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A LOCAL-INTERMITTENCY-BASED REYNOLDS-AVERAGED TRANSITION MODEL FOR TURBULENT MIXING INDUCED BY INTERFACIAL INSTABILITIES

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ABSTRACT

The evolutionary process of mixing induced by Rayleigh-Taylor (RT) and Richtmyer-Meshkov (RM) instabilities typically unfolds in three stages: initial growth of instability, subsequent mixing transition, and final turbulent mixing. Accurately predicting this entire process is crucial for both scientific research and engineering applications. For engineering applications, the Reynolds-averaged Navier-Stokes (RANS) simulation is currently the most viable method. However, existing RANS mixing models are primarily tailored for the fully-developed turbulent mixing stage, and fail to predict the non-equilibrium mixing transition stage. To achieve engineering predictions of mixing transition process, this study proposes a RANS mixing transition model for the first time. Specifically, we extend the intermittency factor (denoted by γ), which is widely used in aerospace engineering for modeling boundary layer transition flows, to mixing problems. Firstly, a γ transport equation is established based on local flow variables to reflect local variations of the flow state. Secondly, the intermittency factor γ is coupled into the well-established K-L turbulence mixing model to constrain the Reynolds stresses in the turbulence model, making it consistent with the flow characteristics of the mixing transition. Lastly, we validate the effectiveness and generalization capability of the proposed model through various test cases. Results demonstrate that the present model not only accurately captures the typical transition processes in RT/RM mixing transition flows, but also predicts the RT/RM turbulent mixing flows well.

INTRODUCTION

When a heavy fluid is accelerated by a light fluid, the Rayleigh-Taylor instability (RTI) arises at the interface between two fluids with different densities. If this acceleration is impulsive, irrespective of its direction, the Richtmyer-Meshkov instability (RMI) manifests. Subsequently, these instabilities rapidly transition to turbulence, with stimulation of the Kelvin-Helmholtz instability, which is induced by shear velocity difference. In practical flows, such as supernova explosion and inertial confinement fusion (ICF), these instabilities frequently interact simultaneously, enhancing the mixing of distinct materials. This mixing process plays an important role on natural phenomena and engineering applications. Therefore, accurately predicting them is of great significance.

At present, the predictions of mixing evolution predominantly relies on numerical simulations, utilizing different methodologies including direct numerical simulation (DNS), large eddy simulation (LES), and Reynolds-averaged Navier-Stokes simulation (RANS). However, DNS and LES are not yet feasible for immediate engineering applications due to their substantial computational demands, particularly at high Reynolds numbers, and the extended duration required to simulate engineering flows. As a result, RANS is currently the most viable method for engineering predictions, due to its low computational cost and acceptable predictive accuracy.

The unsteady mixing evolution typically progresses through three stages: the initial development of instability, subsequent mixing transition and last turbulent mixing. During the initial stage, hydrodynamic instabilities amplify interfacial perturbations. The vorticity entrains the irrotational pure fluid around the interface into a mixing layer, which is characterized by mushroom-shaped bubble and spike structures. At this stage, the growth of instability is linear or weakly nonlinear, attributed to the relatively minor amplitude of the perturbations. Consequently, accurate predictions can be attained using analytical theoretical models (Liu *et al.*, 2022; Zhang & Guo, 2022), particularly when initial interfacial perturbations are single-mode. For cases involving multiple modes, the independent and linear growth of each perturbed mode means that employing single-mode theoretical models remains viable during this first stage. As the perturbation amplitudes grow to become comparable to the dominant wavelength, the characteristic structures start to disintegrate and nonlinear effects intensify, marking the onset of the mixing transition. In this intermediate stage, the stirring action increases the interfacial area, giving rise to multiscale behavior due to the interaction and competition among the various modes. Accordingly, the single-mode models are not applicable any more. To address that, Rollin & Andrews (2013) synthesize Haan's mode coupling equations (Haan, 1991) and Goncharov's interface evolution model (Goncharov, 2002) to construct a modal model capable of tracking the development of multiple modes. This approach holds promise, but there are some problems. Firstly, Haan (1991) emphasize in their original article that the mode coupling equations are only suitable for linear and weakly nonlinear stages, and fall short during the strongly nonlinear stage. Secondly, existing interface evolution models are predominantly designed to handle single-mode perturbations. Their extension to scenarios involving multimode interfaces with intense intermodal interactions remains questionable in terms of reliability. Beyond transition, the flow eventually reaches the stage of turbulent mixing, which is characterized by significant self-similarity and follows well-defined statistical laws that lend themselves to Reynolds-averaged Navier-Stokes (RANS) modeling. With decades of efforts, a variety of RANS models for turbulent mixing have been developed and reported (Zhang *et al.*, 2020; Kokkinakis *et al.*, 2019).

