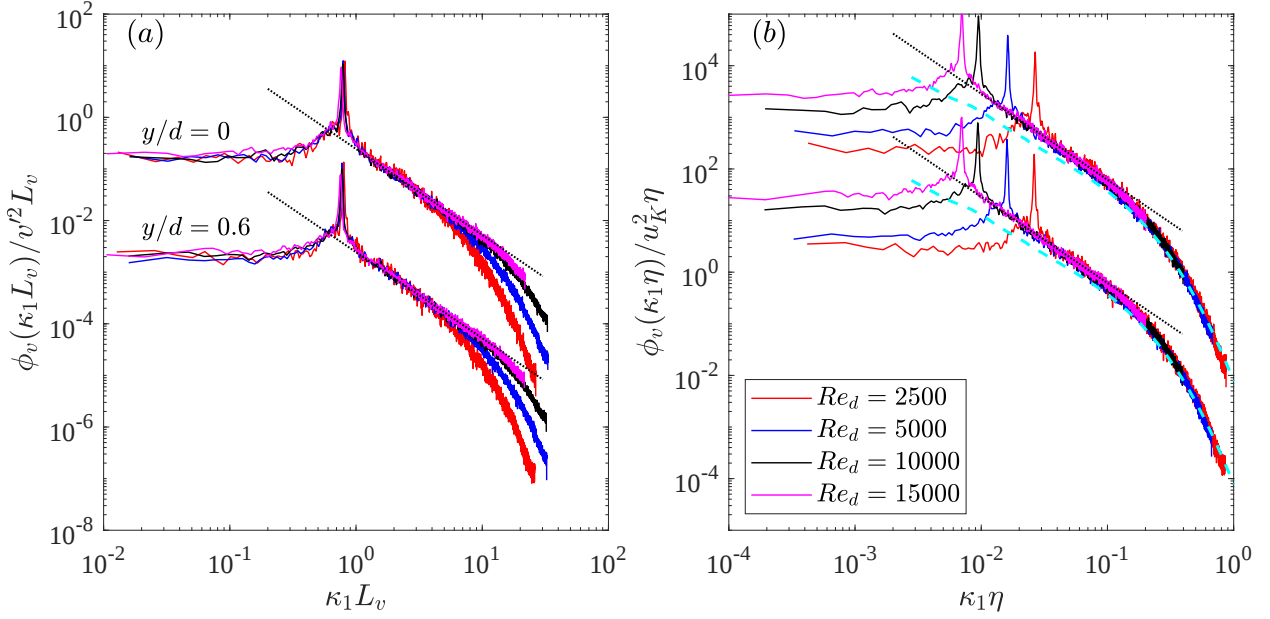


Figure 5. (a) Distributions of $\phi_v(\kappa_1)$ at $x/d = 20$ and $y/d = 0$ and 0.6 , normalized by (v', L_v) . (b) Corresponding distributions normalized by (u_K, η) . Black dotted lines represent the $-5/3$ slope. The cyan curves denote data from Li *et al.* (2008) in homogeneous and isotropic turbulence at $Re_\lambda = 433$. For clarity, the $\phi_v(\kappa_1)$ distributions at $y/d = 0.6$ in these figures are shifted downward by $\log 100$.

Table 1. Main flow parameters at $x/d = 20$ for $Re_d = 2500 - 15000$. Values in parentheses correspond to $y/d = 0$ and 0.6 , respectively. The mean dissipation rate $\bar{\epsilon}$ is estimated using the spectral method of Djenidi & Antonia (2012).

Re_d	v' (ms^{-1})	L_v (mm)	$\bar{\epsilon}$ ($m^2 s^{-3}$)	u_K (ms^{-1})	η (mm)
2500	(0.88, 0.47)	(6.8, 6.7)	(1.35, 1.45)	(0.07, 0.07)	(0.22, 0.22)
5000	(1.60, 0.84)	(6.8, 6.7)	(9.0, 9.5)	(0.11, 0.11)	(0.14, 0.14)
10000	(3.38, 1.82)	(6.7, 6.6)	(76, 80)	(0.18, 0.19)	(0.08, 0.08)
15000	(4.78, 2.66)	(6.7, 6.6)	(230, 235)	(0.24, 0.24)	(0.06, 0.06)

band is typically more extended in ϕ_v than in ϕ_u , while the spanwise component, which is not directly forced by P , is populated through redistribution and nonlinear transfer and often displays intermediate behaviour. Spatially, the wake centreline differs from the shear layers: by symmetry, mean-shear production is weak near $y = 0$, whereas off-centreline locations sample the shear layers where $|\partial U / \partial y|$ and $|\overline{uv}|$ (and thus the SbS production contribution) are largest. Farther away from the wake core, both turbulence intensity and mean-shear forcing decrease and cross-stream transport/diffusion primarily redistributes increment energy, which is consistent with the progressive contraction and eventual departure of the spectra from an approximate $-5/3$ scaling at larger y/d .

We emphasise that the component-dependent extent of the apparent $-5/3$ band in one-dimensional spectra does not contradict the tendency toward local isotropy at sufficiently small scales. In an inhomogeneous wake at finite Reynolds number, outer-range forcing (including shear and coherent shedding) can contaminate different components over different frequency bands, while the approach to inner similarity occurs progressively with increasing Re_d . Therefore, the bandwidth over which each one-dimensional spectrum appears close to $-5/3$ may differ between u , v and w even if the highest-frequency range shows approximate isotropy indicators. This is consistent with interpreting the observed $-5/3$ behaviour as an overlap-scaling phenomenon in a forced, non-equilibrium flow

rather than as a classical inertial subrange.

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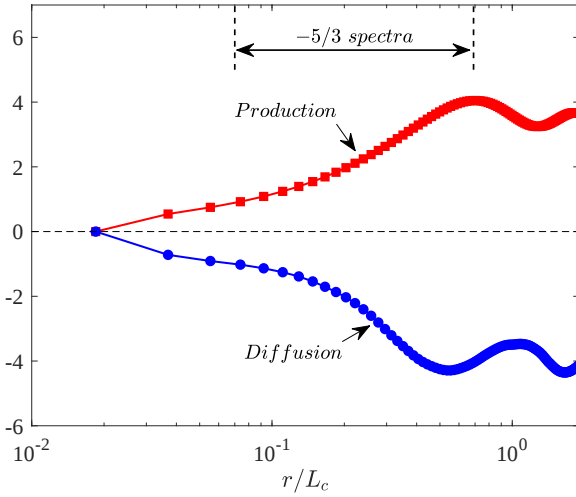


Figure 6. Distributions of the production term $\mathcal{P}(r)$ and the diffusion term $\mathcal{D}(r)$ in the transport equations for $(\Delta q)^2$ (Tang *et al.*, 2016) at $x/d = 20$ and $y/d = 0.6$. Both terms are normalized by $\bar{\epsilon}r$. The horizontal dashed line represents a value of 0. For reference, the range of the $-5/3$ spectra is $0.07 \lesssim r/L_c \lesssim 0.69$, where L_c is the streamwise spacing between two consecutive vortices of the same sign, with $L_c \approx 4.13d$ at $x/d = 20$ (Thiesset *et al.*, 2014).

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