14th International Symposium on Turbulence and Shear Flow Phenomena (TSFP14)
Heidelberg, Germany, July 28–31, 2026

RECURSIVE NEURAL-NETWORK-BASED SUBGRID-SCALE MODEL FOR LARGE EDDY SIMULATION: TURBULENT CHANNEL FLOW

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ABSTRACT

Most neural-network(NN)-based subgrid scale (SGS) models for large eddy simulation (LES) have been trained on low Reynolds number DNS (direct numerical simulation) data and applied to similar Reynolds-number flows. Cho *et al.* (2024) proposed a recursive NN-based SGS model (RNM) to overcome this limitation: (1) an NN is trained on low-Reynolds-number filtered DNS (fDNS) data and then applied to LES at a higher Reynolds number; (2) the NN is trained on augmented data combining the fDNS and filtered LES (fLES) data; and (3) the NN is recursively extended to LES at progressively higher Reynolds numbers by augmenting the training data and retraining the NN. This recursive procedure was successfully applied to forced homogeneous isotropic turbulence (HIT). In the present study, we train an NN on fDNS data of turbulent channel flow at a low Reynolds number ($Re_\tau \approx 395$), and apply the recursive procedure to the flow at high Reynolds numbers ($Re_\tau = 2000$). The recursive NN-based SGS model performs better than the traditional one in predicting the mean velocity and root-mean-square velocity fluctuations.

INTRODUCTION

Large eddy simulation (LES) models the effects of eddies smaller than the grid size through a subgrid-scale (SGS) stress. This approach reduces the computational cost compared to direct numerical simulation (DNS) that requires much finer grids to resolve all scales of eddies. Therefore, it is important to employ a precise SGS model to achieve accurate LES results.

One of the methods to develop an accurate SGS model is to train an artificial neural network (NN) on filtered DNS (fDNS) data. The NN is trained with fDNS data directly, rather than being assumed with analytical or statistical hypothesis. However, previous NN-based SGS models (called NN model hereafter) suffered from limited generalization: a model trained on one flow typically failed when applied to other flows with different topologies at significantly higher Reynolds numbers and with different grid sizes (Choi *et al.*, 2025).

To address one of these challenges (i.e., higher Reynolds numbers), Cho *et al.* (2024) proposed a recursive procedure. They trained an NN on fDNS data of forced homogeneous isotropic turbulence (HIT) at a low Reynolds number, and it was applied to LES of forced HIT at a higher Reynolds number by keeping the LES grid size divided by the Kolmogorov length scale being equal to the filter size. Then, the LES data obtained were filtered based on test-filtering introduced by Germano *et al.* (1991), and the test-filtered LES data were added to the existing training data. The NN was retrained with this augmented training dataset and subsequently applied to LES at even higher Reynolds numbers. The procedure was recursively extended until the target Reynolds number was reached.

The objective of the present study is to apply this recursive procedure (Cho *et al.*, 2024) to a wall-bounded flow, i.e., turbulent channel flow. Its performance is compared with that of a traditional SGS model.

NEURAL NETWORK WITH FDNS DATA

Two NNs are trained: one for SGS normal stresses, and the other for SGS shear stresses (Cho *et al.*, 2024). According to Cho *et al.* (2024), the present NNs do not use bias and batch normalization parameters to satisfy

$$\mathrm{NN}(c\mathbf{A}) = c\mathrm{NN}(\mathbf{A}) \tag{1}$$

for any tensor $\mathbf{A}$ and non-negative scalar c, and use the leaky ReLU function (Maas *et al.*, 2013) as activation function. We use two hidden layers for each NN, and 128 nodes per hidden layer (Fig. 1).

The input of the NNs is $|\gamma|\gamma_{ij}$, where $\gamma_{ij} = (\partial\bar{u}_i/\partial x_k)\bar{\Delta}_j\delta_{jk}$, $|\gamma| = \sqrt{\gamma_{ij}\gamma_{ij}}$, $\bar{u}_i = (\bar{u},\bar{v},\bar{w})$ are the filtered velocity components, $x_i = (x,y,z)$ are the streamwise, wall-normal, and spanwise directions, respectively, and δ_{ij} is the Kronecker delta. The bar $\bar{(\cdot)}$ indicates the filtered variable. The output of the NNs is the anisotropic component of the SGS stress, $\tau^r_{ij} = \tau_{ij} - \frac{1}{3}\tau_{kk}\delta_{ij}$, where $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i\bar{u}_j$.

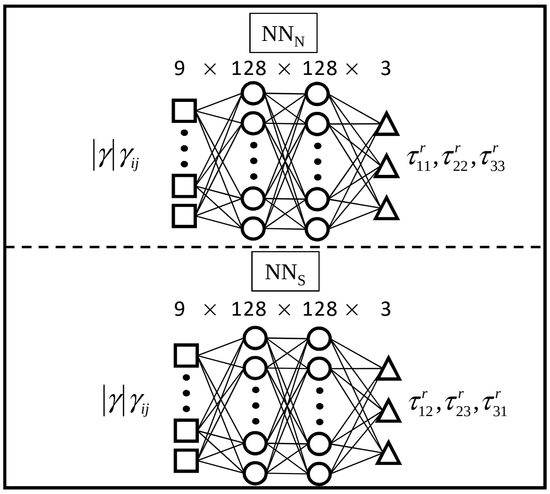

Figure 1. Schematic diagram of the present NN model that consists of a dual NN: $\mathrm{NN_N}$ and $\mathrm{NN_S}$ predict the SGS normal and shear stresses, respectively.

