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LOCAL FLOW INFLUENCE ON INERTIAL PARTICLE DIVERGENCE IN ISOTROPIC TURBULENCE

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ABSTRACT

Inertial heavy particles exhibit a nonuniform spatial distribution in turbulence, which is known as inertial particle clustering. The divergence and convergence behaviors of the particles in homogeneous isotropic turbulence are investigated to gain insight into the clustering behavior of inertial particles beyond the preferential concentration mechanism. A tessellation-based technique is applied to direct numerical simulation data to quantify the velocity divergence for several Stokes numbers. Conditional statistics of the particle velocity divergence show that, for small Stokes numbers, the divergence reflects the local flow structure, obeying the preferential concentration mechanism. Effects of nonlocal clustering mechanism become more significant as the Stokes number increases. The results also show a mixed influence of the preferential concentration and nonlocal clustering mechanisms for $St \approx 1$.

INTRODUCTION

In environmental and industrial flows, one frequently observes inertial heavy particles suspended in turbulent flow: e.g., cloud droplets, aerosols and volcanic ash in atmospheric flows and fuel droplets in spray combustion. Inertial particles show nonuniform spatial distribution, referred to as clustering, in turbulent flows due to deviation of the inertial particle trajectory from the fluid particle trajectory. For recent reviews on the inertial particle dynamics in turbulence, we refer to Refs. Brandt & Coletti (2022); Bec *et al.* (2024). When the relaxation time τ_p of the particle motion is smaller than the flow time scale, the preferential concentration mechanism (Maxey, 1987; Squires & Eaton, 1991) plays a dominant role for the clustering formation: the particles are swept out from high-vorticity regions and concentrated in high-strain regions. In contrast, for larger particle relaxation time, the clustering formation can be explained by the nonlocal mechanism (Gustavsson & Mehlig, 2011; Bragg & Collins, 2014; Bragg *et al.*, 2015), i.e., the particle clustering behavior is not dependent only on the local flow structure. To understand the clustering formation process, the divergence of the particle velocity is an important quantity. However, computation of the divergence is not straightforward because of the unstructured nature of Lagrangian particles. Recently, Oujia *et al.* (2020) proposed a tessellation-based technique to compute the divergence directly based on position and velocity data of discrete particles, and Maurel–Oujia *et al.* (2024) improved the stability and accuracy of the technique. In this work, this novel tessellation-based technique is applied to Lagrangian inertial particle data obtained from direct numerical simulation (DNS) to get insights into the roles of the different clustering mechanisms.

COMPUTATIONAL METHODS

Three-dimensional DNS of particle-laden homogeneous isotropic turbulence is performed to obtain the particle position and velocity data. The fluid flow is governed by the following incompressible Navier–Stokes equations and solved in the Eulerian framework.

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla P + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}, \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0, \tag{2}$$

where $\boldsymbol{u}$ is the fluid velocity, P is the pressure per density, and $\boldsymbol{f}$ is the external forcing to maintain statistically stationary isotropic turbulence. The kinematic viscosity is denoted by v. The particles are assumed to be small, heavy and spherical Stokes particles and tracked in the Lagrangian framework. The equation of particle motion are given as follows:

$$\frac{d\boldsymbol{x}_p}{dt} = \boldsymbol{v}_p, \tag{3}$$
$$\frac{d\boldsymbol{v}_p}{dt} = -\frac{\boldsymbol{v}_p - \boldsymbol{u}(\boldsymbol{x}_p)}{\tau_p}, \tag{4}$$

where $\boldsymbol{x}_p$ and $\boldsymbol{v}_p$ are the position and velocity of a Lagrangian particle, $\boldsymbol{u}(\boldsymbol{x})$ is the fluid velocity at position $\boldsymbol{x}$, and τ_p is the relaxation time of the particle motion. The above governing equations are solved in a cubic computational domain with periodic boundaries. The details of the computational method are described in, e.g., Onishi *et al.* (2011) and Matsuda *et al.* (2021). The simulations are performed with 512^3 grid points and 1.07×10^9 particles. The Taylor-microscale Reynolds number of the obtained turbulent flow is $Re_\lambda = 204$. The particle inertia is represented by the Stokes number defined as $St = \tau_p/\tau_\eta$, where τ_η is the Kolmogorov time.

