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ENERGY CASCADE BASED ON COHERENT VORTICES

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ABSTRACT

Energy cascade is important for understanding the fundamental nature of turbulence and for engineering applications such as turbulence modelings. In the present study, we bridge wavenumber- and real-space views of the energy cascade using DNS of forced homogeneous isotropic turbulence. In wavenumber space, with logarithmic shells, the shell-to-shell transfer rate peaks at the adjacent lower-wavenumber shell, independent of the wavenumber and bandwidth. In real space, the lifetime of vortices scale-decomposed with Fourier band-pass filter ($k/\alpha \leq |\boldsymbol{k}| < k$) drops sharply for $\alpha \lesssim 3$, and plateaus for $\alpha \gtrsim 3$. Thus, $\alpha \approx 3$ suffices to capture coherent vortices with lifetimes consistent with Kolmogorov scaling, and vortices draw energy from the structures about three times larger. These results guide bandwidth choice for scale-resolved analyses and models.

BACKGROUND

The kinetic energy of turbulent flow is transferred from large to small scales. In previous studies, various analyses have been conducted to clarify the details of this energy transfer (the so-called energy cascade).

For example, analyses of energy transfer between different wavenumbers have shown that energy exchange predominantly occurs between wavenumbers of similar magnitude (Domaradzki & Rogallo, 1990; Verma *et al.*, 2005). However, the definition of energy exchange between shells (spherical shells centered on the origin in wavenumber space) is not unified, and the flow field used in the analysis often lacked sufficient resolution to accurately assess self-similar behavior.

On the other hand, attempts to explain energy cascade as a phenomenon in real space have also achieved some success. For example, it has been shown that energy cascade can be understood in terms of the dynamics of hierarchical vortex structures (Goto *et al.*, 2017). In this model, antiparallel vortex pairs stretch to form smaller antiparallel vortex pairs, and this process continues, resulting in a hierarchical transfer of energy from larger to smaller vortices. Nevertheless, a clear linkage between the real-space physical picture and wavenumber-space analyses remains incomplete. Therefore, integrating discussions in both wavenumber space and real space to gain a deeper understanding of the energy cascade remains an urgent task.

DIRECT NUMERICAL SIMULATION

In the present study, we employ two sets of direct numerical simulation (DNS) data of forced homogeneous isotropic turbulence in a periodic box. These two data sets are used for complementary purposes: the wavenumber- and the real-space analysis. The simulations differ in spatial resolution, Reynolds number, and forcing method, as summarized below.

For the wavenumber space analysis, we use single-time snapshot data of high-resolution DNS provided by Ishihara *et al.* (2016). These simulations are performed with a Fourier spectral method in a cubic domain of size 2π, employing 8192^3 grid points. The Taylor-scale Reynolds number of the flow is approximately $\mathrm{Re}_\lambda \approx 1747$, and the resolution parameter satisfies $k_{\max}\eta \approx 1$ where η is the Kolmogorov length scale. The flow is maintained statistically stationary by a kind of forcing (a negative viscosity) acting on modes with wavenumber magnitude $|\boldsymbol{k}| < 2.5$. Details of the numerical procedure and the resulting turbulence statistics are described in Ishihara *et al.* (2016).

For the real space analysis, we perform our own DNS of forced homogeneous isotropic turbulence using the same Fourier spectral method in a periodic cubic domain of size 2π. In this case, the resolution is 1024^3, corresponding to a Taylor-scale Reynolds number of $\mathrm{Re}_\lambda \approx 350$. The small-scale resolution satisfies $k_{\max}\eta \approx 1.5$. The flow is sustained by an energy-injection scheme (Lamorgese *et al.*, 2005) that modulates the forcing amplitude to maintain a constant energy input rate, again restricted to modes with wavenumber magnitude $|\boldsymbol{k}| < 2.5$. This data set is specifically used to examine the energy transfer in terms of the dynamics of coherent vortices in real space.

ANALYSIS IN WAVENUMBER SPACE

To clarify the scale locality (how much energy is transferred to each scale) and self-similarity of energy cascade, we investigated the amount of energy transfer between the shells following the definition of shell-to-shell energy transfer rate

$$\mathcal{T}(k|p) \equiv \sum_{k\le|\boldsymbol{k}|<\alpha k, p\le|\boldsymbol{p}|<\alpha p} \mathrm{Im}(k_j \widehat{u}_i^*(k)\widehat{u}_i(p)\widehat{u}_j(k-p))/(\log_2\alpha)^2 \tag{1}$$

(Verma *et al.*, 2005). This is energy transfer rate from a shell with wavenumber p to a shell with wavenumber k. Here, it should be noted that the width of the shell is proportional to the wavenumber, and the scale factor α determines the proportionality constant. The importance of the logarithmic bands is discussed in Aluie & Eyink (2009). This definition is derived by simplifying triad (three wavenumbers) interaction term, which originates from the nonlinear term in the energy equation in wavenumber space, into an energy exchange process between two wavenumbers.

