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ASSESSMENT OF A NEW CLASS OF TURBULENCE CLOSURES FOR VERY-COARSE NUMERICAL SIMULATIONS

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ABSTRACT

The present work reports an assessment of a new class of turbulence closures recently developed by Cimarelli *et al.* (2025) that, thorugh a temporal filtering procedure, dynamically adjusts the eddy viscosity within the framework of classical RANS turbulence models. The developed dynamic procedure is shown to maximize the amount of information provided by the numerical solution for a given spatial resolution, thus effectively improving the performance of classical RANS models without incurring in a significant increase in computational costs. In particular, the proposed new class of turbulence closures is found to form a bridge between the computational efficiency of RANS and the higher fidelity of LES. A detailed comparison with predictions from the standard $k-\varepsilon$ model and the hybrid LES-RANS $k-\varepsilon-\zeta-f$ model is performed in the settings provided by the transitional flow over a rounded leading edge flat plate and by the flow around a rectangular cylinder with sharp edges. The results show that the developed dynamic closure nicely perform in reproducing the scale-dependent fetaures of the flow, in capturing the laminar to turbulence transition phenomena and in the sensitivity to the free-stream flow conditions.

INTRODUCTION

Turbulent fluid motion characterizes a wide range of CFD applications, with turbulence exhibiting a wide spectrum of spatial and temporal scales. This variety in scales significantly increases the computational cost, making it challenging to accurately resolve all of them. DNS (Direct Numerical Simulation) and LES (Large-Eddy Simulation) approaches, while highly accurate, remain prohibitively expensive for most practical engineering applications due to their high computational demands. That is why RANS (Reynolds-Averaged Navier-Stokes) simulations remain the preferred solution in engineering applications, yet their inability to capture turbulent fluctuations and their difficulties with laminar to turbulence transitional flows represent a hefty trade-off for their low cost. Traditional eddy viscosity models suffer from stress over-prediction in adverse pressure gradients, which severely compromises their ability to predict laminar-to-turbulent transition. They are also incapable of returning scale-resolving solutions when the grid resolution and the numerical settings allow them. This work premiers a new formalism for RANS eddy viscosity models that actually faces these fundamental issues.

A NEW CLASS OF TURBULENCE CLOSURES

We report here a brief introduction to the new class of turbulence closures. The basic assumption is that the smoothing action of RANS turbulence closure can be modeled as a temporal filtering operator implicitly applied to the Navier-Stokes equations, hereafter denoted as $\langle\cdot\rangle_\tau$ where τ represents the filter time scale implicitly imposed by the eddy viscosity. In this framework, the turbulent stresses takes the exact form

$$R_\tau(u_i,u_j) = \langle u_i u_j\rangle_\tau - \langle u_i\rangle_\tau\langle u_j\rangle_\tau \tag{1}$$

A relevant aspect of the use of the filtering approach is given by the fact that some important algebraic properties can be leveraged. In particular, by introducing a second filter operator with a larger characteristic time scale $T > \tau$, it is possible to evaluate the turbulent stresses at a coarser level as

$$R_{T_\tau}(u_i,u_j) = \langle\langle u_i u_j\rangle_\tau\rangle_T - \langle\langle u_i\rangle_\tau\rangle_T\langle\langle u_j\rangle_\tau\rangle_T \tag{2}$$

These turbulent stresses are related with those at the finer level

$$\begin{aligned} R_{T_\tau}(u_i,u_j) =& \langle R_\tau(u_i,u_j)\rangle_T + \\ & \langle\langle u_i\rangle_\tau\langle u_j\rangle_\tau\rangle_T - \langle\langle u_i\rangle_\tau\rangle_T\langle\langle u_j\rangle_\tau\rangle_T \end{aligned} \tag{3}$$

thus allowing us to write one of the most famous identities in turbulence, Germano (1992),

$$\begin{aligned} R_{T_\tau}(u_i,u_j) - & \langle R_\tau(u_i,u_j)\rangle_T = \\ & \langle\langle u_i\rangle_\tau\langle u_j\rangle_\tau\rangle_T - \langle\langle u_i\rangle_\tau\rangle_T\langle\langle u_j\rangle_\tau\rangle_T \end{aligned} \tag{4}$$

This identity can be exploited to develop a dynamic procedure for the computation of the model coefficient c_μ classically appearing as a constant in RANS eddy viscosity models, Wilcox *et al.* (1998).

