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TOWARDS HIGH-REYNOLDS NUMBER SHOCK-TURBULENCE INTERACTION VIA ONE-DIMENSIONAL TURBULENCE

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ABSTRACT

The canonical problem of the passage of high-Reynolds number homogeneous isotropic turbulence through a normal shock wave is studied from the perspective of one-dimensional turbulence (ODT) approach. Initial shock-turbulence interaction (STI) by Ribner Ribner (1953, 1954) proposed analytical scaling in the thin shock limit via linear interaction analysis. More recent computational works, using direct numerical simulations, have been limited to the lower-Reynolds number regimes due to the computational limitation of the approach. Here, we propose to model STI by using a new compressible formulation of ODT. The approach combines stochastic and structural aspects of the interaction and seeks to accurately model turbulence amplification through the shock at high Reynolds number. The compressional straining by the eddies downstream of the shock produce the trends of turbulent kinetic energy (TKE) jump while creating peak in dissipation rate and drop in Kolmogorov length scale just downstream of the shock location. Further tuning of the ODT STI model has the potential to capture the TKE jump with variation in upstream Mach number and turbulent intensity.

INTRODUCTION

High-speed flows give rise to significant modelling challenges in the accurate representation of turbulence. Many of these high-speed turbulent flows interact with shock waves, which results in a nearly discontinuous jump in kinematic and thermophysical properties. This jump is also accompanied by a systematic amplification of turbulent kinetic energy across the shock. As the subsequent post-shock turbulent decay directly affects the mixing and thus large-scale flow features, accurate prediction of the turbulent kinetic energy (TKE) jump is of critical importance for performance prediction in shock-turbulence interaction (STI) in many applied problems. Although experimental setups are necessary, much of the characterization of the TKE amplification has relied on direct numerical simulation. At high Reynolds numbers, the turbulence amplification through the shock can be analytically determined from Linear Interaction Analysis Ribner (1953, 1954). The combination of experimental, analytical, and numerical results can then be used to inform modelling strategies to be used within Reynolds Averaged Navier-Stokes (RANS) or Large-Eddy Simulation (LES) solvers. Classical RANS-based turbulence models do not adequately account for the TKE jump through the production terms due to the inherent mesh dependence at the discontinuity due to the gradient computation in the TKE production terms; thus *ad hoc* models must be adopted, for example, in the work by Sinha *et al.* (2003); Lacombe *et al.* (2021).

Reduced-order models though often are less accurate, enable a valuable representation of isolated aspects of the STI problem. The study of STI via three-dimensional DNS is inherently Reynolds number-limited given the computational costs (see e.g. Chen & Donzis (2019)). Using reduced-order models, the effect on eddies that pass through the stationary shock can be quantified in the shock-normal direction, for direct inputs to RANS-like models. This order reduction is accompanied by decreased fidelity of the model, for example, an inadequate accounting of shock-curvature and undulation; and the phenomena in shock-turbulence interaction that occur in the shock plane directions. The scaling of the TKE jump with the upstream turbulence conditions: Re_λ,M_t and M, have been proposed by Lacombe *et al.* (2021) and Chen & Donzis (2019). Chen and Donzis Chen & Donzis (2019) proposed a universal scaling for turbulence amplification through a shock that showed good agreement with previously published experimental and numerical results. These studies relate the observed amplification to the ratio of the turbulence time-scale to the time-scale associated with the shock wave. Reduced-order models that reproduce the TKE jump based on such scalings can be retrofitted into RANS-like simulations for physics-based generation of TKE on the interaction of shock with turbulence. Similar approach was taken for integrating ODT based turbulence models into LES for sub-grid scale modelling (Schmidt *et al.*, 2010).