First, figure 1 (a) shows the plot of $\mathcal{T}(k|p)$ when the scale factor α is set to a very small value ($\alpha = \sqrt[16]{2}$). This result shows that, regardless of the wavenumber, most energy is transferred from the immediately adjacent (lower wavenumber) shell, indicating that energy transfer occurs scale-locally and self-similarly. Next, figure 1 (b) shows the result when the wavenumber of the shell receiving energy is fixed ($k = 32$), while the scaling factor α is varied. This result indicates that, regardless of the value of α, the energy is received from the immediately adjacent shell; in other words, it means that the energy is received from a scale that is α times larger, and scale locality can be said to depend on the scale factor α.

ANALYSIS IN REAL SPACE

In the previous section, we have shown that the bandwidth for scale decomposition is crucial in the energy cascade analysis. Therefore, in this section, we identify the appropriate bandwidth (scale factor α) for decomposing turbulent flow based on the characteristics of coherent vortices in real space. The lifetime of a vortex characterized by a specific wavenumber k can be estimated as $\tau_k \equiv (\varepsilon k^2)^{-1/3}$ through the dimensional analysis based on Kolmogorov's theory, where ε represents the average dissipation rate. Accordingly, we determine the appropriate scale factor α by comparing the vortex lifetime obtained through scale decomposition using various scale factors with the value of τ_k.

The lifetime of vortices is evaluated using the Lagrangian time correlation function of vorticity. Let $\boldsymbol{x}_\mathrm{L}(t|\boldsymbol{x}_0,t_0)$ be the trajectory initialized at $\boldsymbol{x}_\mathrm{L}(t_0) = \boldsymbol{x}_0$ and advected by the flow

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{x}_\mathrm{L}(t|\boldsymbol{x}_0,t_0) = \boldsymbol{u}(\boldsymbol{x}_\mathrm{L}(t|\boldsymbol{x}_0,t_0),t). \tag{2}$$

Vorticity is decomposed into different scales using a Fourier bandpass filter that retains only Fourier modes within a spherical shell $(k/\alpha \le |\boldsymbol{k}| < k)$. Assuming statistical homogene-

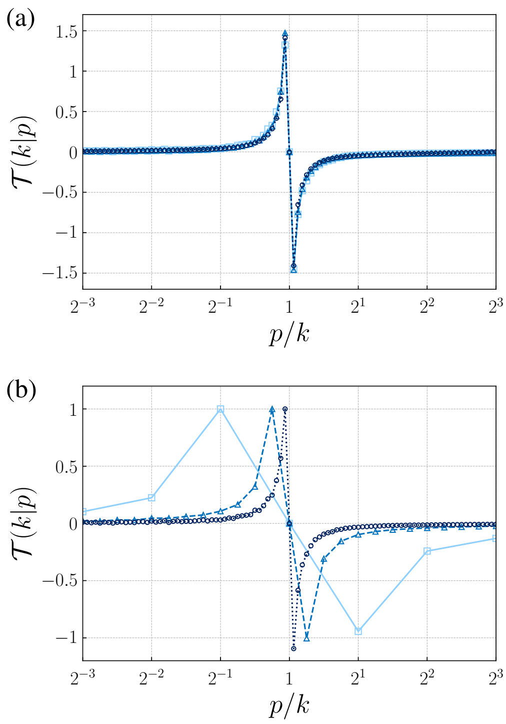

Figure 1: (a) Energy transfer rate $\mathcal{T}(k|p)$ evaluated with $\alpha = \sqrt[16]{2}$. The three lines represent $k = 32$ ($k\eta \approx 0.0083$, circles), 64 ($k\eta \approx 0.017$, triangles), and 128 ($k\eta \approx 0.033$, squares). (b) Energy transfer rate $\mathcal{T}(k|p)$ with $k = 32$ normalized by its maximum value. The three lines represent $\alpha = 2$ (circles), $\sqrt[4]{2}$ (triangles), and $\sqrt[16]{2}$ (squares).