In the context of eddy viscosity models the turbulent stresses at the two filter levels can be written as

$$\begin{aligned} R_\tau(u_i,u_j) &= -2\nu_\tau \langle S_{ij}\rangle_\tau \\ R_{T_\tau}(u_i,u_j) &= -2\nu_{T_\tau}\langle\langle S_{ij}\rangle_\tau\rangle_T \end{aligned} \tag{5}$$

where $S_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2$ is the rate of deformation tensor and the two diffusion coefficients, ν_τ and ν_{T_τ}, are the eddy viscosity associated to the two filter levels. Without loss of generality, by considering the classical $k-\varepsilon$ approach, the two eddy viscosity can be written as

$$\begin{aligned} \nu_\tau &= c_\mu \frac{k_\tau^2}{\varepsilon_\tau} \\ \nu_{T_\tau} &= c_\mu \frac{k_{T_\tau}^2}{\varepsilon_{T_\tau}} \end{aligned} \tag{6}$$

Here, k_τ and ε_τ are the transported turbulent kinetic energy and turbulent dissipation whose classical evolution equation is

$$\begin{aligned} &\frac{\partial k_\tau}{\partial t} + \frac{\partial k_\tau \langle u_j\rangle_\tau}{\partial x_j} = \\ &2\nu_\tau \langle S_{ij}\rangle_\tau \langle S_{ij}\rangle_\tau - \varepsilon_\tau + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_\tau}{\sigma^k}\right)\frac{\partial k_\tau}{\partial x_j}\right] \end{aligned} \tag{7}$$

$$\begin{aligned} &\frac{\partial \varepsilon_\tau}{\partial t} + \frac{\partial \varepsilon_\tau \langle u_j\rangle_\tau}{\partial x_j} = \\ &\frac{\varepsilon_\tau}{k_\tau}\left(2C_{\varepsilon_1}\nu_\tau \langle S_{ij}\rangle_\tau \langle S_{ij}\rangle_\tau - C_{\varepsilon_2}\varepsilon_\tau\right) + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_\tau}{\sigma^\varepsilon}\right)\frac{\partial \varepsilon_\tau}{\partial x_j}\right] \end{aligned} \tag{8}$$

where σ^k, σ^ε, C_{ε_1} and C_{ε_2} are adjustable model coefficients, Launder & Spalding (1983). On the other hand, k_{T_τ} and ε_{T_τ} are the turbulent kinetic energy and turbulent dissipation associated with the flow solution at the test filter level and can be computed as

$$\begin{aligned} k_{T_\tau} &= \langle k_\tau\rangle_T + \mathscr{L}_{ii}/2 \\ \varepsilon_{T_\tau} &= \langle \varepsilon_\tau\rangle_T + \nu \mathscr{D}_{ii} \end{aligned} \tag{9}$$

where the tensor $\mathscr{L}_{ij}$ directly comes from identity (4),

$$\begin{aligned} \mathscr{L}_{ij} &\equiv R_{T_\tau}(u_i,u_j) - \langle R_\tau(u_i,u_j)\rangle_T = \\ &\langle\langle u_i\rangle_\tau \langle u_j\rangle_\tau\rangle_T - \langle\langle u_i\rangle_\tau\rangle_T \langle\langle u_j\rangle_\tau\rangle_T \end{aligned} \tag{10}$$

on the other hand, $\mathscr{D}_{ij}$ derives from the same arguments but applied to the velocity gradient,