It is concluded that the most pressing challenge in current predictive endeavors is the modeling of the mixing transition stage, which presents formidable obstacles due to its highly nonlinear dynamics. The mixing transition unfolds as a temporal process characterized by pronounced spatial variations at different locations. These characteristics cannot be accurately described by existing RANS mixing models, which are generally based on the assumption of equilibrium turbulence and tailored for fully developed turbulent flows. As elucidated in the high-fidelity studies conducted by Livescu *et al.* (2009) and Morgan & Greenough (2015), the eddy viscosity closure and gradient diffusion approximation (GDA) used in existing RANS mixing models fall short in describing the mixing transition.

For accurate prediction of transition, two key factors need to be considered. Firstly, it is essential to accurately identify the onset of transition. However, existing transition criteria are constructed using global metrics, such as the outer-scale Reynolds number (Dimotakis, 2000; Zhou *et al.*, 2003) and mixing mass (Wang *et al.*, 2022). These global measures fall short in capturing the local nuances of the transition and do not satisfy the demands for real-time predictions. Secondly, accurately predicting the progression of transition process itself is crucial. While several promising methods have been reported, each approach has its limitations. For instance, in the context of reshocked RM mixing issues, Haines *et al.* (2013) employed an empirical method that relies on controlling the activation of the turbulence model based on the timing of shock waves interacting with the interface. Although this scheme has proven effective in tracking the evolution of the mixing transition, it is heavily reliant on empirical judgment and manual intervention. A more sophisticated and ideal methodology would enable the model to autonomously and accurately capture both the commencement and development of the transition. Recognizing that transition to turbulence is driven by large scale coherent vortical-structure, Grinstein *et al.* (2020) have delved into a dynamic hybrid RANS/LES bridging approach. Similarly, Pereira *et al.* (2021) put their hopes on the hybrid RANS/LES modeling strategy as well, utilizing the partially averaged Navier-Stokes (PANS) simulation scheme to tackle the challenges of RT mixing problems. Despite these innovative efforts, these approaches remain in their nascent stages and are steeped in empirical practices, posing significant hurdles to their application in engineering predictions.

To summarize, a reasonable and practicable engineering prediction scheme for mixing transition flows has not been proposed yet. This is particularly true for RANS models designed to capture mixing transitions, as there is a notable absence of precedents in the literature to guide such efforts. To address that, we draw on a transition modeling idea from other fields. The physical process of mixing evolution tells that, at the early stage of the interface development, the flow at any specific local point across the interface is either laminar or non-turbulent. As perturbations grow, the mixing zone expands outward from the interface, intensifying the mixing and fluctuations. Consequently, regions that were initially laminar or non-turbulent gradually transition to turbulence. This characteristic bears a striking resemblance to the intermittency phenomenon observed in boundary layer flows. In the study of boundary layer, researchers have long utilized an intermittent factor γ, to locally and quantitatively describe this behavior. Specifically, $\gamma = 0$ corresponds to laminar flow, $\gamma = 1$ indicates fully turbulent flow, and intermediate values reflect the transition between these two states. This concept of intermittent factor has seen widespread application in the aerospace industry, where it has been successfully incorporated into RANS modeling of boundary layer transition flows and has shown encouraging results. Motivated by this success, we aim to adapt this well-established concept to address mixing problems, with the goal of developing a dedicated RANS mixing transition model. As will be demonstrated, this novel modeling approach proves to be quite effective.