In this work, we attempt to capture the TKE jump in STI in the framework of one-dimensional turbulence, wherein, stochastic linear eddies represent the compressional straining and the real three-dimensional turbulent eddies. Jozefik developed an ODT model for the TKE amplification caused by passage of turbulence over a sharp density jump in context of Richmeyer-Meshkov (RM)instability. Gao *et al.* (2023) used the eddy model from Jozefik *et al.* (2016) in a compressible formulation of ODT with a shock-capturing scheme and reported an improvement in performance of the ODT method to model RM instability. The approach for eddy models with reference frame acceleration in the work by Jozefik *et al.* (2016), is extended in this work to model the TKE jump in STI. The aforementioned reference frame acceleration produces turbulent kinetic energy by virtue of increased compressional straining by eddies. Physics-based explanations for TKE jump with respect to the the local compression at the shock front causing the abrupt increase in transverse vorticity leading to a TKE jump slightly downstream of the shock, form the basis of eddy models in this work.

We present a compressible formulation of ODT coupled with a STI eddy model based on jump conditions across a shock wave. A brief introduction to salient aspects of ODT with the proposed STI model is followed by the results obtained with regard to TKE amplification. The study is conducted with varying STI model parameters at a nominal turbulent Mach number $M_t = 0.223$ and shock Mach number $M = 1.5$ at a Taylor scale Reynolds number of ~ 170.

METHODOLOGY

ODT provides a potential framework for the representation of the two important aspects of STI problem in one dimension, namely: the generation of HIT at elevated turbulent Mach numbers and the passage of the generated turbulent flow onto a stationary shock. The multiscale triplet mapping of ODT, which mirrors the three-dimensional turbulent eddies of real flows, reproduces the energy cascade in the HIT by virtue of the complete resolution of temporal and spatial scales. The high-Reynolds-number turbulence is sustained through a forcing term applied to the large scales in the flow. In addition, the consideration of the triplet-map energetics allows for a physics-based representation of post-shock TKE jump when turbulence interacts with a shock.

Modelling compressible turbulence and shock wave in the ODT domain necessitates extension of ODT framework to a compressible formulation, where the fully conservative form of the Navier-Stokes equations (ρ, ρu_i, and ρe) is solved in one dimension. Furthermore, at the nearly discontinuous jump, numerical schemes must minimize dispersive and dissipative errors while affording numerical stability to the problem. The WENO $5th$-order scheme is used in conjunction with the HLLC solver, as described in the work of Houim & Kuo (2011) and a third-order Runge-Kutta scheme is used for temporal integration. An acoustic-CFL criterion is used to set the time-step based on the computed flow velocity (u) and the isentropic speed of sound (c). The present work uses an Eulerian implementation of the open-source ODT code developed by Stephens & Lignell (2021), which was based on a Lagrangian framework. The turbulence model of ODT is also modified to account for conservative variables.

Turbulence model

The turbulent eddies that embody the mechanistic and structural aspects of turbulence form the core of the ODT approach to turbulence modelling. In ODT, one-dimensional stochastic linear eddies model the three-dimensional turbulent eddies, originally proposed in the seminal work by Kerstein (1991, 1999). The turbulent stirring occurs along the ODT line using a predefined mapping known as a triplet mapping (see equation 1), which causes compressional straining causing the energy to cascade from largest to the smallest flow scales. The mapping is measure-preserving for all the mapped variables and their moments, and is defined in a manner such that it does not introduce jump discontinuities. Three copies of the eddy region profile are made, compressed, and then stitched together, with the middle segment inverted, as depicted in Figure 1. Owing to the compressible formulation of the current work, we map the conservative quantities ρ, ρu_i, and ρe using:

$$f(x) = x_o + \begin{cases} 3(x-x_o), & x_o \leq x \leq x_o + \frac{1}{3}l \\ 2l - 3(x-x_o), & x_o + \frac{1}{3}l \leq x \leq x_o + \frac{2}{3}l \\ 3(x-x_o) - 2l, & x_o + \frac{2}{3}l \leq x \leq x_o + l \\ x - x_o, & \textit{otherwise} \end{cases} \tag{1}$$

The choice of the eddy size l and the start location of eddy along the ODT line x_o is carried out stochastically following the probability distribution functions $g_1(x_o)$ and $g_2(l)$. The net displacement caused by the eddy is quantified in terms of a kernel function ($K(x) = x - f(x)$), which integrates to zero over the eddy region. The kernel function provides a mechanism to quantify the transport of mapped variables caused by the ODT eddy. In addition to the triplet map $f(x)$, the velocity components are subject to a transformation to simulate the tendency towards isotropy by mutual energy transfer among the components (Ashurst & Kerstein, 2005).