$$\begin{aligned} \mathscr{D}_{ij} \equiv & R_{T_\tau}\left(\frac{\partial u_i}{\partial x_k}, \frac{\partial u_j}{\partial x_k}\right) - \left\langle R_\tau\left(\frac{\partial u_i}{\partial x_k}, \frac{\partial u_j}{\partial x_k}\right)\right\rangle_T = \\ & \left\langle \frac{\partial\langle u_i\rangle_\tau}{\partial x_k}\frac{\partial\langle u_j\rangle_\tau}{\partial x_k}\right\rangle_T - \frac{\partial\langle\langle u_i\rangle_\tau\rangle_T}{\partial x_k}\frac{\partial\langle\langle u_j\rangle_\tau\rangle_T}{\partial x_k} \end{aligned} \tag{11}$$

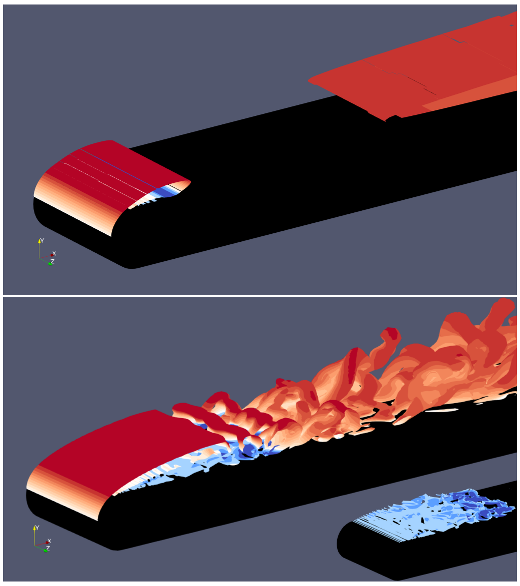

Figure 1. Transitional flow over a rounded leading-edge flat plate, the T3L test case. Iso-surfaces of the Q-criterion $Q = 0.15$, Hunt *et al.* (1988), colored with the streamwise velocity for the standard $k-\varepsilon$ model (left) and for the new dynamic RANS procedure (right). In the inset plot of the right panel, isosurfaces with positive streamwise velocity are hidden to reveal the structures of the reverse boundary layer within the main recirculating bubble.

By substituting modeling assumptions (6) into the Boussinesq hypothesis (5), the identity (4) can be rewritten as

$$c_\mu = \frac{\mathscr{M}_{ij}\mathscr{L}_{ij}}{\mathscr{M}_{ij}\mathscr{M}_{ij}} \tag{12}$$

where

$$\mathscr{M}_{ij} = 2\left\langle \frac{k_\tau^2}{\varepsilon_\tau}\langle S_{ij}\rangle_\tau \right\rangle_T - 2\frac{k_{T_\tau}^2}{\varepsilon_{T_\tau}}\langle\langle S_{ij}\rangle_\tau\rangle_T \tag{13}$$

and

$$\mathscr{L}_{ij} = \langle\langle u_i\rangle_\tau \langle u_j\rangle_\tau\rangle_T - \langle\langle u_i\rangle_\tau\rangle_T \langle\langle u_j\rangle_\tau\rangle_T \tag{14}$$

Equation (12) represents a new class of turbulence closures that has been recently derived by Cimarelli *et al.* (2025). Indeed, the tensor $\mathscr{M}_{ij}$ has been derived here specifically for the $k-\varepsilon$ model but can be equivalently written for any other two-equation eddy viscosity model. This new class of models (12) will be hereafter called dynamic RANS closures.

