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DATA-DRIVEN TRANSIENT MODAL ANALYSIS FOR EXTREME VORTEX-GUST AIRFOIL INTERACTIONS WITH TIME-DEPENDENT BASES

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ABSTRACT

Extreme gust-wing interactions generate highly transient flow dynamics that are challenging to analyze due to their unsteadiness and the absence of steady base states. Here, we perform a data-driven, time-dependent modal analysis of such interactions. The present approach, a data-driven, time-dependent bases framework, represents the flow with time-evolving modes and their energies (singular values), estimated from the time derivatives of streamed snapshots. Applied to large-eddy simulations at $Re_c = 5000$ with a Taylor-vortex gust, time-dependent bases reconstructs the vortical flow fields through the strong transient dynamics. The leading modes capture the translating separation region and subsequent roll-up. In particular, after the vortex impingement, non-dominant modes gain energy, as indicated by increasing singular values and a narrower gap between the leading and remaining singular values. The change in singular values over time aligns with the lift-coefficient overshoot and decay indicating the connection between voritcal structures and aerodynamic force. The current time-dependent bases approach provides modal content and reconstructions for strongly unsteady aerodynamic flows.

INTRODUCTION

Understanding gust–wing interactions is critical because small aircraft frequently encounter high gust ratios ($G \geq 1$) in urban canyons, mountainous terrain, and severe turbulence. Here, the gust ratio is defined as $G = u_g/u_\infty$, where u_g is the gust velocity and u_∞ is the cruise velocity. These encounters produce violent load excursions as the gust reorganizes separation and the wake (Taira, 2026; Jones *et al.*, 2022). The response is strongly nonlinear and transient — characterized by vortex breakdown, fine-scale turbulence, and rapidly fluctuating forces — with three-dimensionality that intensifies at higher G, complicating interpretation.

Data-driven reduced-order models project high-dimensional flows onto low-dimensional, interpretable subspaces. Proper orthogonal decomposition (Sirovich, 1987) and dynamic mode decomposition (Schmid, 2010) perform well for statistically stationary or mildly unsteady flows and reveal coherent structures. However, gust encounters feature rapidly translating, short-lived separated/vortical structures. Therefore, fixed spatial bases or globally stationary dynamics cannot resolve transport-dominated transients. Time-dependent subspace models address this challenge by allowing the bases to evolve over time. These frameworks appear in both model-driven and data-driven forms. In model-driven approaches, time-dependent bases are obtained by projecting the governing equations into a low-rank, time-varying subspace with orthonormal bases and dynamical orthonormality constraints, reducing the cost of large-scale stochastic modeling (Patil & Babaee, 2020; Sapsis & Lermusiaux, 2009). In addition to stochastic modeling, these models are applied to various problems, including combustion (Ramezanian *et al.*, 2021), sensitivity analysis (Donello *et al.*, 2022), and flow stability analysis (Zhong *et al.*, 2025). Time-dependent bases can also be constructed from data, updating the bases from instantaneous temporal variation, e.g., a finite difference derivative from streamed snapshots, without full-history storage, preserving spatial and temporal variation and enabling efficient compression and modal analysis (Zamani Ashtiani *et al.*, 2022).

Time-dependent bases are well-suited for transient, transport-dominated flows, such as vortex gust–airfoil interactions. This study exploits this property for modal analysis of extreme gust encounters. To this end, we apply time-dependent bases to large-eddy simulations of gust–airfoil interactions at a chord-based Reynolds number of $Re_c = 5000$ (Fukami *et al.*, 2025), where the gust is modeled as a Taylor vortex with intensity $G = 2$ and 4. Both cases lie in the regime $G > 1$, which is traditionally considered unflyable (Fukami *et al.*, 2024; Fukami & Taira, 2023).

