STUDY ON THE EFFECT OF MHD INTERACTION ON TURBULENT TAYLOR-COUETTE FLOW WITH END WALLS

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ABSTRACT

The effect of magnetohydrodynamic (MHD) interaction on turbulent Taylor-Couette (TC) flow is investigated using large-eddy simulation (LES). An annular container is filled with the electrically conductive fluid. The inner cylinder rotates, and the outer cylinder and the top and bottom walls, i.e., end walls, are at rest. The axial magnetic field is applied to the fluid, so that the produced Lorentz force acts against the flow and suppresses turbulence. When the Hartmann number, proportional to the magnetic flux density, increases, high- and low-speed streaks near the inner wall owing to Görtler vortices are suppressed. As a result, high-speed triplet streaks are observed near the inner wall. These streaks consist of a centerline streak and two spiral streaks. The strength of the MHD interaction near the inner wall is influenced by the ratio of the Hartmann number to the Taylor number.

INTRODUCTION

A flow in a rotating co-axial cylinder is well-known as Taylor-Couette (TC) flow, which is one of the fundamental and important flows in fluid mechanics. Typical operation is that the inner cylinder is rotating and the outer one is at rest. As the inner cylinder's angular velocity increases, the flow transits to turbulence accompanying Taylor vortex cells owing to the strong shear stress on the inner wall, and high- and low-speed streaks are observed near the inner wall (Grossmann et al. 2016).

Although numerical simulations for the TC flow are usually conducted using the periodic boundary condition in the axial direction (Leng et al. 2020), we use a container with upper and lower walls referred to as end walls. In addition, the fluid with electrical conductivity is assumed as a working fluid, and the magnetic field is applied in the axial direction (Takana and Tanida 2017, Hasebe et al. 2023). As the magnetic field increases, it is well known that turbulence is suppressed by the Lorentz force that acts against the flow (Kobayashi et al. 2012).

In the present study, we study the effect of magnetohydrodynamic (MHD) interaction on turbulent TC flow using large-eddy simulation (LES). We focus on the criterion of MHD interaction determined from streaks in the vicinity of the inner wall.

GOVERNING EQUATIONS AND NUMERICAL CONDITIONS

Under the conditions of incompressible MHD flows, the governing equations consist of the continuity equation, the Navier-Stokes equations with the Lorentz force, the Maxwell equations at low magnetic Reynolds number, and the generalized Ohm's law. The second-order Adams-Bashforth method and a second-order central finite difference method are used for time matching and spatial discretization, respectively. The SMAC scheme is utilized for coupling between velocity and

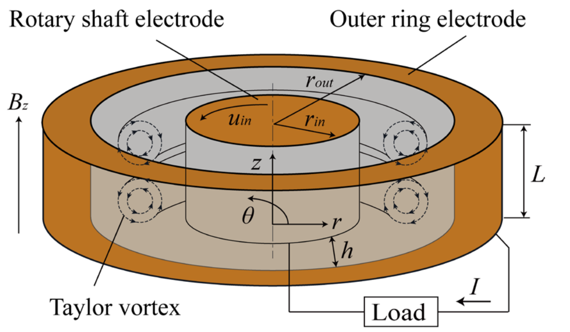

Figure 1. Schematic of MHD Taylor-Couette flow with a co-axial rotating cylinder and stationary upper and lower walls.

