DRAG REDUCTION THROUGH REDUCED VORTEX STRETCHING IN SHEAR FLOW

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ABSTRACT

In recent work we extended von Kármán's logarithmic velocity profile to turbulence with reduced vortex stretching and validated the resulting prediction using numerical simulations of channel flow. This generalized framework is based on two key assumptions: (i) the presence of an inertial layer where viscous effects can be neglected, and (ii) the applicability of a mixing-length model for turbulent momentum and vorticity transport. In this work, employing Direct Numerical Simulations, we test these assumptions and analyze the mean-velocity profile in both turbulent plane Couette and channel flows.

1 DRAG REDUCTION BY POLYMERS AND TURBULENCE WITH REDUCED VORTEX STRETCHING

Adding small amounts of polymer to a liquid flow can reduce drag in pipes and channels by up to 70% Toms (1949), a phenomenon known for 75 years but still not fully understood. Two key features of polymer-induced drag reduction are (1) a modified mean-velocity profile and (2) an asymptotic maximum drag reduction (MDR) state (Virk, 1975).

A novel theoretical angle to examine system was recently proposed (Bos *et al.*, 2025). The idea is to bypass the modeling of the polymer dynamics and to directly take into account their influence on the vorticity dynamics. The new approach thereby heavily relies on the theoretical considerations developed in the framework of turbulence without vortex stretching (Bos, 2021; Wu & Bos, 2021).

The system we consider is the following,

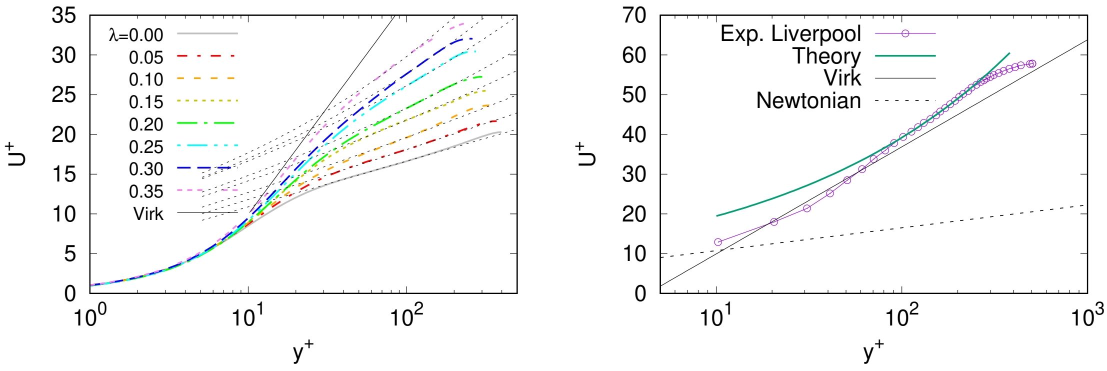
$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{\omega} = (1-\lambda)\boldsymbol{\omega} \cdot \nabla \boldsymbol{u} - \lambda \nabla P_\omega + \nu \Delta \boldsymbol{\omega} \quad (1)$$

where the vorticity $\boldsymbol{\omega}$ is the curl of the velocity $\boldsymbol{u}$, ν the kinematic viscosity and P_ω a Lagrange multiplier, ensuring $\nabla \cdot \boldsymbol{\omega} = 0$. This formulation allows to consider, for $\lambda = 0$ the classical Navier-Stokes vorticity equation, and for $0 < \lambda \leq 1$ a system with reduced vortex stretching. The case $\lambda = 1$ was introduced in (Bos, 2021; Wu & Bos, 2021) and studied in statistically homogeneous turbulence, where it was shown that the absence of vortex stretching allows both enstrophy and helicity to be conserved (Wu & Bos, 2022; Wu *et al.*, 2023).

Let us now turn to inhomogeneous flow. For the special case of parallel shear flow, such as plane channel or plane Couette flow, we can derive in a statistically steady state, from Eq. (1), the exact mean vorticity balance,

$$\frac{\partial \langle \omega_z \rangle}{\partial t} = 0 = \lambda \frac{d\langle \omega_z' v' \rangle}{dy} + (\lambda - 1)\frac{d^2 \langle u'v' \rangle}{dy^2} + \nu \frac{d^3 U}{dy^3}. \quad (2)$$

Following von Kármán von Kármán (1930), using a classical mixing length assumption for both unclosed quadratic terms



(a) (b)

Figure 1. (a) Modified velocity profile in turbulent channel flow with reduced vortex stretching (in wall units). λ indicates the reduction of vortex stretching imposed on the system. Dotted lines are theoretical prediction. (b) Comparison of the theory with an experimentally measured velocity profile of visco-elastic turbulence in a pipe.