METHODS

We represent the streaming field $v(x,y,z,t)$ by a low-rank reconstruction $\hat{v}(x,y,z,t)$ with its reconstruction error ϵ as $v(x,y,z,t) = \hat{v}(x,y,z,t) + \epsilon$, and define the reconstruction as follows:

$$\hat{v}(x,y,z,t) = \sum_{i_1=1}^{r}\sum_{i_2=1}^{r} T_{i_1 i_2}(t)\,\phi_{i_1}(x,y,t)\,\psi_{i_2}(z,t). \quad (1)$$

Here, $\phi(x,y,t)$, $\psi(z,t)$, $T(t)$, and r denote the in-plane orthonormal modes, spanwise orthonormal modes, the reduced covariance matrix, and the number of modes respectively. The time-dependent bases framework is developed for multidimensional tensor data (Zamani Ashtiani *et al.*, 2022). In this work, we implement it with in-plane two-dimensional modes $\phi(x,y,t)$ to focus on sectional flow structure. To this end, we matri-

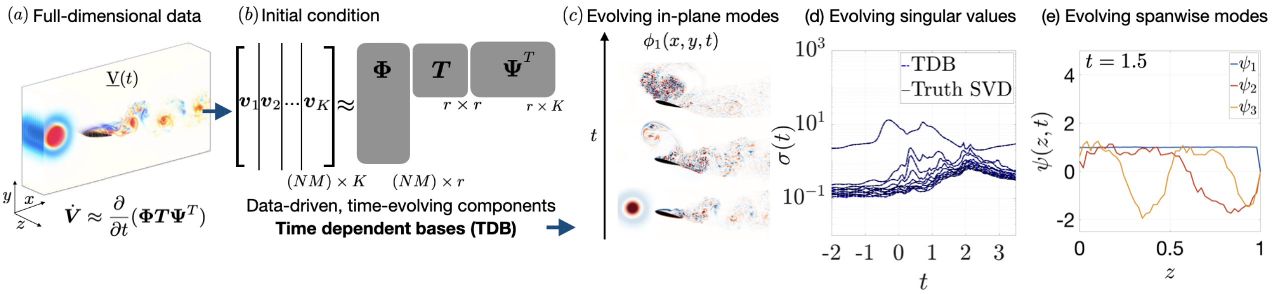

Figure 1. Time-dependent bases (TDB) framework. (a) Streaming data $\underline{V}(t)$. (b) Matricizing each snapshot in z direction $\mathbf{V} = \left[\mathbf{v}_{z_1} | \cdots | \mathbf{v}_{z_{N_z}}\right]$, where each column is the vectorized in-plane (x, y) field at location z. (c–e) Evolve $\mathbf{\Phi}$, $\mathbf{\Psi}$, and $\mathbf{T}$ using time derivative $\dot{\mathbf{V}}$.

cize the tensor along the spanwise (z) direction to form the matrix $\mathbf{V} \in \mathbb{R}^{N_x N_y \times N_z}$, as shown in figures 1(a) and (b), and initialize the decomposition by performing a singular value decomposition at the initial time step. In figure 1, the time-dependent bases framework is shown in discrete notation for simplicity, where

$$\mathbf{\Phi} = \left[\phi_1(x,y,t) \mid \cdots \mid \phi_r(x,y,t)\right] \in \mathbb{R}^{N_x N_y \times r},$$
$$\mathbf{\Psi} = \left[\psi_1(z,t) \mid \cdots \mid \psi_r(z,t)\right] \in \mathbb{R}^{N_z \times r},$$
$$\hat{\mathbf{V}} = \mathbf{\Phi}\mathbf{T}\mathbf{\Psi}^\top \in \mathbb{R}^{N_x N_y \times N_z}.$$

For simplicity, we adopt discrete notation hereafter, with orthonormality conditions defined as:

$$\mathbf{\Phi}^\top \mathbf{W}_{xy} \mathbf{\Phi} = \mathbf{I},$$
$$\mathbf{\Psi}^\top \mathbf{W}_z \mathbf{\Psi} = \mathbf{I}.$$

With $\mathbf{W}_{xy} = \mathrm{diag}\big(\boldsymbol{w}_y \otimes \boldsymbol{w}_x\big)$ and $\mathbf{W}_z = \mathrm{diag}(\boldsymbol{w}_z)$, $\boldsymbol{w}_x \in \mathbb{R}^{N_x}$, $\boldsymbol{w}_y \in \mathbb{R}^{N_y}$, and $\boldsymbol{w}_z \in \mathbb{R}^{N_z}$ representing the quadrature weights in the x, y, and z directions, respectively.

