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INTERACTION BETWEEN QUANTUM TURBULENCE AND NORMAL-FLUID TURBULENCE IN SUPERFLUID HELIUM

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ABSTRACT

Turbulence phenomena caused by the interaction between superfluid and normal-fluid in superfluid helium are investigated by using two-way coupled numerical simulation via mutual friction. In homogeneous isotropic turbulence, quantized vortices winded by the normal-fluid vortex tubes are accumulated inside and around the vortex tubes, which corresponds to quasi-classical turbulence. In contrast, in a thermal counterflow, two large normal-fluid tornadoes with dense quantized vortices are induced in the streamwise direction, which corresponds to ultra-quantum turbulence. Those interactions between quantum turbulence and normal-fluid turbulence are revealed.

INTRODUCTION

Superfluid helium below 2.17 K is composed of inviscid superfluid and viscous normal-fluid. The two fluids interact via mutual friction. As decreasing the temperature, the ratio of the superfluid increases whereas the ratio of the normal-fluid decreases. This concept is the so-called two-fluid model.

A quantized vortex, namely, a vortex line where a circulation of velocity is quantized, exists in the superfluid. Superfluid exists as quantized vortices and those tangles are called quantum turbulence. In experiments, homogeneous isotropic turbulence of superfluid helium is produced downstream behind the fluctuating grid. The energy spectrum of superfluid velocity is observed as $-5/3$ power of wave number. his is the so-called quasi-classical turbulence.

Another experimental setup is the so-called thermal counterflow (Gao et al., 2017). As shown in Fig. 1, two baths filled with superfluid helium are connected with a duct. When the left surface of the left bath is heated by the heater, the normal-fluid moves to the right bath, and the superfluid moves to the left bath to conserve the mass. As increasing the heat flux of the left bath, the relative velocity between superfluid and normal-fluid increases. The quantized vortex is stretched and becomes tangled so that the superfluid transits to quantum turbulence in more than the critical relative velocity. As increasing the heat flux further, normal-fluid transits to turbulence as well. The turbulence in less than mean vortex line spacing is called ultra-quantum turbulence.

In this study, those interactions between quantum turbulence and normal-fluid turbulence are numerically examined

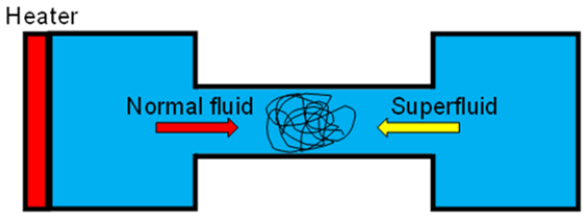

Figure 1. Schematic of thermal counterflow.

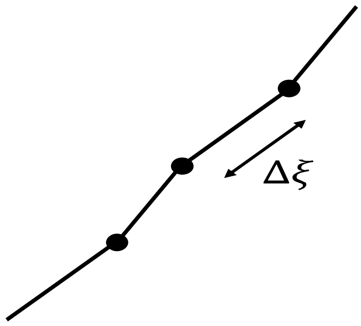

Figure 2. Discretized vortex filament.

to reveal the dynamics.

NUMERICAL METHOD

The quantized vortex is divided into finite vortex filaments (Schwarz, 1985; 1988) as shown in Fig. 2. The induced velocity is obtained by integrating the induced velocities generated from all of the other filaments via the Biot-Savart law (Adachi et al., 2010; Yui et al., 2022). The integration is a many-body problem of O(N2) where N denotes the number of filaments. To speed up the calculation of quantum vortices, the fast multipole method (FMM) (Yokota et al., 2007; Yokota et al., 2009) is used to be O(N), and about 100 times speed-up is achieved. The position of the filament is calculated using the 4th-order Runge-Kutta method.

As for the normal-fluid, Navier-Stokes equations with the mutual friction force which is the interaction force between two fluids are solved with the 2nd order FDM (Yui et al., 2018; Yui et al., 2020). A periodic box of D = 1mm cubic is used and the grid points are set to 120^3. In this study, two types of simulations are performed: one is the flow with the following external force (Goto et al., 2017) at 1.9 K:

$$\boldsymbol{f} = (-0.25\sin\frac{2\pi x}{D}\cos\frac{2\pi y}{D}, 0.25\cos\frac{2\pi x}{D}\sin\frac{2\pi y}{D}, 0). \quad (1)$$

The other is the thermal counterflow with the normal-fluid velocity of 9 mm/s and the external force of ABC flow of 1:-1:5 at 1.9 K for wavenumber k = 1 and 2.