The divergence of particle velocity $\nabla \cdot \boldsymbol{v}$ is a crucial quantity to understand the clustering formation, where $\boldsymbol{v}$ denotes the particle velocity field in the continuous setting. To calculate the particle velocity divergence $\nabla \cdot \boldsymbol{v}$ based on the discrete particle data, we apply the novel tessellation-based technique (Oujia *et al.*, 2020; Maurel–Oujia *et al.*, 2024). When we apply the Voronoi tessellation to the particles distributed in space, each cell segmented by the Voronoi diagram has volume V_p that approximately corresponds to the inverse of the local number density. The time change of this volume thus permits to determine the divergence of the particle velocity.Therefore, the divergence $\nabla \cdot \boldsymbol{v}$ can be approximately determined by

$$\nabla \cdot \boldsymbol{v} \approx \mathscr{D}(\boldsymbol{x}_p) = \frac{1}{V_p}\frac{DV_p}{Dt}. \tag{5}$$

Positive divergence values correspond to void formation, while negative values indicate cluster formation.

RESULTS AND DISCUSSION

Figure 1 shows spatial distributions of the particle velocity divergence $\mathscr{D}$ for $St = 0.1$, 1.0 and 5.0. For $St = 0.1$, positive divergence values (red) are observed in the periphery of void regions, and negative divergence values (blue) are mostly found inside cluster regions. For larger St, some cluster regions exhibit both positive and negative divergence values. We now examine the relationship of such divergence distribution with the preferential concentration mechanism.

The preferential concentration mechanism is explained based on Maxey's approximation (Maxey, 1987), in which the particle velocity divergence is given by

$$\nabla \cdot \boldsymbol{v} = 2\tau_p Q + O(\tau_p^2), \tag{6}$$

where Q is the second invariant of the velocity gradient tensor of the fluid flow field. Positive and negative Q values represent rotational and strain dominated regions of the fluid flow, respectively. This approximation is valid for $St \ll 1$. Figure 2 shows the joint probability density functions (PDFs) $P(n_p, Q)$ (left column) of the invariant Q and the number density $n_p = 1/V_p$ and the conditionally averaged particle velocity divergence $\langle\mathscr{D}\rangle$ (right column) for given values of Q and

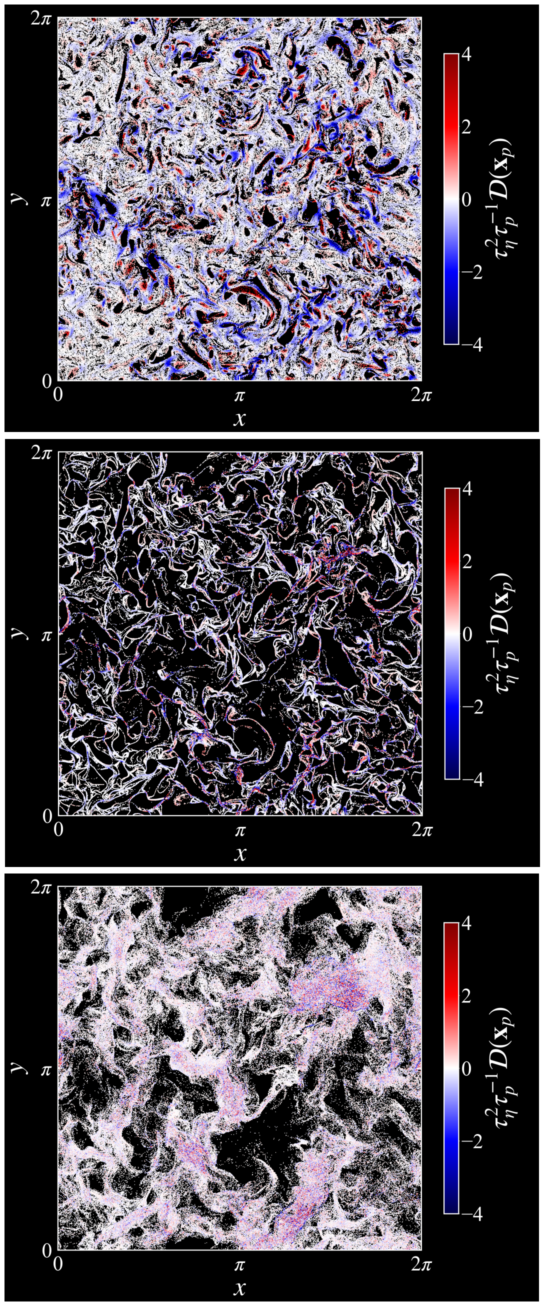
(a)
(b)
(c)