The time-dependent bases framework evolves the decomposed components in time. To this end, following the time-dependent bases formulation (Zamani Ashtiani *et al.*, 2022; Patil & Babaee, 2020; Naderi & Babaee, 2023), a dynamic orthogonality condition is applied. This condition enables the derivation of closed-form evolution equations. Specifically, dynamic orthogonality means that the temporal variation of each subspace is orthogonal to the subspace spanned by its corresponding modes:

$$\mathbf{\Phi}^\top \mathbf{W}_{xy} \dot{\mathbf{\Phi}} = \mathbf{0},$$
$$\mathbf{\Psi}^\top \mathbf{W}_z \dot{\mathbf{\Psi}} = \mathbf{0}.$$

By applying these conditions (Zamani Ashtiani *et al.*, 2022), the closed system of evolution equations for time-dependent bases is obtained as:

$$\dot{\mathbf{T}} = \mathbf{\Phi}^\top \mathbf{W}_{xy} \dot{\mathbf{V}} \mathbf{W}_z \mathbf{\Psi} \tag{2}$$
$$\dot{\mathbf{\Phi}} = \left(\mathbf{I} - \mathbf{\Phi}\mathbf{\Phi}^\top \mathbf{W}_{xy}\right) \dot{\mathbf{V}} \mathbf{W}_z \mathbf{\Psi} \mathbf{T}^{-1}, \tag{3}$$
$$\dot{\mathbf{\Psi}} = \left(\mathbf{I} - \mathbf{\Psi}\mathbf{\Psi}^\top \mathbf{W}_z\right) \dot{\mathbf{V}}^T \mathbf{W}_{xy} \mathbf{\Phi} \mathbf{T}^{-1}. \tag{4}$$

In these equations, $\dot{\mathbf{V}}$ is computed numerically from the streaming snapshots. Using the fourth-order Runge–Kutta time integration, $\mathbf{\Phi}$, $\mathbf{\Psi}$, and $\mathbf{T}$ evolve at each time step, as illustrated in figures 1(c–e).

The time-dependent bases approach is equivalent to performing an instantaneous singular value decomposition at each time instant, but at a substantially lower computational cost. This connection can be established by taking the singular value decomposition of the reduced covariance matrix $\mathbf{T} = \mathbf{U}\mathbf{\Sigma}\mathbf{Y}^\top$, where $\mathbf{\Sigma}$ is a diagonal matrix containing the singular values in descending order $\sigma_1 > \sigma_2 > \cdots > \sigma_r$. The singular values quantify the contribution of each mode and recover the same instantaneous modal energy content as the singular value decomposition of the instantaneous data. The singular values obtained from time-dependent bases agree with this per-time-step benchmark, as presented in figure 1(d). Here, we focus on the evolution of the in-plane modes and their corresponding singular values for time-varying modal analysis.