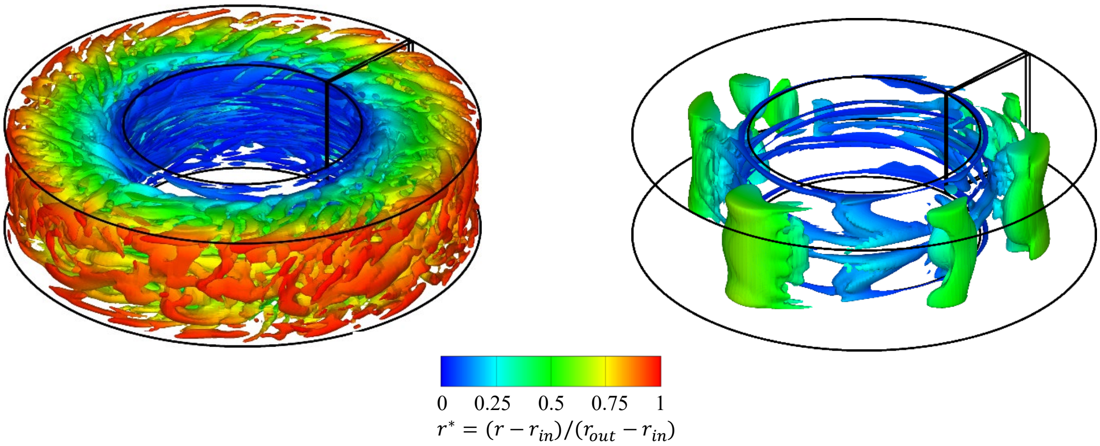

Figure 2. Instantaneous iso-surfaces of the second invariant of velocity gradient tensor Q=10 for Re=8000 at Ha=0 (left) and 100 (right). Color shows the distance from the inner wall. r^*, r, r_{in}, and r_{out} denote the normalized radius, radius, inner radius, and outer radius, respectively

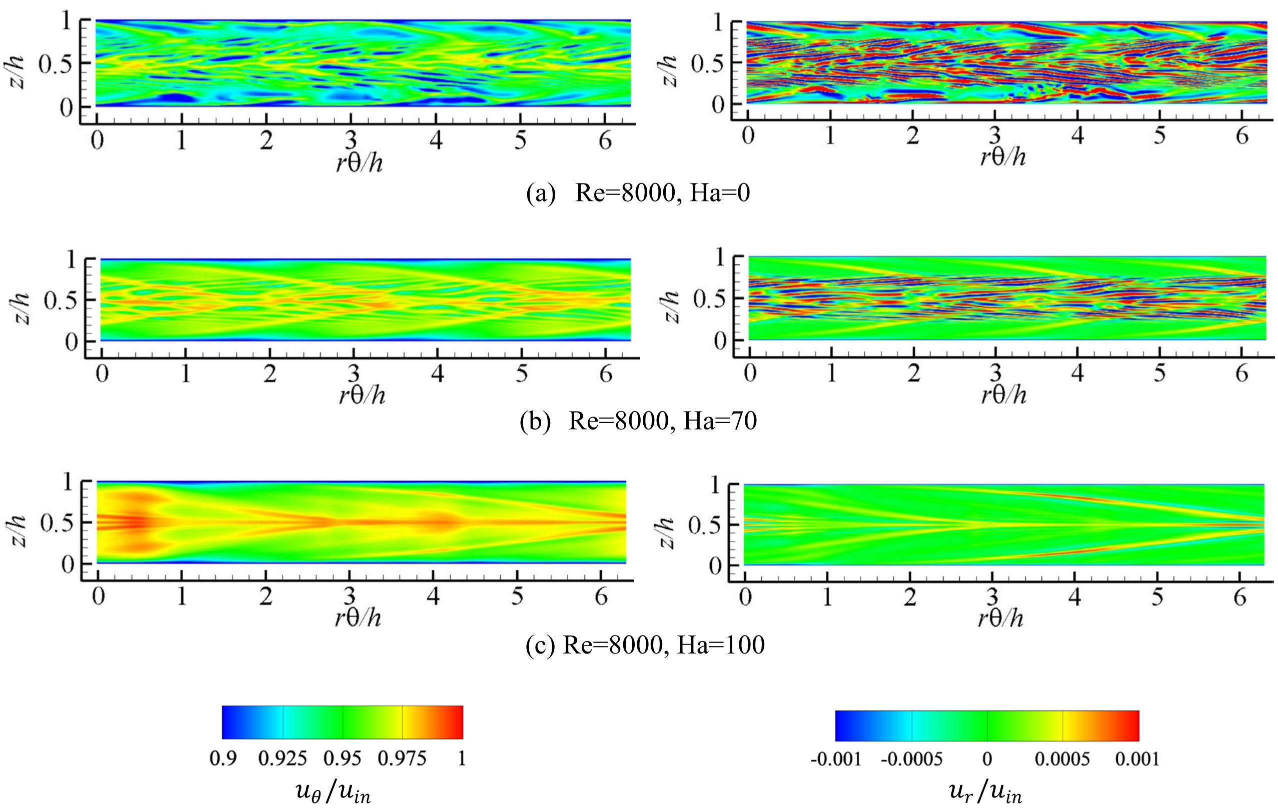

Figure 3. Instantaneous azimuthal (left) and radial velocity (right) distributions for Re=8000 at (a) Ha=0, (b) Ha=70, and (c) Ha=100 in the vicinity of the inner wall.