METHODOLOGY

Governing equations and baseline model

The multicomponent RANS equations are solved. The transport equations for the mean density ρ, velocity u_i, total energy E of the mixture, and mass fraction Y_α of specie α are presented as follows:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_j}{\partial x_j} = 0, \tag{1}$$

$$\frac{\partial \bar{\rho} \tilde{u}_i}{\partial t} + \frac{\partial \bar{\rho} \tilde{u}_i \tilde{u}_j}{\partial x_j} + \frac{\partial \bar{p}}{\partial x_i} - \bar{\rho} g_i = -\frac{\partial \tau_{ij}}{\partial x_j} + \frac{\partial \bar{\sigma}_{ij}}{\partial x_j}, \tag{2}$$

$$\frac{\partial\bar{\rho}\tilde{E}}{\partial t}+\frac{\partial(\bar{\rho}\tilde{E}+\bar{p})\tilde{u}_j}{\partial x_j}-\bar{\rho}\tilde{u}_i g_i=D_E+D_K+\frac{\partial\psi}{\partial x_j}, \quad (3)$$

$$\frac{\partial\bar{\rho}\tilde{Y}_\alpha}{\partial t}+\frac{\partial\bar{\rho}\tilde{Y}_\alpha\tilde{u}_j}{\partial x_j}=\frac{\partial}{\partial x_j}\left(-\bar{\rho}\widetilde{u_i''Y_\alpha''}+\bar{\rho}\bar{D}\frac{\partial\tilde{Y}_\alpha}{\partial x_j}\right). \quad (4)$$

Here $\psi=-\tau_{ij}\tilde{u}_i+\bar{\sigma}_{ij}\tilde{u}_i-\bar{q}_c-\bar{q}_d$, the viscous stress tensor $\bar{\sigma}_{ij}=2\bar{\mu}(\tilde{S}_{ij}-\tilde{S}_{kk}\delta_{ij}/3)$, $\tilde{S}_{ij}=(\partial\tilde{u}_i/\partial x_j+\partial\tilde{u}_j/\partial x_i)/2$. The overbar and tilde represent the Reynolds and Favre averaged fields, respectively. The double prime denotes Favre fluctuation. The heat flux $\bar{q}_c$ is defined by the Fourier's law as $\bar{q}_c=-\bar{\kappa}\partial\tilde{T}/\partial x_j$. The interspecies diffusional heat flux $\bar{q}_d$ is given by $\bar{q}_d=-\sum\bar{\rho}\bar{D}C_{p,\alpha}\tilde{T}\partial\tilde{Y}_\alpha/\partial x_j$. The $\bar{\mu}$, $\bar{D}$, $\bar{\kappa}$, $C_{p,\alpha}$ and g_i represent dynamic viscosity, mass diffusivity, thermal conductivity, constant-pressure specific heat of specie α, and gravitational acceleration of the i direction, respectively. The τ_{ij} is the Reynolds stress, and the terms $D_E=-\partial(\bar{\rho}\widetilde{u_j''e''}+\overline{pu_j''})/\partial x_j$, $D_K=-\partial(\bar{\rho}\widetilde{u_i''u_i''u_j''}/2)/\partial x_j$, and $-\bar{\rho}\widetilde{u_i''Y_\alpha''}$ represent the turbulent diffusion terms of the total energy, turbulent kinetic energy (TKE) $\tilde{K}$, and mass fraction, respectively. It should be emphasized that Eqs.(1)∼(4) are deduced based on the concept of ensemble averaging and are theoretically applicable to the three stages of mixing evolution. Once the unclosed terms are appropriately modeled, the equation array can be solved by coupling it with the equation of state for ideal gas.