RESULTS

Let us apply the time-dependent bases framework to extreme-vortex gust–airfoil interactions at chord-based Reynolds number $Re_c = 5000$ (Fukami *et al.*, 2025). The datasets are generated by large-eddy simulations of a NACA0012 airfoil at $\alpha = 14^\circ$, yielding a turbulent separated wake. The computational domain spans $(x,y,z)/c \in [-20,25] \times [-20,20] \times [0,1]$ with the leading edge at the origin, using a spanwise extent of $1c$ (chord length) with periodic boundaries to resolve three-dimensional structures (Rolandi *et al.*, 2024, 2025). To impose an extreme disturbance, a Taylor-vortex gust is introduced upstream at $x_0/c = -2$ (Taylor, 1918), where x_0 is the streamwise position of the gust center relative to the wing. The gust ratio defined as $G = u_{\theta,\max}/u_\infty$ where $u_{\theta,\max}$ is the maximum tangential velocity of the Taylor vortex and u_∞ is the freestream velocity. We examine $G = 2$ and 4, values traditionally avoided in flight due to strong unsteadiness for small and medium sized aircraft (Fukami *et al.*, 2024; Fukami & Taira, 2023). The non-dimensional gust size and vertical offset are fixed at $D = 2R/c = 0.5$, where R is the vortex core radius, and $Y = y_0/c = 0.1$, where y_0 is the vertical position of the gust center relative to the wing, respectively.

We first consider the case with $G = 2$, as shown in figure 2. The vortex impinges on the wing and rolls up into a concentrated leading-edge vortex. This structure then convects downstream toward the trailing edge,

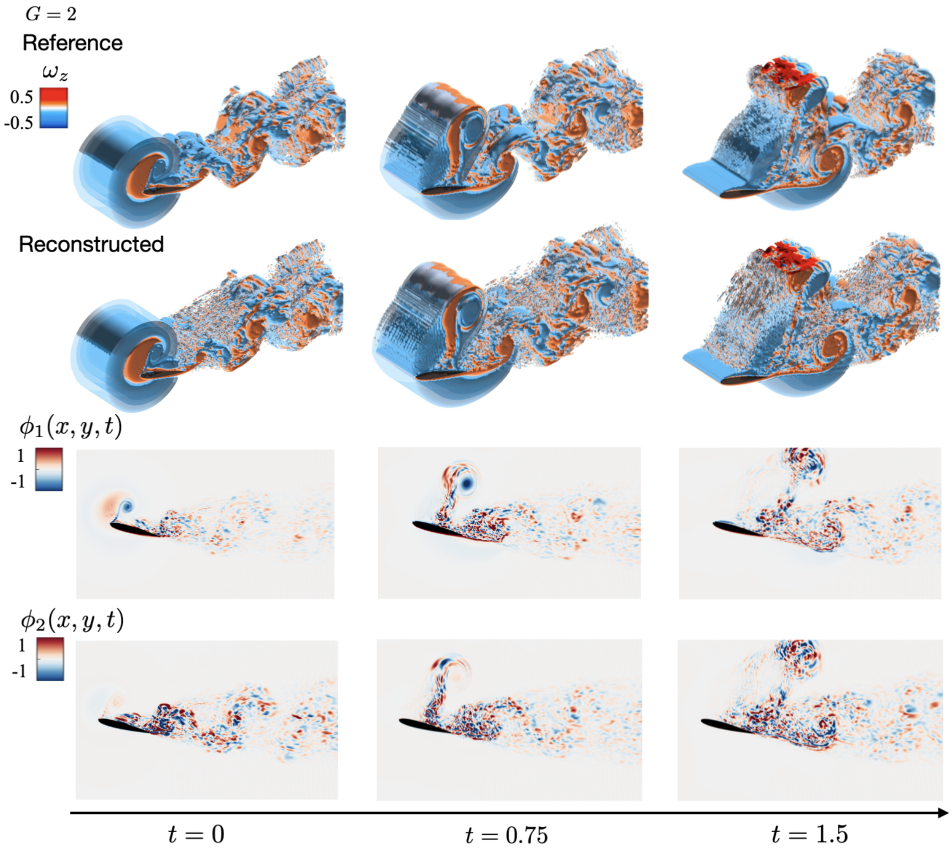

Figure 2. Vorticity fields, time-dependent bases reconstructions, and the corresponding two leading in-plane modes for gust intensity $G = 2$.