The energy available for post-triplet-map velocity transformation in each of the velocity components, E_{kin}, helps associate a time-scale to the considered eddy:

$$\frac{1}{\tau} = C\sqrt{\frac{2KK}{\rho_{KK} l^3}(E_{kin} - ZE_{vp} + E_p)} \tag{2}$$

where $KK = \int_{x_o}^{x_o+l} K^2(x)dx$ and $\rho_{KK} = \int_{x_o}^{x_o+l} \rho K^2(x)dx$. C is a control parameter to achieve the desired turbulence intensity and Z is a coefficient of the viscous penalty E_{vp}, which in turn sets a lower limit on the allowable eddy size as defined by Ashurst & Kerstein (2005). The viscous penalty adds a mechanism for viscous dissipation in addition to the Navier-Stokes viscous diffusion terms.

A multitude of eddies is tested and invoked stochastically in time, with each eddy event being an instantaneous occurrence. However, the acceptance of eddies is restricted by defining an eddy acceptance probability, p_e, as a function of the eddy time scale, τ and the mean simulation time-step Δt_e:

$$p_e = \frac{\Delta t_e}{l^2 g_1(x_o) g_2(l) \tau} \tag{3}$$

Forced HIT

We generate forced HIT on the one-dimensional ODT domain by applying an external forcing in conjunction with the turbulence model of ODT which produces the energy cascade. The external forcing overcomes the energy loss due to the viscous dissipation at the Kolmogorov scales, hence, leading to sustained levels of turbulence intensity. Forcing adds energy to the largest scale, which in turn cascades down to the smallest scales due to the ODT eddies.

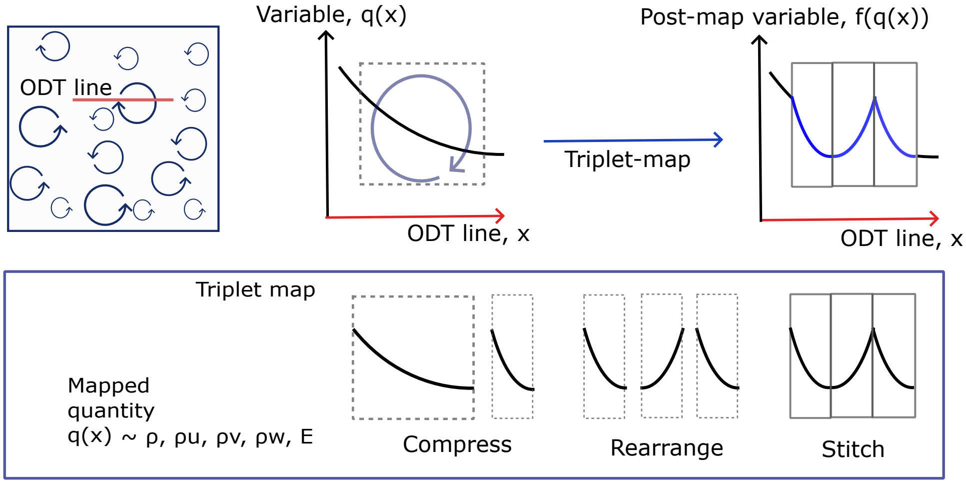

Figure 1. Triplet-map on variable $q(x)$ over an ODT line along x.

The forcing method used by Fistler *et al.* (2020) is extended to variable density. A predetermined amount of kinetic energy Δk is added to the velocity components at each forcing event on an average. The added energy is a direct function of the production rate P, forcing time scale, τ_{ke}, and domain volume, V_d such that: $\Delta k = P\tau_{ke}V_d$. Production rate P is estimated based on the desired Taylor Reynolds number, Re_λ, and turbulent velocity magnitude, u'.