In previous studies, the majority of RANS simulations focus on fully developed turbulence. Consequently, numerous turbulent mixing models have been proposed, such as the K-ε, K-L, Besnard-Harlow-Rauenzahn (BHR) models and so on. In this study we take the well-developed K-L model as an example to briefly illustrate how turbulent flow is modeled. Specifically, the turbulent transport terms are modeled by the GDA, i.e. $-\bar{\rho}\widetilde{u_i''f''}=\frac{\mu_t}{N_f}\frac{\partial\tilde{f}}{\partial x_i}$, where f denotes an arbitrary physical variable and N_f is a model coefficient. The μ_t is the turbulent viscosity, which is described by the TKE $\tilde{K}$ and the turbulent length scale $\tilde{L}$

$$\mu_t=C_\mu\bar{\rho}\tilde{L}\sqrt{2\tilde{K}}. \quad (5)$$

With the Boussinesq eddy viscosity hypothesis, the Reynolds stress is modeled as

$$\tau_{ij}=C_P\bar{\rho}\tilde{K}\delta_{ij}-2\mu_t(\tilde{S}_{ij}-\tilde{S}_{kk}\delta_{ij}/3), \quad (6)$$

Additionally, the closed transport equations of the TKE and $\tilde{L}$ are

$$\frac{\partial\bar{\rho}\tilde{K}}{\partial t}+\frac{\partial\bar{\rho}\tilde{u}_j\tilde{K}}{\partial x_j}=-\tau_{ij}\frac{\partial\tilde{u}_i}{\partial x_j}+\frac{\partial}{\partial x_j}(\frac{\mu_t}{N_K}\frac{\partial\tilde{K}}{\partial x_j})+S_{Kf}-D_r, \quad (7)$$

$$\frac{\partial\bar{\rho}\tilde{L}}{\partial t}+\frac{\partial\bar{\rho}\tilde{u}_j\tilde{L}}{\partial x_j}=\frac{\partial}{\partial x_j}(\frac{\mu_t}{N_L}\frac{\partial\tilde{L}}{\partial x_j})+C_L\bar{\rho}\sqrt{2\tilde{K}}+C_C\bar{\rho}\tilde{L}\frac{\partial\tilde{u}_j}{\partial x_j}, \quad (8)$$

where $S_{Kf}=C_B\bar{\rho}\sqrt{2\tilde{K}}A_{Li}g_i$ with $A_{Li}=C_A\tilde{L}(\partial\bar{\rho}/\partial x_i)/\bar{\rho}$, $D_r=C_D\bar{\rho}\left(\sqrt{2\tilde{K}}\right)^3/\tilde{L}$, C_μ, C_P, C_A, C_B, C_C, C_D, and C_L are model coefficients. More details about the K-L model, including the value of model coefficients, can be found in Zhang *et al.* (2020).

The aforementioned K-L model is not suitable for the mixing transition stage since all closures are built for fully developed turbulence. The next section will introduce the general idea of constructing the mixing transition model based on the baseline K-L model.

Modeling strategy for the mixing transition model

It is important to seek for a suitable flow variable to quantificationally capture the dramatic variations of flow state and the inherent pronounced local spatio-temporal dependencies in the transition process. Inspired by the concept of the intermittent factor γ, we attempt to extend it to mixing problems.

Following the original definition (Dopazo, 1977), the concept of the intermittent factor is recovered based on the ensemble-averaged approach, i.e.

$$\gamma\equiv\frac{1}{N}\sum_{n=1}^{N}I(x,y,z,t,n)=\gamma(x,y,z,t), \quad (9)$$

where N represents the sample size. The intermittent function $I(x,y,z,t,n)$ depends on time, space and sample parameter n, and is assigned $I=0$ for laminar flow and $I=1$ for turbulence. This definition (9) theoretically indicates γ varying from 0 to 1.

The Reynolds stress play a crucial role for modeling turbulence. Given that the flow undergoes a gradual transition from laminar to turbulent, it is imperative to remodel the Reynolds stress to more accurately reflect the underlying physics of this progression. In accordance with the definition (9), a new expression for the Reynolds stress τ_{ij}^{new} can be written as

$$\tau_{ij}^{new}=(1-\gamma)\tau_{ij}^{non-tur}+\gamma\tau_{ij}^{tur}. \quad (10)$$

Here $\tau_{ij}^{non-tur}$ and τ_{ij}^{tur} represent the non-turbulent and turbulent parts of the Reynolds stresses, respectively, with $\tau_{ij}^{non-tur}$ being negligible. The τ_{ij}^{tur} is closed based on the Eq.(6). Given that the normal stress component of the Reynolds stress is determined by TKE, which has been closed with the transport equation (7), γ only acts on the turbulent viscosity μ_t to modify the deviatoric stress part, i.e.