RESULTS

Figure 3 shows the snapshots of quantum vortices (black line) and vortex tubes (green) in turbulence driven by Eq. (1). The four large vortex tubes of normal-fluid is generated by the force of Eq. (1) and the smaller vortex tubes are produced perpendicular to the four large vortex tubes periodically. The large four vortex tubes wind up and stretch the quantized vortices so

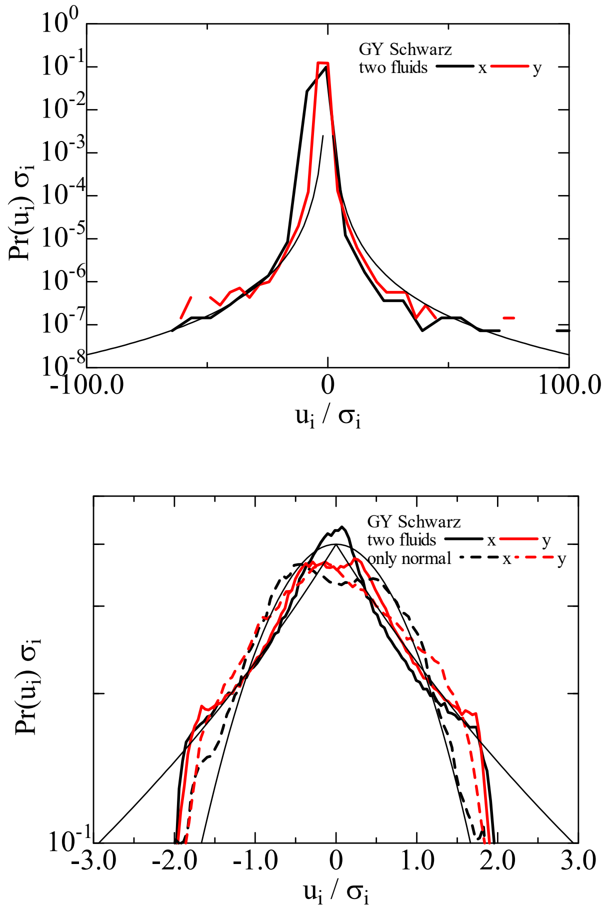

Figure 4. Pdfs of velocity of superfluid (top) and normal-fluid (bottom) in turbulence driven by Eq. (1).

that the superfluid transits to quantum turbulence. The quantized vortices concentrate inside and around the vortex tubes. In experiments (Gao et al., 2017), it is known that quantum turbulence has an energy spectrum with a $-5/3$ power law. These bundles of quantized vortices generated by the normal-fluid turbulence may contribute to the power law.

The pdfs of superfluid and normal-fluid velocities for Fig. 3 are shown in Fig. 4. The pdfs of superfluid velocity have a -3 power tail distribution (Paoletti et al., 2008). The velocity pdfs of only the normal-fluid yield the Gaussian distribution. However, the normal-fluid velocity pdfs in two fluids give the Gaussian distribution around the center and -3 power tail distribution owing to the interaction. All pdfs are symmetrical in the x and y directions.

Figure 5 shows the snapshots of quantum vortices (black line) and vortex tubes (green) of normal-fluid in the flow direction of a counterflow. The vortex tubes become fine when compared with the case of no quantized vortices because the fine vortex tubes are produced as the wake generated from the quantized vortex. Interestingly, two large tornado vortex cells are generated around the upper-left and lower-right in the streamwise direction. These large vortex cell structures are never seen without the quantized vortices.

The pdfs of superfluid and normal-fluid velocities for Fig. 5 are shown in Fig. 6. The pdfs of superfluid velocity have -3 power tail distribution and are symmetrical in the x and y directions. The normal-fluid velocity pdf in the x direction shows the Gaussian distribution around the center and negatively skewed -3 power tail owing to the interaction with the superfluid. The double-peak pdf emerges in the y direction

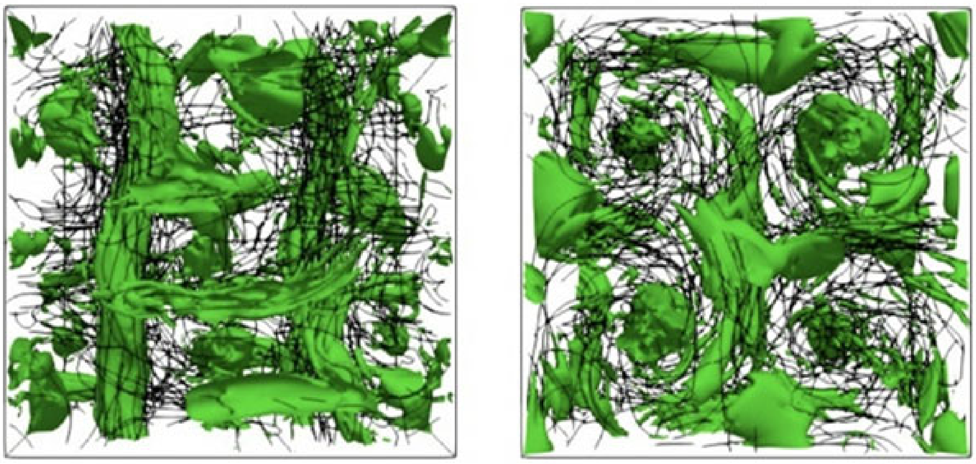

Figure 3. Snapshots of quantum vortices (black line) and vortex tubes (green) in turbulence driven by Eq. (1) (left: side view, right: top view).