pressure. The Poisson equations for pressure and electric potential are solved by using a Bi-CGSTAB method. For LES, the subgrid-scale stress tensor is modeled by the coherent structure model (Kobayashi 2005).

Figure 1 shows the schematic of MHD Taylor-Couette flow with a co-axial rotating cylinder and stationary upper and lower walls. In the present study, the directions of r, θ, and z are determined as the radial, azimuthal (streamwise), and axial directions, respectively. The computational domain in the azimuthal direction is set to 2π. The aspect ratio is set to 1.0, and the radius ratios χ are set to 0.25, 0.5, and 0.75. The Hartmann number Ha is defined as the ratio of the Lorentz force to the viscous force and is proportional to the magnetic field. The Reynolds number Re is set to 4000, 5000, 8000, and 10000. The

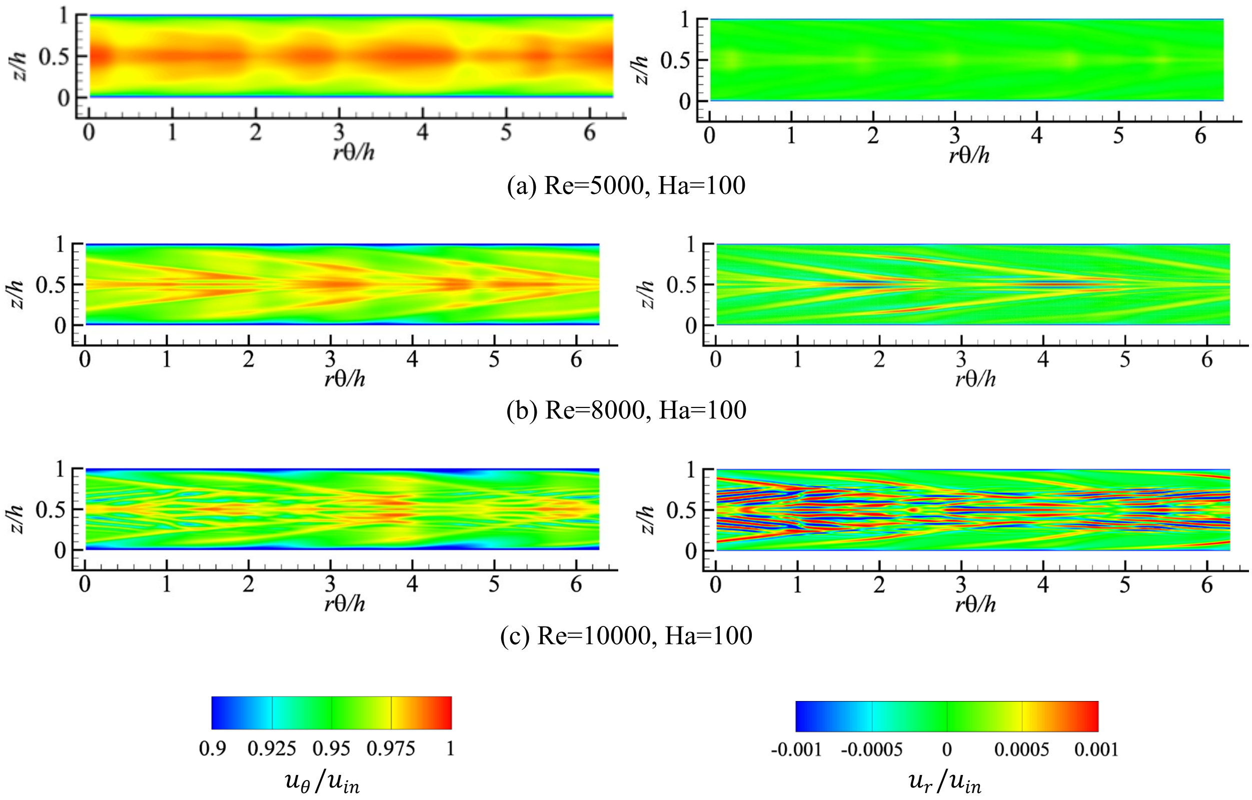

Figure 4: Instantaneous azimuthal (left) and radial velocity (right) distributions for Ha=100 at (a) Re=5000, (b) Re=8000, and (c) Re=10000 in the vicinity of the inner wall.

periodic boundary condition is used in the azimuthal direction, and no-slip boundary conditions are applied to the top, bottom, inner, and outer walls. All walls are assumed to be insulating walls.

RESULTS AND DISCUSSION

Figure 2 visualizes the instantaneous iso-surfaces of the second invariant of velocity gradient tensor Q=10 for Re=8000 at Ha=0 and 100. At Ha=0, turbulent vortex tubes elongated in the azimuthal direction are seen in the entire channel. At Ha=100, the Lorentz force suppresses the fine turbulent vortex tubes, and the quasi-two-dimensional vortices emerge and align in the axial direction.

Figure 3 shows the instantaneous azimuthal and radial velocity distributions for Re=8000 at Ha=0, 70, and 100 in the area near the inner wall. High- and low-speed streaks are seen in Fig. 3(a) left, and are produced by a pair of vortices with inflow and outflow as shown in Fig. 3(a) right. At Ha=70, the Lorentz force diminishes the fine streaks near the upper and lower walls. At Ha=100, those fine streaks are completely curbed, and high-speed spiral streaks are observed from the upper and lower walls. We can understand that two types of streaks exist in the TC flow in a container at Ha=0 of Fig. 3(a).