$$\tau_{ij}^{new}=C_P\bar{\rho}\tilde{K}\delta_{ij}-2\mu_{new}(\tilde{S}_{ij}-\tilde{S}_{kk}\delta_{ij}/3), \quad \mu_{new}=\gamma\mu_t=C_\mu\gamma\bar{\rho}\tilde{L}\sqrt{2\tilde{K}}. \quad (11)$$

Accordingly, in the turbulent diffusion term modeled by GDA, μ_t is replaced by μ_{new}.

The Eq.(11) implies that when the intermittency factor γ is very small, the mixing and turbulence will be suppressed. As the flow develops and γ gradually increases, the associated shear effects correspondingly intensify the degree of mixing and the level of turbulence.

Transport equation for the intermittent factor

In this section, a transport equation for γ is built to describe the local spatio-temporal dependency of the mixing transition flow.

Following the framework (Menter *et al.*, 2002) of constructing the boundary layer transition models based on the intermittent factor, we present the γ transport equation:

$$\frac{\partial\bar{\rho}\gamma}{\partial t}+\frac{\partial\bar{\rho}\tilde{u}_j\gamma}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\bar{\mu}+\frac{\mu_{new}}{N_\gamma}\right)\frac{\partial\gamma}{\partial x_j}\right]+P_\gamma-\varepsilon_\gamma. \quad (12)$$

Here P_γ and ε_γ represent the product and dissipation terms, respectively. The actual physical process of mixing evolution tells that the flow eventually reaches a fully developed turbulent state, indicating that the new Reynolds stress τ_{ij}^{new} should ultimately approach to the closure (6). Correspondingly, γ should increase to 1, then fluctuate around 1 due to the fluctuating nature of turbulence. It implies that $P_\gamma-\varepsilon_\gamma$ should be greater than 0 to provide a positive net increment, then tend to 0 when the fully developed turbulence is generated. This requirement is met by setting $\varepsilon_\gamma=\gamma P_\gamma$.

The production term P_γ is the key to decide the performance of the model. A transition model should have two basic functions: identifying the onset of transition and predicting the subsequent flow evolution accurately. These functions can be achieved by reasonably modeling the production term. In the present model P_γ is expressed as

$$P_\gamma=F_{onset}G_r. \quad (13)$$

Here F_{onset} serves as a transition switch, while G_r describes the growth rate of γ. Specifically, F_{onset} is given by

$$F_{onset}=1-exp(-Re_t), \quad Re_t=\frac{\tilde{L}\sqrt{2\tilde{K}}}{\nu}. \quad (14)$$

Table 1. Setup of the present RANS simulations.

	Cases	Δ (cm)	$\tilde{K}_0$	$\tilde{L}_0$	γ_0
Transitional	RTI [Livescu *et al.* (2021),$A_t = 0.5$]	0.0196	$A_t g \tilde{L}_0$	0.039	0
	RTI [Livescu *et al.* (2021),$A_t = 0.75$]	0.0196	$A_t g \tilde{L}_0$	0.039	0
	Inverse Chevron [Hahn *et al.* (2011)]	0.0625	0.001	0.0625	0
Turbulent	Tilted-rig [Denissen *et al.* (2014)]	0.1	$0.01 A_t g \tilde{L}_0$	0.01	1
	Reshocked RM [Poggi *et al.* (1998)]	0.025	$0.001(A_t \Delta\tilde{v})^2$	0.065	1

* The last four columns give the grid scale Δ and the initial values of turbulent variables. The $\Delta\tilde{v}$ represents the variation of the interfacial velocity after shock waves passing through the interface.

Here Re_t is the local turbulent Reynolds number. It is easy to understand the mechanism of F_{onset}. During the initial stage of instability development, flows keep stagnant or a very weak fluctuant state, Re_t approaches 0, resulting in $F_{onset} \to 0$. It makes sure that transition does not occur, thus γ approaching to 0. As instabilities develop, the TKE $\tilde{K}$ and the turbulent scale $\tilde{L}$ begin to evolve, Re_t growing accordingly. It makes F_{onset} rise to 1 rapidly, resulting in that the model is completely activated.