Figure 1. Spatial distribution of the inertial particles colored by the particle velocity divergence $\mathscr{D}$ for (a) $St = 0.1$, (b) $St = 1.0$ and (b) $St = 5.0$.

n_p, i.e., $\langle\mathscr{D}\rangle(n_p, Q) = \int_{-\infty}^{+\infty} \mathscr{D}P(\mathscr{D}, n_p, Q)d\mathscr{D}/P(n_p, Q)$, where $P(\mathscr{D}, n_p, Q)$ is the joint PDF of $\mathscr{D}$, n_p and Q. Here Q is normalized by using τ_η while n_p is normalized by the mean number density $\overline{n_p} = (2\pi)^3/N_p$, where N_p is the number of particles. The conditionally averaged divergence $\langle\mathscr{D}\rangle$ is normalized by using τ_η and τ_p. For $St = 0.1$, the conditionally averaged divergence $\langle\mathscr{D}\rangle$ shows positive values for $Q > 0$ and negative values for $Q < 0$. The positive $\langle\mathscr{D}\rangle$ values are skewed on the void side ($n_p/\overline{n_p} < 1$), while the negative $\langle\mathscr{D}\rangle$ values are on the cluster side ($n_p/\overline{n_p} > 1$) in agreement with the joint PDF. These results indicate that the preferential concentration mechanism plays the dominant role for the clustering for $St = 0.1$. For $St = 5.0$, the conditionally averaged divergence $\langle\mathscr{D}\rangle$ does not show clear dependence on Q, and large variance of the divergence is observed on the cluster side. These results indicate that the nonlocal clustering mechanism plays an important role instead of the preferential concentration mechanism. For $St = 1.0$, similar to $St = 5.0$, large variance of the divergence is observed on the cluster side, indicating the contribution of the nonlocal clustering mechanism. On the void side, however, we can also observe positive values of $\langle \mathscr{D} \rangle$ for $Q > 0$. This indicates that the preferential concentration mechanism partially contributes to the void formation, while the nonlocal clustering mechanism is significant in cluster regions.

CONCLUSIONS

The divergence of velocity of inertial heavy particles in homogeneous isotropic turbulence has been quantified by applying the novel tessellation-based technique to DNS data to get insights into the particle clustering formation process. The joint PDFs and the conditionally averaged divergence of particle velocity are quantified based on the tessellation-based analysis. The results show that the preferential concentration mechanism plays the dominant role for the clustering for $St = 0.1$, and this mechanism becomes less significant as the Stokes number increases. The results also show that for $St = 1.0$, the preferential concentration mechanism partially contributes to the void formation, while the nonlocal clustering mechanism plays an important role in cluster regions, which becomes even more pronounced for $St = 5.0$.

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REFERENCES

Bec, J, Gustavsson, K & Mehlig, B 2024 Statistical models for the dynamics of heavy particles in turbulence. *Annual Review of Fluid Mechanics* **56** (1), 189–213.

Bragg, A. D. & Collins, L. R. 2014 New insights from comparing statistical theories for inertial particles in turbulence: I. spatial distribution of particles. *New J. Physics* **16**, 055013.

Bragg, A. D., Ireland, P. J. & Collins, L. R. 2015 On the relationship between the non-local clustering mechanism and preferential concentration. *J. Fluid Mech.* **780**, 327–343.

Brandt, Luca & Coletti, Filippo 2022 Particle-laden turbulence: progress and perspectives. *Annual Review of Fluid Mechanics* **54** (1), 159–189.

Gustavsson, K. & Mehlig, B. 2011 Ergodic and non-ergodic clustering of inertial particles. *Europhysics Letters* **96** (6), 60012.

Matsuda, K., Schneider, K. & Yoshimatsu, K. 2021 Scale-dependent statistics of inertial particle distribution in high Reynolds number turbulence. *Phys. Rev. Fluids* **6**, 064304.

Maurel–Oujia, T., Matsuda, K. & Schneider, K. 2024 Computing differential operators of the particle velocity in moving particle clouds using tessellations. *J. Comput. Phys.* **498**, 112658.

Maxey, M. R. 1987 The gravitational settling of aerosol particles in homogeneous turbulence and random flow fields. *J. Fluid Mech.* **174**, 441–465.

Onishi, R., Baba, Y. & Takahashi, K. 2011 Large-scale forcing with less communication in finite-difference simulations of steady isotropic turbulence. *J. Comput. Phys.* **230**, 4088–4099.

Oujia, T., Matsuda, K. & Schneider, K. 2020 Divergence and convergence of inertial particles in high Reynolds number turbulence. *J. Fluid Mech.* **905**, A14.