As illustrated in Fig. 3(c), the triplet high-speed streak, consisting of two spiral streaks and a centerline streak, serves as an effective marker for understanding the criteria of MHD interaction. We investigate what determines the criterion hereafter. Figure 4 exhibits the instantaneous azimuthal and radial velocity distributions for Ha=100 at Re=5000, 8000, and 10000 near the inner wall. At Re=5000, the high-speed region of azimuthal velocity seems diffused, and the radial velocity distribution looks uniform. This result shows that the Lorentz force is strong against the inertia force, producing turbulence, and the triplet streaks are laminarized. At Re=10000, fine streaks emerge in the middle of the flow and overlaid on the triplet streaks as shown in Fig. 3(b). This result means that the inertia force is strong against the Lorentz force. We can consider that the criterion is determined by the ratio of the Hartmann number to the Reynolds number.

Figure 5 depicts the instantaneous azimuthal and radial velocity distributions at Re=4000 and Ha=50, Re=5000 and Ha=62.5, and Re=10000 and Ha=125 near the inner wall. We can see the triplet streaks for all the figures, although the width of the streak becomes finer for higher Reynolds numbers. The criterion where the triplet steaks are generated is defined as

$$\mathrm{H_r} = \frac{\mathrm{Ha}}{\mathrm{Re}} \times 10^4 = 125 \quad (1)$$

Therefore, we can conclude that the strength of MHD interaction is expressed by $\mathrm{H_r}$ for the MHD turbulent state.

Let us investigate the effect of radius ratio χ on the turbulent TC flow. There are a lot of definitions for the Taylor number Ta. We use the following definition with a linear relation with the Reynolds number.