The function G_r dominates evolution of the intermittent factor γ, and its formation should reflect two aspects: the dynamic characteristics of the targeted flow system and the growth trend of γ. For the former, the shear and buoyancy effects, which are depicted with the first and third terms in the right side of the TKE eq.(7), are the essential dynamic mechanisms. For the latter, it is important to choose an appropriate function to describe the evolutionary trend of γ well. However, for mixing flows concerned here, studies on the intermittent factor have not been reported yet. To alleviate this problem, we employ the formula (Wang & Fu, 2011) that depicts the development of γ of boundary layer transition flows. Finally, G_r is expressed as:

$$G_r = \sqrt{-\ln(1-\gamma)} \left(\bar{\rho}\sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}} + \gamma\bar{\rho}A_{Li}g_i/\sqrt{\tilde{K}} \right). \qquad (15)$$

Here, the $\sqrt{-\ln(1-\gamma)}$ depicts the development of γ, while $\bar{\rho}\sqrt{2\tilde{S}_{ij}\tilde{S}_{ij}}$ and $\gamma\bar{\rho}A_{Li}g_i/\sqrt{\tilde{K}}$ represent the shear and buoyancy effects, respectively.

MODEL VALIDATION

The performance of the proposed model is verified by two different kinds of flow cases: one exhibiting pronounced transitional characteristics, and the other swiftly developing into fully developed turbulence, rendering the transition stage negligible. The details of the RANS simulation setups for these cases are listed in Table 1.

Predictions for mixing transition flows

In this section, the ability of the proposed model to predict mixing transition is verified by transitional RT/RM mixing flows. In particular, the RT flows contain two different density ratios of 3:1 and 7:1.

Fig.1 presents the temporal evolutions of mixing width for RT cases with density ratios of 3:1 and 7:1. The mixing width (mixing quantity) is the most essential quantity for engineering applications, and thus serves as the benchmark against which we assess the performance of our proposed model. The computational results are normalized using the the characteristic length scale $L_r = 2\pi/32 \approx 0.196$, velocity scale $U_r = \sqrt{A_t g L_r}$, and time scale $t_r = \sqrt{L_r/(A_t g)}$. Within these evolutionary curves, some transition characteristics are discernible. Notably, significant inflection points signaling transition appear

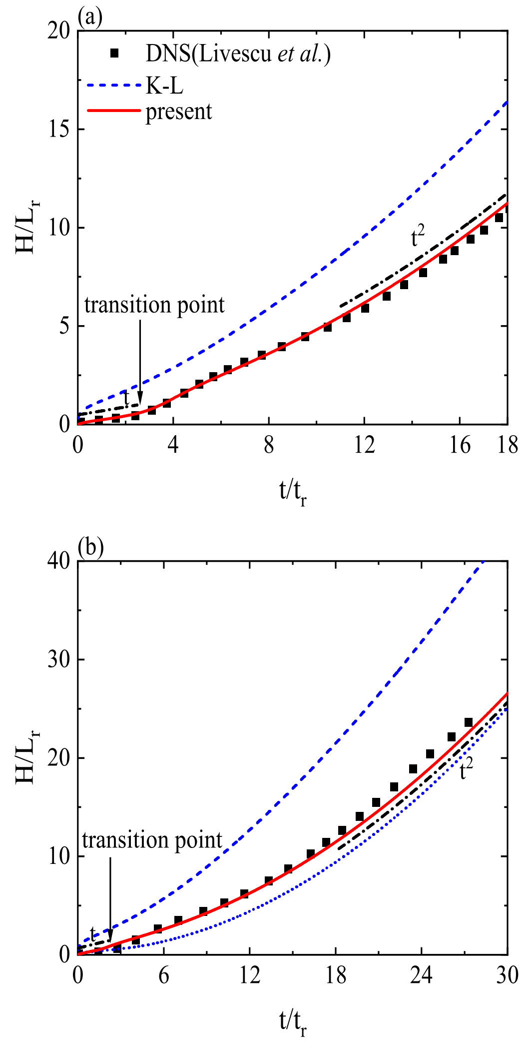

Figure 1. Temporal evolution of the mixing width for the two RT cases with density ratios 3:1 (a) and 7:1 (b), respectively.