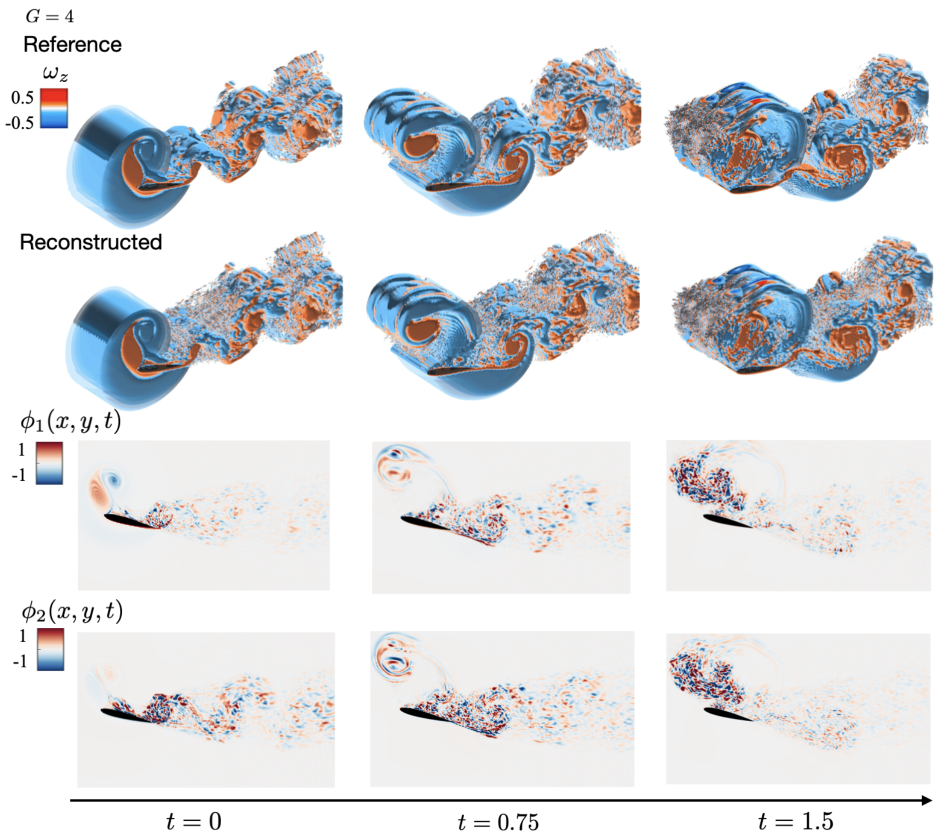

Figure 3. Vorticity fields, time-dependent bases reconstructions, and the corresponding two leading in-plane modes for gust intensity $G = 4$.

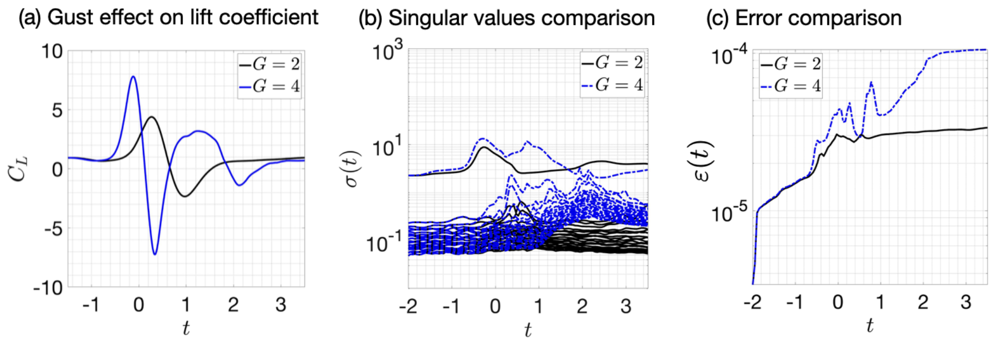

Figure 4. (a) Gust impact on lift, $C_L(t)$. (b) The evolution of singular values $\sigma(t)$ for $G = 2$ and $G = 4$. (c) Reconstruction error, $\varepsilon(t) = \|\mathbf{V}(t) - \widehat{\mathbf{V}}(t)\|_2/\ (N_x N_y N_z)$.