$\langle \omega_z' v' \rangle$ and $\langle u' v' \rangle$, allows to derive for the mean velocity profile (see details in Bos *et al.* (2025)),

$$U^+(y^+) = \frac{1}{\kappa}\frac{{y^+}^\Gamma - 1}{\Gamma} + U_0^+, \tag{3}$$

where all quantities are expressed in wall units. The exponent Γ is given by

$$\Gamma = \frac{\lambda}{2Pr_T - \lambda(2Pr_T - 1)}. \tag{4}$$

where the turbulent Prandtl number measures the ratio of the turbulent diffusivity of velocity and vorticity,

$$Pr_T \equiv \frac{\nu_T^{(u)}}{\nu_T^{(\omega)}}. \tag{5}$$

It was shown in Bos *et al.* (2025) that in channel flow a unity value of Pr_T, so that $\Gamma = \frac{\lambda}{2-\lambda}$, allowed to well describe the results for the mean-velocity profile in simulations of Eq. (1) (see Fig. 1(a)), but also of experimental results of polymer-laden turbulent pipe flow (Owolabi *et al.*, 2017) (Fig. 1(b)).

2 NUMERICAL RESULTS FOR COUETTE FLOW

We use direct numerical simulation (DNS) to examine incompressible Couette flow between two parallel plates. Boundary conditions in the streamwise and spanwise directions are periodic. No-slip and no-penetration boundary conditions are applied at the walls.

In Fig. 2 we show visualizations of instantaneous values of spanwise vorticity in Couette flow at 4 different time-instants (A,B,C,D) for values of the reduction parameter $\lambda = 0; 0.1; 0.2; 0.3$. Clearly, the enstrophy diminishes when λ increases.

In Fig. 3(a) we show time-series of the wall shear-stress in Couette flow for different values of λ. The simulations with $\lambda = 0.5$ and $\lambda = 1$ relaminarize. For the other runs, after a drop from the initial turbulent profile, the system reaches a statistically steady state at a level which decreases with increasing value of λ.

In Fig. 3(b) we show cross-stream profiles of the turbulent Prandtl number. It is observed that, while plateaus are not observed for the larger values of λ, all curves converge around a value of $Pr_T \approx 2$ around $y^+ = 80$.

We used therefore the value $Pr_T = 2$ to determine Γ, using expression (4). In Fig. 4(a) we confirm the validity of the modified law of the wall (Eq. (3)) in turbulent Couette flow with reduced vortex-stretching, where U_0 was adapted to give an optimal fit in the center of the domain. Subsequently, in Fig. 4(b) we determine the amount of drag-reduction obtained by reducing the vortex stretching as a function of λ. It is observed that determining the drag-reduction assuming the velocity profile to be given by Eq. (3) allows to well predict the values observed in the simulation.

3 RELATION WITH THE MAXIMUM DRAG REDUCTION REGIME

In our simulations of both Channel and Couette flow, we observe that for $\lambda \gtrapprox 0.35$ the flow relaminarizes. We expect this to correspond to the Maximum Drag Reduction (MDR) state. Indeed the limiting condition with minimal vortex stretching required to sustain wall turbulence should determine the MDR asymptote. In our system the flow does not remain turbulent beyond this value, in contrast to experimental observation. This is due to the fact that in our system λ is imposed, so that it remains fixed in the laminar state, while polymer molecules would relax back to their equilibrium in a relaminarized (hibernating) state, lowering the value of λ locally, and retriggering transition to turbulence.

4 CONCLUSION

Reducing the vortex-stretching of the Navier-Stokes equations, allows to reduce drag. The key question is how to relate this to the physics of a realistic flow. Comparison with polymer-laden flow shows a clear agreement for the mean-velocity profile (Fig. 1(b)). For this system the agreement can be intuitively understood by the tendency of polymer molecules to align with the vorticity vector (ur Rehman *et al.* (2022)) and the property of polymers to increase the extensional viscosity. The MDR regime is also explained by com-

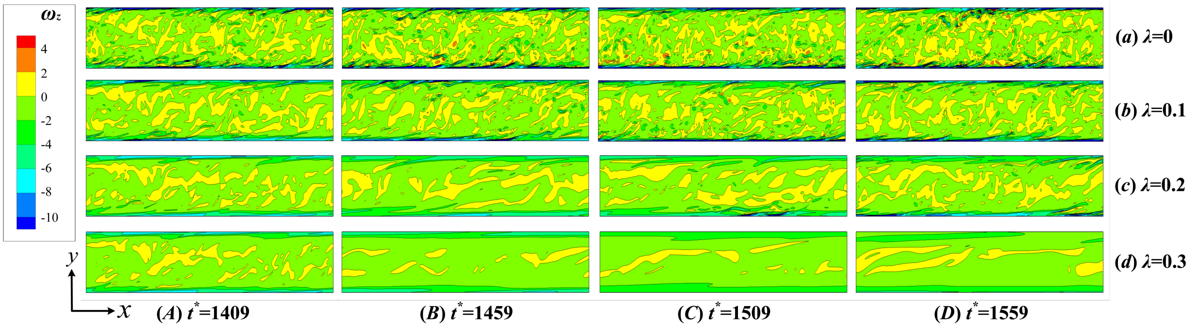


Figure 2. Visualization of spanwise vorticity in Couette flow at 4 different time-instants (A,B,C,D) for values of the reduction parameter $\lambda = 0; 0.1; 0.2; 0.3$ (from top to bottom).