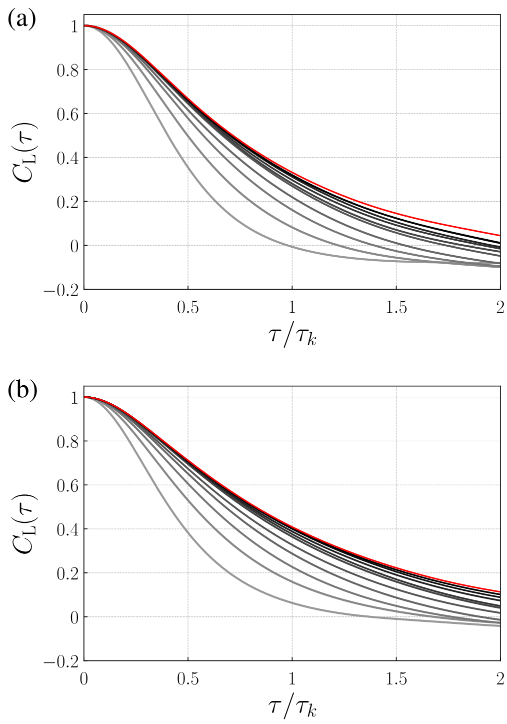

Figure 2: The Lagrangian time-correlation function $C_\mathrm{L}(\tau)$ of vorticity scale-decomposed at (a) $k = 16$ ($k\eta \approx 0.050$) and (b) 32 ($k\eta \approx 0.099$). Darker grayscale indicates larger scale factor. Red curves indicate the Lagrangian time-correlation function of vorticity low-pass filtered at the corresponding cutoff wavenumber k. Here, τ is normalized by the characteristic timescale τ_k.

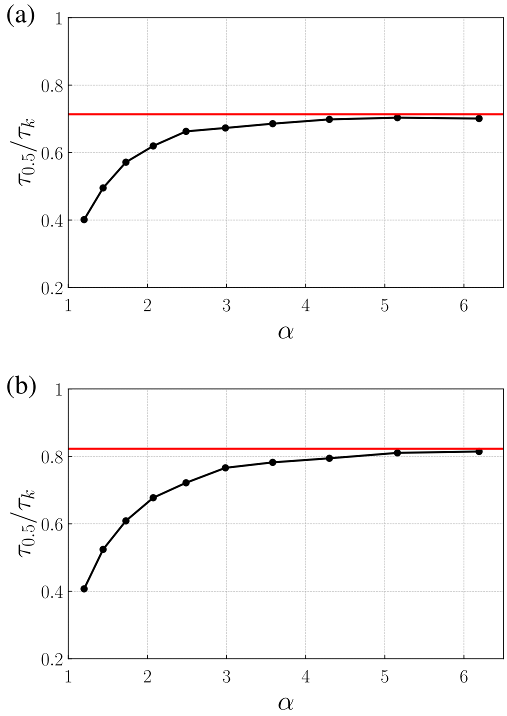

Figure 3: Scale-factor dependence of the correlation half-time for the Lagrangian time-correlation function of the vorticity scale-decomposed at (a) $k = 16$ ($k\eta \approx 0.050$) and (b) 32 ($k\eta \approx 0.099$). The red lines indicate the results for vorticity low-pass filtered at the corresponding cutoff wavenumber k. Here, $\tau_{0.5}$ is normalized by the characteristic timescale τ_k.

ity and isotropy (therefore the mean is zero), the Lagrangian time correlation function of the x-component of the scale-decomposed vorticity is defined as

$$C_{\mathrm{L}}(\tau) \equiv \frac{\left\langle \widetilde{\omega}^{k}_{\mathrm{L},x}(t|\boldsymbol{x}_0,t_0)\,\widetilde{\omega}^{k}_{\mathrm{L},x}(t+\tau|\boldsymbol{x}_0,t_0)\right\rangle}{\left\langle \left(\widetilde{\omega}^{k}_{\mathrm{L},x}(t|\boldsymbol{x}_0,t_0)\right)^2 \right\rangle}, \qquad (3)$$

where $\langle\cdot\rangle$ denotes statistical average.

Figure 2 shows $C_{\mathrm{L}}(\tau)$ for wavenumber $k = 16$ and 32, with varying scale factor α. The horizontal axis is normalized by the characteristic time τ_k derived by the dimensional analysis. This figure shows that, for both k, increasing α leads to a longer correlation time of the scale-decomposed vortices, and $C_{\mathrm{L}}(\tau)$ approaches the correlation of a low-pass filtered vorticity field as $\alpha \to \infty$ (red curves).

Furthermore, to quantify the relationship between bandwidth and vortex lifetime, we define the correlation half-life $\tau_{0.5}$ as the time at which the correlation decays to 0.5 ($C_{\mathrm{L}}(\tau_{0.5}) = 0.5$). Figure 3 plots $\tau_{0.5}$ versus α. These results show that the correlation time decreases sharply for $\alpha \lesssim 3$ and then approaches a plateau for larger α. This implies that when the shell is too narrow (small α), the coherent vortex structure consistent with Kolmogorov scaling cannot be captured. In other words, a finite bandwidth with $\alpha \approx 3$ is necessary to analyze turbulence based on the coherent vortices.

Combining these findings with the wavenumber-space results suggests that turbulent vortices predominantly receive energy from vortices roughly three times larger than themselves.

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