RESULTS

The present section reports the first attempt to assess the properties of the proposed dynamic procedure. Different flow cases have been considered. Here, we focus on the transitional flow over a rounded leading edge flat plate under different free-stream turbulence conditions, the so-called T3L test case series of the ERCOFTAC suite, Yang & Voke (2001); Bassi *et al.* (2020); Cimarelli *et al.* (2020), and on the flow around

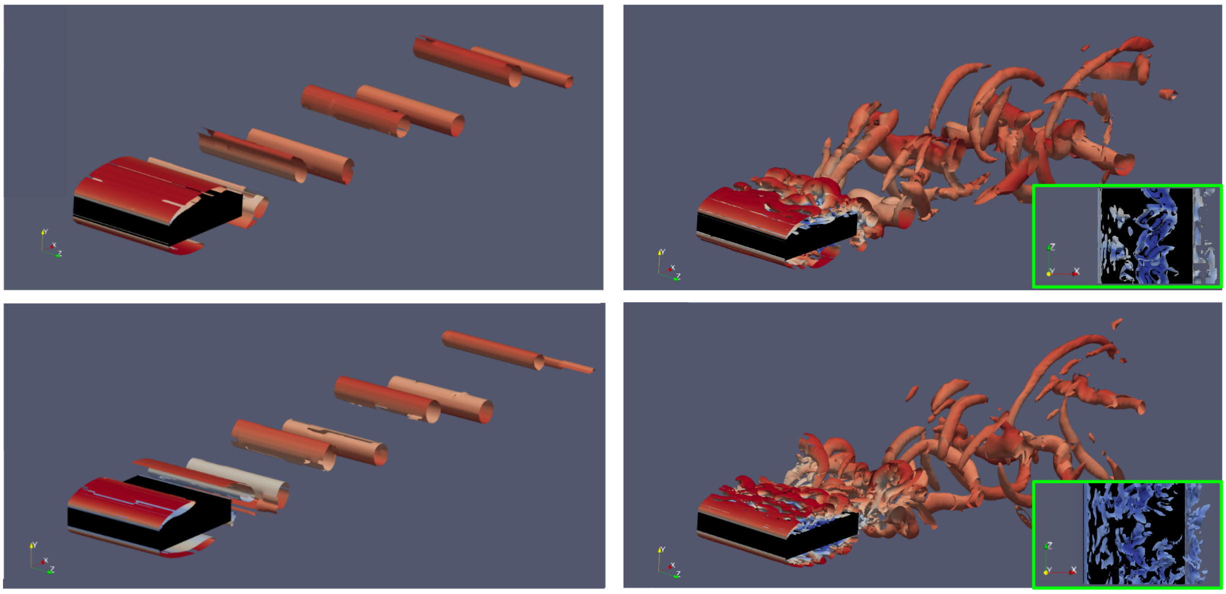

Figure 2. Separating and reattaching flow around a rectangular cylinder, the BARC test case. Iso-surfaces the Q-Criterion $Q = 0.15$, Hunt *et al.* (1988), colored with the streamwise velocity for the standard $k-\varepsilon$ model (left) and for the new dynamic procedure (right). Top and bottom panels show the solutions obtained in a finer and coarser grid. In the inset plot of the right panels, isosurfaces with positive streamwise velocity are hidden to reveal the structures of the reverse boundary layer within the main recirculating bubble.

a rectangular cylinder with aspect ratio 5, the so-called BARC (Benchmark on the Aerodynamics of a Rectangular 5:1 Cylinder) Bruno *et al.* (2014).

The T3L case is chosen because it explicitly concerns flow phenomena of separation, transition and reattachment, which are intimately linked with the management of the near-wall physics, of the laminar to turbulence transition and of the physical dependence of such phenomena on the values of free-stream turbulence, the main prospects for the developed dynamic procedure. Simulations are performed by varying the grid resolution and the free-stream turbulence levels by keeping a fixed Reynolds number, $Re = U_0 D/\nu = 3333$ where U_0 is the free-stream velocity, D the plate thickness and ν the kinematic viscosity.

The BARC flow case is selected because of the availability of several high-fidelity numerical studies that can be used for comparison. Among others, here we consider as reference the DNS of Cimarelli *et al.* (2024) because of the fairly high Reynolds number considered, $Re = U_0 D/\nu = 14000$. The distinguishing feature of the BARC case compared to the T3L case, is the laminar boundary layer separation that is fixed at the sharp leading-edges of the body.