$$\mathrm{Ta} = \mathrm{Re}\left(\frac{1}{\chi} - 1\right)^{1/2} \quad (2)$$

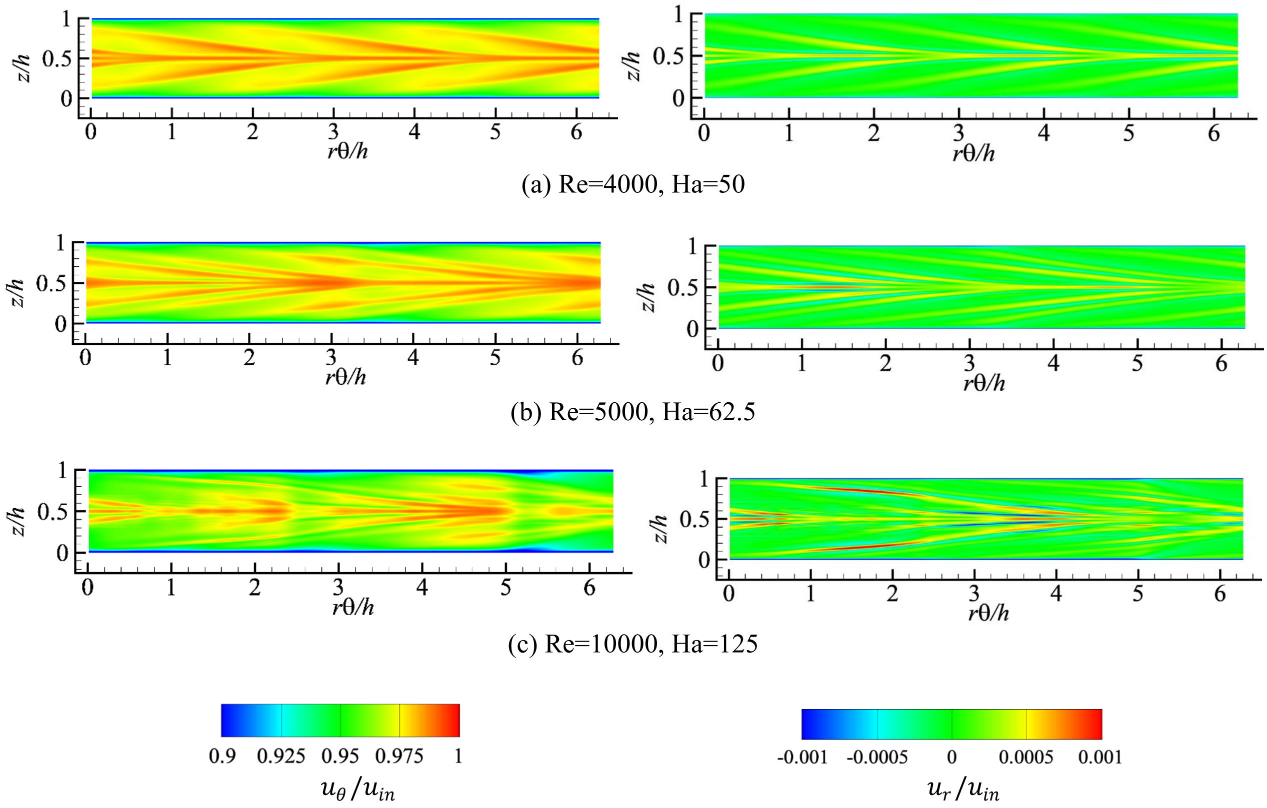

Figure 5. Instantaneous azimuthal (left) and radial velocity (right) distributions at (a) Re=4000 and Ha=50, (b) Re=5000 and Ha=62.5, and (c) Re=10000 and Ha=125 in the vicinity of the inner wall.

By using the Taylor number instead of the Reynolds number, we finally propose the following criteria, $\mathrm{H_T}$.

$$\mathrm{H_T} = \frac{\mathrm{Ha}}{\mathrm{Ta}} \times 10^4 = 125 \quad (3)$$

Figure 6 displays the instantaneous azimuthal and radial velocity distributions for Re=8000 at χ=0.75 and Ha=58, χ=0.50 and Ha=100, and χ=0.25 and Ha=173 near the inner wall. A high radius ratio leads to the reduction of the curvature effect. As shown in Fig. 6(a), a low Hartmann number is balanced with the corresponding Taylor number. However, a low radius ratio causes an increase in the centrifugal instability. As a result, as shown in Fig. 6(c), high Hartmann number matches the corresponding Taylor number. The number of triplet steaks increases with the radius ratio.