To obtain DNS data for training, DNS of turbulent channel flow is performed at $Re_b = u_b(2h)/\nu = 14000$ (corresponding to $Re_\tau = u_\tau h/\nu \approx 395$), where u_b is the bulk velocity, u_τ is the wall-shear velocity, h is the channel half-height, and ν is the kinematic viscosity. The computational domain size for DNS is $L_x \times L_y \times L_z = 2\pi h \times 2h \times \pi h$. Uniform grids of $\Delta_x^+ = 13.0$ and $\Delta_z^+ = 6.5$ are used in x and z directions, respectively, and stretched grids with $\Delta_{y,\min}^+ = 0.13$ are applied in y direction. A dealiased spectral method in homogeneous directions and second-order central difference method in y direction are employed for spatial discretization. For time advancement, a third-order Runge-Kutta method and second-order Crank–Nicolson method are used for convective and diffusive terms, respectively.

The DNS data is filtered using a cut-Gaussian filter (Zanna & Bolton, 2020; Guan *et al.*, 2022, 2023; Pawar *et al.*, 2023; Cho *et al.*, 2024) in homogeneous directions and a box filter in y direction to obtain training data. The filter size is $\bar{\Delta}_x^+ = 78.0, \bar{\Delta}_z^+ = 39.0$, and $\bar{\Delta}_{y,\min}^+ = 0.90$. To obtain more training data and facilitate the training by setting the mean of input and output to be zero, a rotational transformation is applied to the filtered data. For each NN, the number of training data is 2,764,800, and the rms (root-mean-square) value of the output is computed along the wall-normal direction. The maximum rms value is used as the scaling factor for non-dimensionalization.

The NN model trained on the fDNS data is applied to LES at the same Re_b (the same numerical scheme and computational domain size are used) but with the grid sizes of $\bar{\Delta}_x^+ = 73.5, \bar{\Delta}_z^+ = 36.8$, and $\bar{\Delta}_{y,\min}^+ = 0.91$, which are similar to the filter sizes of fDNS. For comparison, LESs using the dynamic Smagorinsky model (DSM; Germano et al., 1991; Lilly, 1992) with the same grids or finer grids ($\bar{\Delta}_x^+ = 25.8, \bar{\Delta}_z^+ = 12.9$, and $\bar{\Delta}_{y,\min}^+ = 0.18$) are also performed, where spectral cut-off filtering is used as test filtering as it yields better performance for DSM than Gaussian filtering or cut-gaussian filtering (Gullbrand, 2001).

Figure 2 shows the mean streamwise velocity, rms velocity fluctuations, and Reynolds shear stress predicted by the present NN model, together with fDNS data and those from DSM with two grid cases. As shown, the present NN model outperforms DSM in predicting the mean velocity and rms of velocity fluctuations.

NEURAL NETWORK WITH FILTERED LES DATA

To begin the recursive procedure, we apply the trained NN to LES at a higher Reynolds number, Re_b = 36,444 (corresponding to $Re_\tau \approx 950$). The LES grid sizes are $\bar{\Delta}_x^+ = 73.5$, $\bar{\Delta}_z^+ = 36.8$, and $\bar{\Delta}_{y,min}^+ = 0.89$, which are similar to those used for LES at $Re_b = 14,000$. Then, the cut-Gaussian filter is applied to the LES results in homogeneous directions to obtain filtered LES (fLES) data. The test-filter sizes are $\tilde{\bar{\Delta}}_x^+ = 147.0$, $\tilde{\bar{\Delta}}_z^+ = 73.5$, and $\tilde{\bar{\Delta}}_{y,min}^+ = 0.89$, and the number of fLES data is 1,382,400. The NN is trained again on the expanded dataset combining the fDNS and fLES data.

Now, the NN model is applied to a higher Reynolds number of $Re_b = 87,000$ ($Re_\tau \approx 2,000$). Uniform grids with $\bar{\Delta}_x^+ = 130.9$ and $\bar{\Delta}_z^+ = 65.4$ are used, and stretched grids with $\bar{\Delta}_{y,\min}^+ = 0.91$ are applied in wall-normal direction. The LES grid sizes are similar to the test-filter size of the previous fLES. Figure 3 shows the results of LES with the present NN model, DSM, and DNS (Lee & Moser, 2015), together with those of DSM by Montecchia *et al.* (2017) ($Re_\tau \approx 2000$, $\bar{\Delta}_x^+ = 157$, $\bar{\Delta}_z^+ = 63$ and $\bar{\Delta}_{y,\min}^+ = 0.29$). The present model predicts the mean velocity accurately but DSM overestimates it. Figure 2 and 3 show that, as indicated by Montecchia *et al.* (2017), DSM can accurately predict mean velocity profiles when the LES grids are sufficiently fine in the homogeneous directions.