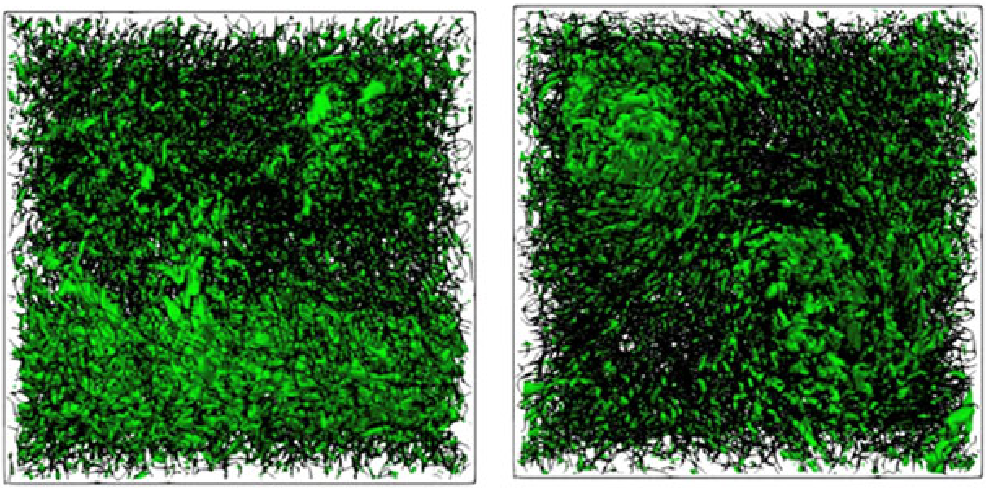

Figure 5. Snapshots of quantum vortices (black line) and vortex tubes (green) in thermal counterflow (left: side view, right: front view).

owing to the two large tornadoes.

SUMMARY

Turbulence phenomena in superfluid helium were investigated by considering the interaction between superfluid and normal-fluid. Two-way coupled numerical simulation was carried out via mutual friction. The major conclusions are as follows.

In the quasi-classical turbulence obtained by forced homogeneous isotropic turbulence, quantized vortices winded by the normal-fluid vortex tubes are accumulated inside and around the vortex tubes. The pdf of normal-fluid velocity is affected by the superfluid velocity whose pdf has a -3 power tail. In ultra-quantum turbulence produced in a thermal counterflow, two large normal-fluid tornadoes with dense quantized vortices are induced in the streamwise direction, and small-scale vortex tubes are generated by the quantized vortices. The pdf of normal-fluid velocity depends on the flow direction and transverse direction.

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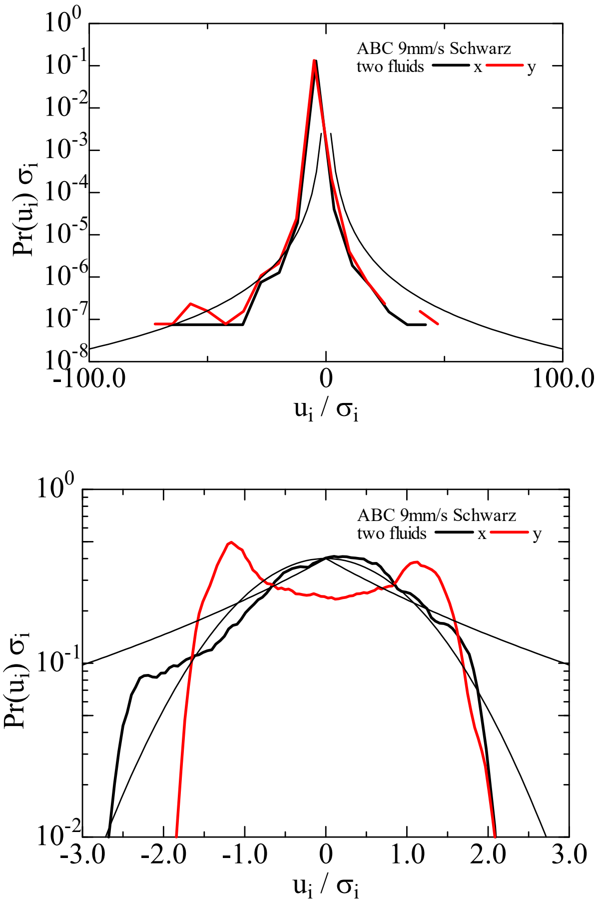

Figure 6. Pdfs of velocity of superfluid (top) and normal-fluid (bottom) in thermal counterflow.

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