CONCLUDING REMARKS

We investigated the effect of MHD interaction on turbulent TC flow in the co-axial rotating cylinder with end walls. When applying the magnetic field in the axial direction, the Lorentz force suppresses the fine streaks, and high-speed spiral streaks appear near the inner wall. The spiral streaks emerge under the balance between the turbulence production by the inertia force and the laminarization by the Lorentz force. The strength of the MHD interaction was found to be indicated by the ratio of the Hartmann number to the Taylor number.

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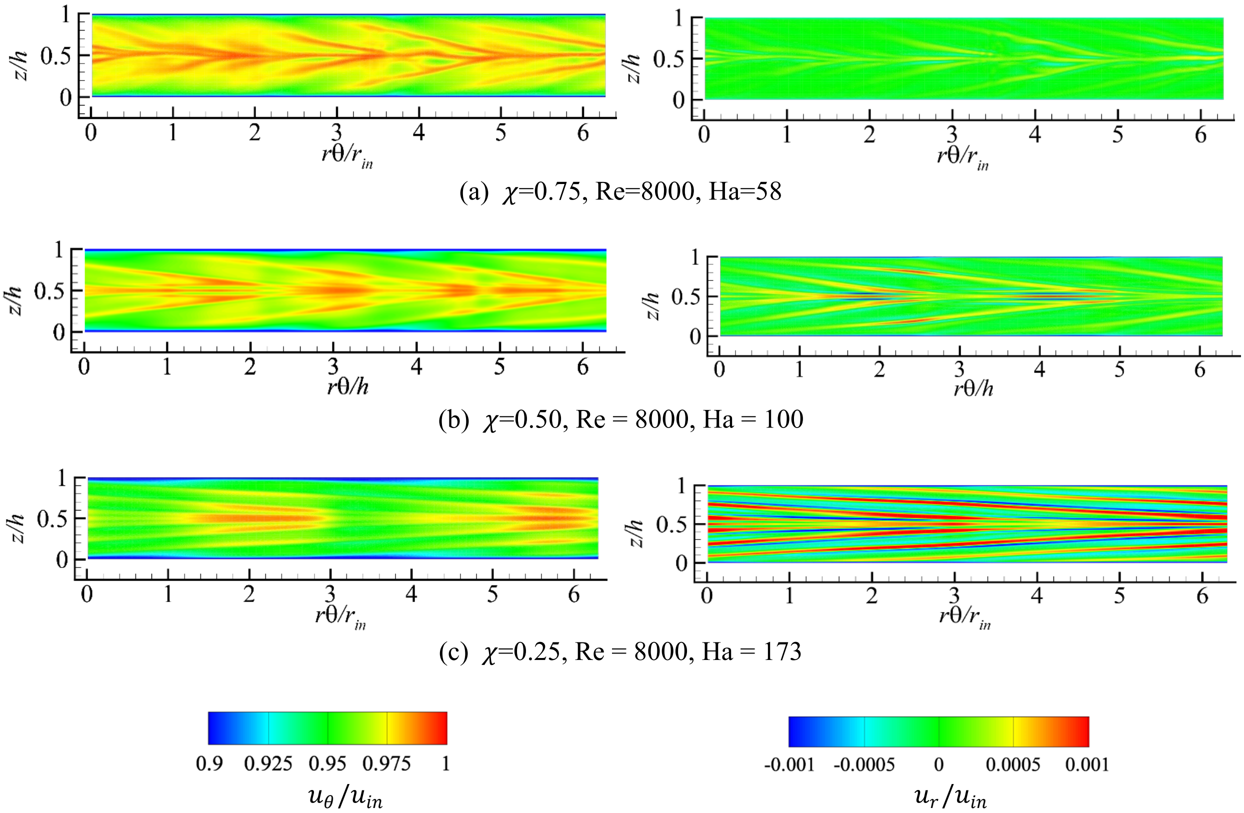

Figure 6. Instantaneous azimuthal (left) and radial velocity (right) distributions for Re=8000 at (a) χ=0.75 and Ha=58, (b) χ=0.50 and Ha=100, and (c) χ=0.25 and Ha=173 in the vicinity of the inner wall.