at approximately t/t_r of 2.9 and 2.5 for density ratios 3:1 and 7:1, respectively. These points are precisely captured by the present K-L-γ model, while the baseline K-L model fails to do that. Prior to the inflection points, the mixing width exhibits a linear growth pattern with a relatively shallow slope, indicative of a low-mixed state before the onset of the transition. The K-L-γ model adeptly describes this stage with a narrowly growing mixing width that evolves slowly, aligning closely with DNS results. In contrast, the K-L model's predictions display a steep initial increase, resulting in an abrupt rise akin to a cliff edge. As the perturbed interface area expands, the marked increase in the mixing level and the sharp rise in the mixing width signal the flow's entry into the transition stage. The good agreement between the predictions of K-L-γ model and DNS signifies the success of the present modeling approach. Subsequently, the mixing evolution develops into a self-similar state where the mixing width increases according to a determined quadratic law. Here too, the K-L-γ model maintains a satisfactory predictive accuracy, while the K-L model consistently overestimates the progression of the mixing width.

We further test the present model in the transitional RM mixing case originated from the experiments (Hahn *et al.*, 2011) conducted at the linear shock tube facility of AWE (Aldermaston). This setup features a block of dense SF6 gas sur-

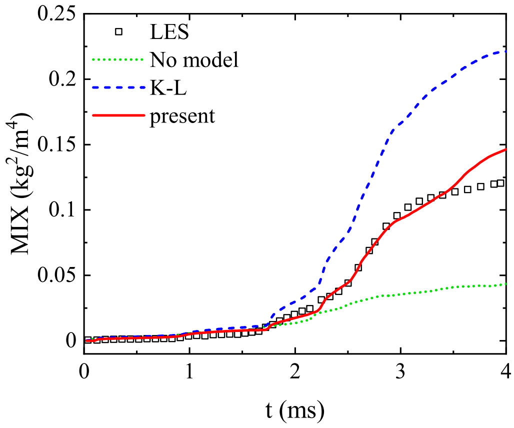

Figure 2. Total mixing (MIX) vs time for the reshocked RM mixing with inverse chevron interface.

rounded by air within a shock tube. Apart from the RM effect, the uniquely designed inverse chevron interface initiates an early KH instability, which expedites the transition to turbulence. Moreover, the reflective boundary makes that the shock wave, upon reflecting off the right wall, re-impacts the mixing region. This reshock phenomenon intensifies the mixing and renders the transitional effects more pronounced.

For mixing flows, the integral quantity MIX is resistant to statistical noise and offers a straightforward yet effective means of quantifying the total amount of mixing in in complex flows (Hahn *et al.*, 2011). Fig.2 illustrates the temporal evolution of MIX. It shows that, at approximately $0.1ms$ and $0.85ms$, the incident shock wave sequentially impacts on the planar and chevron interfaces, resulting in a slight increase in MIX. At around $1.7ms$, a marked surge in total mixing is observed when the mixing region is firstly reshocked by the reflected wave. A subsequent reshock occurs around $2.2ms$, catalyzing another swift increase in MIX. The evolutionary process observed and the results of LES indicate that prior to reshocking, the mixing is at a low-mixed level. Both RANS models effectively predict the low-mixed state before reshocking. However, after the mixing region experiences reshocking, the flow enters a significant transitional stage, where both the onset of transition and the subsequent evolution are accurately delineated by the K-L-γ model. Conversely, the K-L model presents an absurd overprediction. The good agreement between LES and the present model affirms the efficacy of the modeling strategy employed.