CONCLUSION

Cho *et al.* (2024) developed a recursive procedure for 3D homogeneous isotropic turbulence, which enables NNs to be effectively applied even to flows at untrained much higher Reynolds numbers. We extended this recursive procedure to turbulent channel flow to confirm generalization of the recursive procedure. First, filtered DNS data at $Re_b = 14,000 (Re_\tau \approx 395)$ were used to train an NN. The NN was proved to predict the mean velocity and rms velocity fluctuations better than DSM. This NN was applied to LES at $Re_b = 36,444 (Re_\tau \approx 950)$. The resulting LES data were filtered based on the test-filtering concept. The filtered LES data as well as the fDNS data were used to train an NN. The NN was applied to LES at $Re_b = 87,000 (Re_\tau \approx 2000)$. The LES grids adopted were significantly coarse so that DSM failed to predict the mean velocity accurately. In contrast, although it was initially trained on low Reynolds number fDNS data, the NN-based SGS model constructed by the recursive procedure demonstrated good performance in LES of turbulent channel flow at high Reynolds numbers.

ACKNOWLEDGEMENT

This work is supported by the National Research Council of Science & Technology of Korea (no. GTL24031-000) and by the National Research Foundation of Korea (no. RS-2025-00522345).

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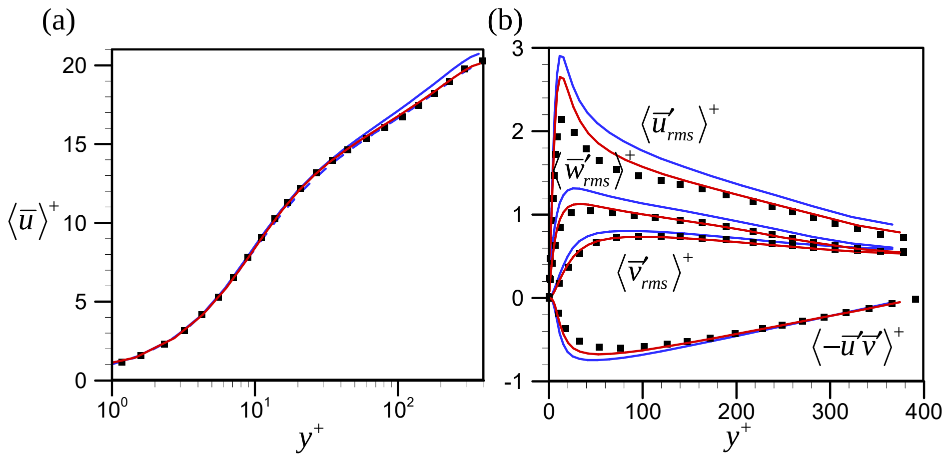

Figure 2. Turbulence statistics of turbulent channel flow at $Re_b = 14,000$: (a) mean velocity; (b) rms velocity fluctuations and resolved Reynolds shear stress. ■, fDNS; ——, present NN model; ——, DSM; - - - -, DSM with fine grids.

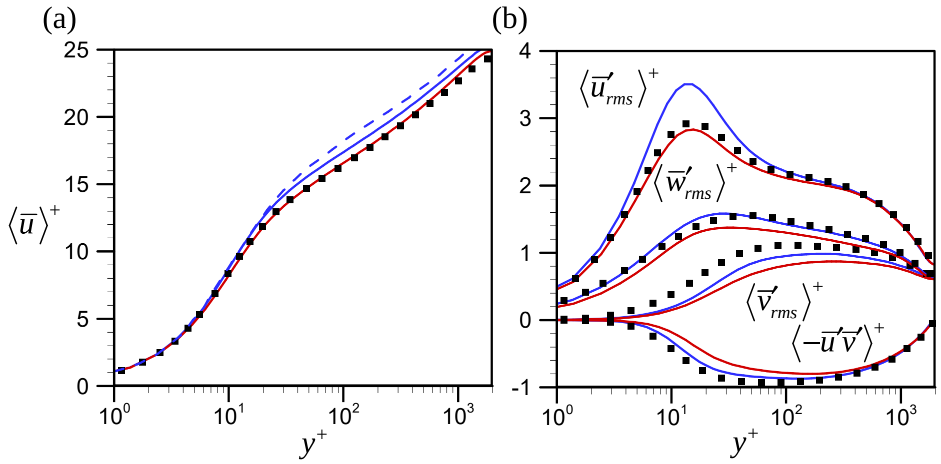

Figure 3. Turbulence statistics of turbulent channel flow at $Re_b = 87,000$:(a) mean velocity; (b) rms velocity fluctuations and resolved Reynolds shear stress. ■, DNS; ——, present NN model; ——, DSM; - - - -, DSM (Montecchia *et al.*, 2017)

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