Squires, K. D. & Eaton, J. K. 1991 Measurements of particle dispersion obtained from direct numerical simulations of isotropic turbulence. *J. Fluid Mech.* **226**, 1–35.

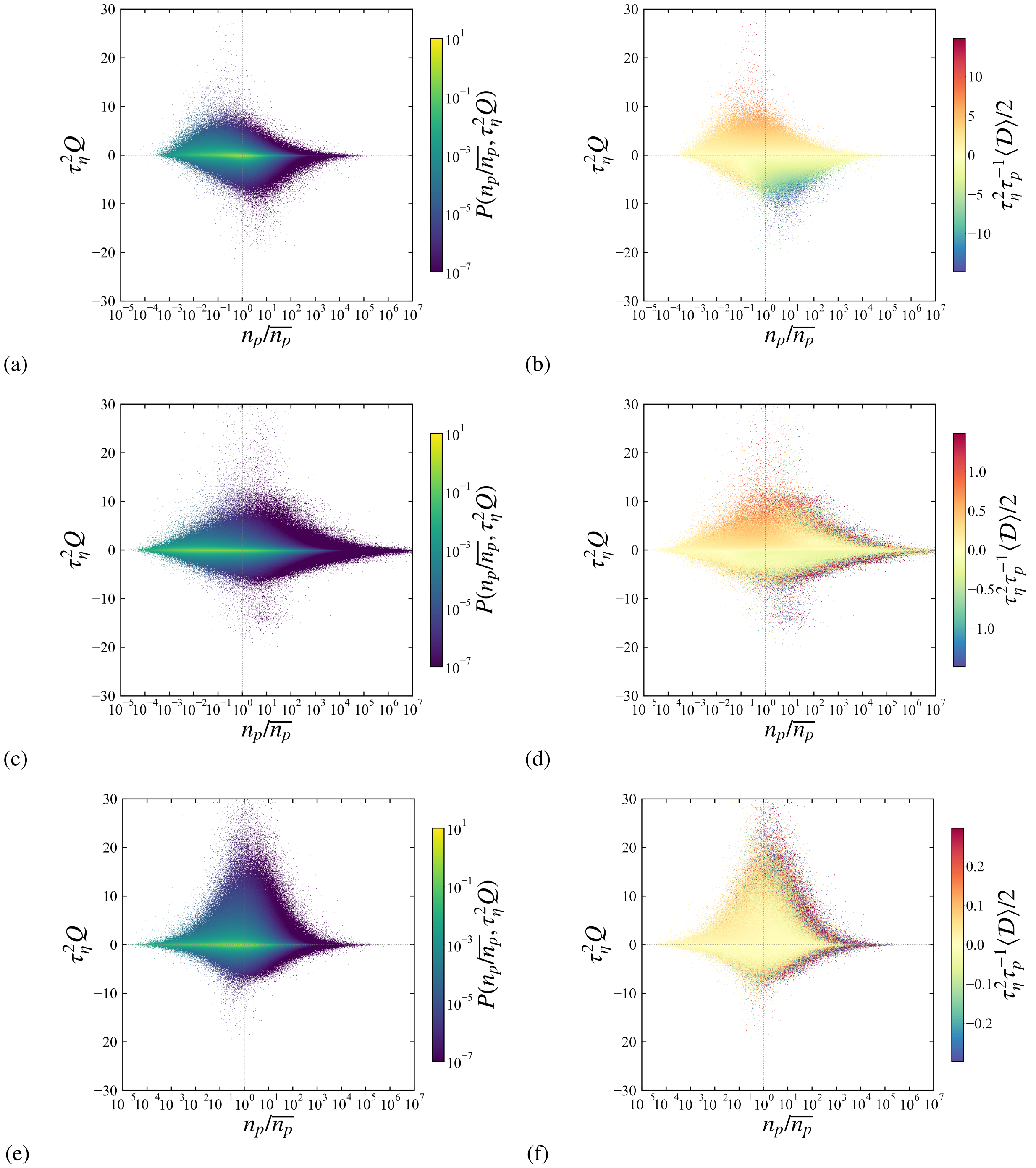

Figure 2. (a, c, e) Joint PDFs of n_p and Q and (b, d, f) conditionally averaged particle velocity divergence $\langle \mathscr{D} \rangle$ for given values of n_p and Q for (a, b) $St = 0.1$, (c, d) $St = 1.0$ and (e, f) $St = 5.0$.