Predictions for turbulent mixing flows

In certain scenarios, the transition effect can be ignored as flows rapidly develop into the fully developed turbulence. In such cases, it is imperative for the K-L-γ model to quickly adapt to that. This section uses two fully developed RT/RM mixing cases to verify whether the proposed K-L-γ model possesses this capability. It should be added that the K-L model has been shown to provide accurate predictions for these types of scenarios, with detailed results available in Zhang *et al.* (2020). In the present study we only show the results from the K-L-γ model. Simulation setups of the cases are listed in the turbulent part of Table 1, involving the RT mixing with initial tilted interface (named as tilted-rig cases) and the planar reshocked RM mixing.

The good agreement between the present predictions and the reference data indicates that the K-L-γ model still has good accuracy for the fully developed turbulent mixing flows.

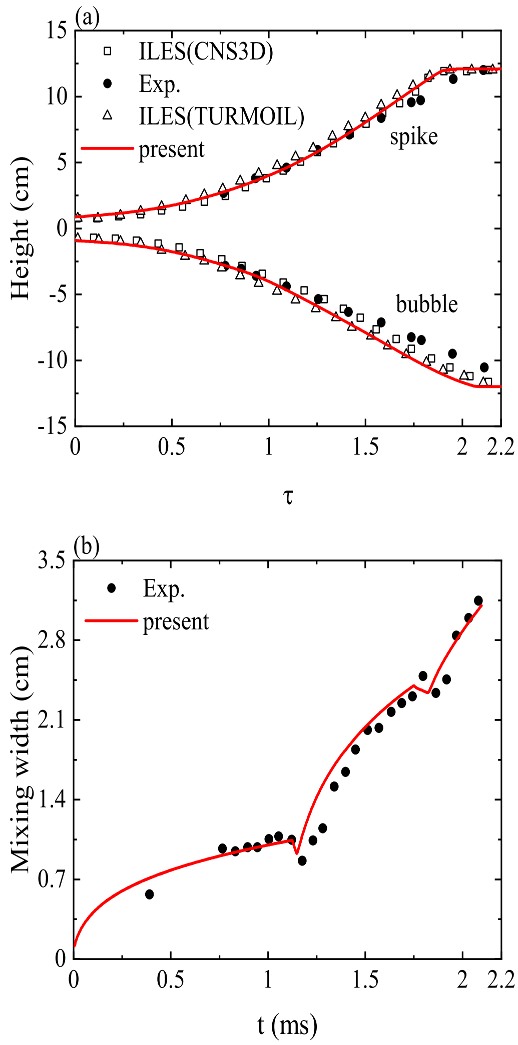

Figure 3. Temporal evolution of mixing width for the tilted-rig RT turbulent mixing (Denissen *et al.*, 2014) and the planar reshocked RM turbulent mixing conducted by Poggi *et al.* (1998).

Conclusions and discussions

The local spatio-temporal dependence and non-equilibrium nature of mixing transition are challenging for RANS predictions. Drawing on the concept of the intermittency factor, which is extensively utilized for boundary layer transitions modeling in aerospace engineering, this study extends this well-developed modeling strategy to mixing problems induced by interfacial instabilities. Specifically, we build a transport equation for the intermittent factor γ and couple it with the K-L model to modify the key Reynolds stress. The efficacy of the proposed model is then assessed using two distinct types of mixing flows. The good predictive results for the mixing transition flows confirm that the present model can accurately capture the onset of the transition and depict its subsequent progression. Further testing on turbulent mixing flows reveals that the model remains appropriate even as the flow rapidly evolves into fully developed turbulence.

To the best of our knowledge, this is the first study proposing a RANS mixing transition model, showcasing a potential for accurately predicting the entire process of mixing evolution. Moreover, the modeling framework presented here is flexible and exhibits a promising potential for more sophisticated modeling of mixing transition. We anticipate that the current framework can be expanded to incorporate other turbulent mixing models, such as the K-ε, BHR models, and so on.

Nonetheless, a broader array of test cases is required to thoroughly validate the performance of the proposed model. There is, however, a notable shortage of benchmark cases that exhibit significant transition effects, underscoring the need for additional experiments and high-fidelity simulations to address this gap. In the future, it is necessary to develop more advanced models to improve predictions of mixing transition, such as the hybrid LES/RANS modeling, which can provide much richer information than RANS methods.

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