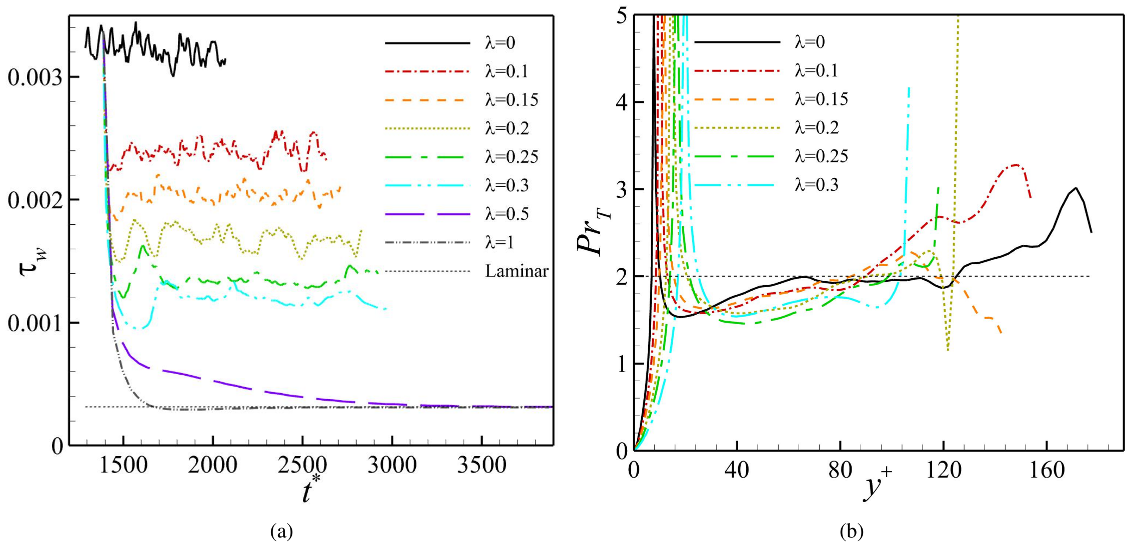


Figure 3. (a) Time-series of the wall shear-stress in Couette flow for different values of λ. (b) Cross-stream profiles of the turbulent Prandtl number.

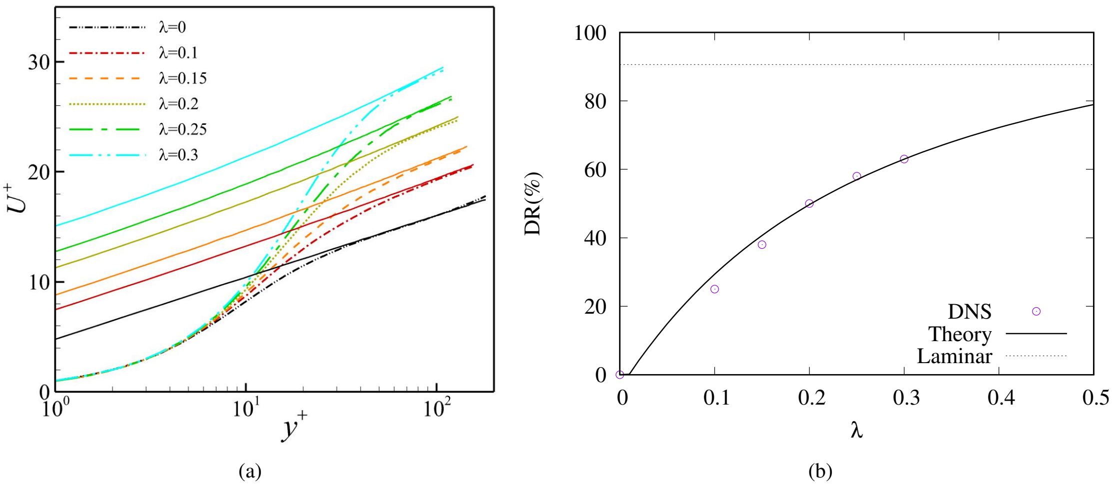


Figure 4. (a) Modified velocity profile in turbulent Couette flow with reduced vortex stretching (in wall units). λ indicates the reduction of vortex stretching imposed on the system. Dotted lines are theoretical prediction. (b) Determination of the amount of drag-reduction (DR).

paring the systems. To have a marginally stable turbulent state that does not relaminarize for $\lambda > 0.35$ elastic effect should most probably be taken into account (Dubief *et al.* (2023)), since it is the elasticity which allows to relax the polymers when local relaminarization takes place.

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