All the simulations have been perfomed by using the solver T-Flows (https://github.com/DelNov/T-Flows) Hadžiabdić *et al.* (2022). T-Flows is based on a finite volume method for collocated, unstructured and arbitrary grids. The transport scheme for momentum and turbulent quantities is SMART, a high-order upwind-biased scheme which maximises accuracy within the limits required to preserve boundedness. Time integration is performed with the Backward Euler scheme in first-order form for the stresses and in a second-order variant, used in iterative fashion, for the inertial term. The time step is adapted to obtain a condition $CFL \approx 1.5$.

As shown in figures 1 and 2, the new class of closures based on the dynamic procedure (12) show to enable well-defined scale-resolving features in both the T3L and BARC test cases contrary to the baseline $k-\varepsilon$ model that reproduces a 2D flow solution. As shown in the bottom panels of figure 2, these scale-resolving features are reproduced also when coarsening the grid resolution. A more deep analysis will be reported at the conference talk and shows that a significant improvement of the quality of the statistics is also obtained by applying the dynamic procedure to the baseline model.

CONCLUSIONS

A new class of turbulence closures is developed that is based on a dynamic procedure recently developed by Cimarelli *et al.* (2025) for the computation of the model coefficient c_μ commonly employed in eddy viscosity models. The procedure is based on a formalism that describes the smoothing action of closures as a temporal filtering operator. A detailed analysis of the results will be reported at the conference talk. We anticipate here that the dynamic procedure applied to the $k-\varepsilon$ model allows to capture scale-resolving flow features that are supported by the grid resolution and to nicely predict challenging phenomena such as turbulence transition and the dependence of the flow solution on the free-sream turbulence conditions. Such important properties are obtained without an increase of the computational costs and are completely missed by the baseline $k-\varepsilon$ model.

REFERENCES

Bassi, F., Botti, L., Colombo, A., Crivellini, A., Franciolini, M., Ghidoni, A. & Noventa, G. 2020 A p-adaptive matrix-free discontinuous galerkin method for the implicit les of incompressible transitional flows. *Flow, Turbulence and Combustion* **105**, 437–470.

Bruno, L., Salvetti, M. V. & Ricciardelli, F. 2014 Benchmark on the aerodynamics of a rectangular 5:1 cylinder: an overview after the first four years of activity. *J. Wind Eng. Ind. Aerodyn.* **126**, 87–106.

Cimarelli, A, Corsini, R & Stalio, E 2024 Reynolds number effects in separating and reattaching flows with passive scalar transport. *J. Fluid Mech.* **984**, A20.

Cimarelli, A, Franciolini, M & Crivellini, A 2020 Numerical experiments in separating and reattaching flows. *Phys. Fluids* **32** (9).

Cimarelli, A, Marras, N, Niceno, B & Urrecha, Y Tessier 2025 Unifying formalism and closures for coarse-grained approaches to turbulence. *arXiv preprint arXiv:2512.19274* .

Germano, M. 1992 Turbulence: the filtering approach. *J. Fluid Mech.* **238**, 325–336.

Hadžiabdić, M., Hafizović, M., Ničeno, B. & Hanjalić, K. 2022 A rational hybrid rans-les model for cfd predictions of microclimate and environmental quality in real urban structures. *Building and Environment* **217**, 109042.

Hunt, J. CR, Wray, A. A. & Moin, P 1988 Eddies, streams, and convergence zones in turbulent flows. *Center for Turbulence Research Report* **CTR-S88**, 193.

Launder, B.E. & Spalding, D.B. 1983 The numerical computation of turbulent flows. In *Numerical prediction of flow, heat transfer, turbulence and combustion*, pp. 96–116. Elsevier.

Wilcox, David C *et al.* 1998 *Turbulence modeling for CFD*, , vol. 2. DCW industries La Canada, CA.

Yang, Z. & Voke, P. R. 2001 Large-eddy simulation of boundary-layer separation and transition at a change of surface curvature. *J. Fluid Mech.* **439**, 305–333.