where it interacts with the wake region. The time-dependent bases reconstruction shows good agreement with the reference field. At $t = 1.5$, minor noise is observed above the upper surface, due to the error growth over time. To further characterize the dominant flow dynamics, the first two leading in-plane modes are also shown in figure 2. The first mode captures the vortex structure from impingement through its downstream convection toward the trailing edge. The second mode, however, does not prominently capture the vortex at $t = 0$, but gains energy after impingement, capturing the shear layer roll-up as the interaction intensifies. This suggests that small-scale structures become significant as the flow evolves.

Let us then examine the case with a stronger gust ratio of $G = 4$. As shown in figure 3, increasing the gust intensity strengthens the vortex–wing interaction, resulting in a more pronounced leading-edge roll-up followed by reimpingement on the wing surface. The vorticity field and its time-dependent bases reconstruction remain in agreement. The first mode captures the dominant leading-edge vortex structure as it reimpinges and convects downstream toward the trailing edge, while the second mode gains energy after impingement to represent the finer-scale features associated with vortex roll-up and reimpingement on the wing surface.

The stronger gust case of $G = 4$ shows earlier modal growth and a slower decay, whereas the milder case of $G = 2$ remains more compact and returns to its baseline state more quickly. This behavior is consistent with the lift-coefficient histories shown in figure 4(a), where both cases exhibit a sharp overshoot at impact. Compared with $G = 2$, the case with $G = 4$ reaches a higher peak earlier and then decays more slowly. Consistently, figure 4(b) shows that the singular values for $G = 4$ remain elevated for longer after impact, whereas those for $G = 2$ decay more rapidly, indicating more persistent post-impact dynamics in the stronger gust case. Moreover, for $G = 4$, the gap between the leading singular value and the lower-ranked singular values becomes smaller after impact, indicating that the smaller scale structures gain more energy than in the $G = 2$ case. The reconstruction error in figure 4(c) rises more strongly after impact for $G = 4$ than for $G = 2$. This indicates that the stronger gust produces additional flow complexity that is not fully represented by the chosen rank, and therefore a larger number of modes, or equivalently more singular values, is required to capture the post-impact energy accurately. This observation agrees with the findings of the previous study (Fukami *et al.*, 2025), reporting that the interaction for $|G| \geq 4$ becomes dynamically more violent and gives rise to turbulent fine-scale structures in the flow. Overall, the present rank captures the timing and dominant structures well for $G = 2$, and performs sufficiently for $G = 4$. Nevertheless, for the stronger gust case, a modest increase in rank, or an adaptive-rank approach as proposed in Zamani Ashtiani *et al.* (2022), would reduce the post-impact reconstruction error further.

CONCLUDING REMARKS

We performed modal analysis of vortex–gust airfoil interaction using time-dependent bases on two cases, $G = 2$ and $G = 4$. Increasing gust intensity alters the modal behavior, as non-dominant modes gain energy to resolve finer structure, and their decay is slower than in the milder gust. This aligns with the general trend that stronger gusts introduce more complex flow and slower convergence to the undisturbed quasi-periodic dynamics.

In summary, the present time-dependent bases framework provides a time-dependent modal description of extreme vortex–gust interactions while maintaining flow reconstruction through strongly transient dynamics. For both gust strengths, the modes capture the evolution of the dominant vortex structures. As the gust intensity increases to $G = 4$, the modal analysis reveals a more persistent and complex post-impact state, characterized by slower decay, reduced gap between leading and lower-ranked singular values, and increased contribution from smaller-scale structures. The results demonstrate that time-dependent bases is a useful framework for analyzing unsteady aerodynamic flows with rapidly evolving multiscale features. Further details are referred to Zamani Ashtiani & Fukami (2025).

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