In the present work, we model the forced HIT on a one-dimensional periodic domain of length 2π. The smallest eddy size is defined as $l_{min} = 3dx_{min}$. Therefore, for achieving complete scale-resolution, the minimum cell size in ODT is set to be one-third of the Kolmogorov length scale. An adaptive meshing scheme is used, which increases local mesh resolution in regions of high velocity gradient. The simulations are carried out for a sufficient total duration to sample enough of the forcing events and attain stable turbulence characteristics over multiple eddy turn-over times.

The forcing events modify the velocity components–similar to the ODT eddy event–but without the triplet map: $(\tilde{u}_i \longrightarrow u_i + d_i K(x))$. The imparted kinetic energy is related to the coefficient d_i by:

$$\begin{aligned} \tfrac{1}{3}\Delta k &= \tfrac{1}{2}\int_{x_o}^{x_o+l} \rho(\tilde{u}_i{}^2 - u_i^2)dV \\ &= \tfrac{1}{2}\int_{x_o}^{x_o+l} \rho((u_i + d_i K(x))^2 - u_i^2)dV \end{aligned} \tag{4}$$

The coefficients d_i are computed as: $d_i = \frac{1}{I}\left(-J_i \pm \sqrt{J_i^2 + 2I\Delta k/(3A)}\right)$, where $I = \int_{x_o}^{x_o+l} \rho K(x)^2 dx$ and $J_i = \int_{x_o}^{x_o+l} \rho u_i K(x) dx$.

Shock-turbulence interaction model

STI model developed in this work is based on the TKE amplification mechanism using an approach based on energy balance with regard to eddies. The turbulence passing through the shock wave is subjected to abrupt compression at the shock location, causing a sharp increase in transverse vorticity downstream of the shock Larsson *et al.* (2013). The increased vorticity causes a surge in the turbulent oscillations in transverse velocity at the shock location. This energy is transferred to the longitudinal velocity oscillations slightly downstream of the shock due to the tendency of turbulence towards isotropy. In this work, we attempt to build the STI model to simulate this post-shock turbulent energy generation.

Though the streamwise ODT domain cannot capture kinematic effect of the transverse shape of the shock-front on the incoming turbulence, the ODT model provides an approach based on energy balance, to model the influence exerted by the shock-front on the passing turbulent eddies Jozefik *et al.* (2015). The base flow causes three-dimensional effects on the considered eddy over a finite time duration in a fixed reference frame. In ODT framework, there is no direct acceleration of eddies caused by the time-dependent flow equations, as the ODT eddies are instantaneous occurrences. In lieu of such three-dimensional spatial and temporal effects on eddies, ODT models the acceleration using reference-frame acceleration using Einstein's equivalence principle (Jozefik *et al.*, 2015). This approach treats any acceleration mechanism as an analog of the gravitational force. Similar approach has been used elsewhere for treating Rayleigh-Taylor instability E.D. Gonzalez-Juez & Lignell (2013), Darrieus-Landau instability Jozefik *et al.* (2015) and the Richtmyer-Meshkov instability (Jozefik *et al.*, 2016).

The reference-frame acceleration is implemented by an ad-hoc partition of the ODT time-advancement pertaining to the eddy-time scale and the forcing due to the base flow where eddies are applied, by introducing a potential energy term associated with the latter in equation 2. The potential energy term increases the TKE by amplifying the eddy likelihood. In addition, if the potential energy is treated as arriving from external source, the term is also added during the energy-redistribution post-triplet map. The potential energy term is given in equation 5, where the reference frame-acceleration is: $a_{STI} = C_{STI}(Mc\Delta u)/l$. Here, c is the speed of sound and M are computed downstream of the shock and Δu is the velocity jump across the shock (Jozefik *et al.*, 2016). A parameter C_{STI} is used to control the TKE amplification intensity. The potential energy term is activated if the considered eddy is at a distance of about θ times the eddy size, away from the shock. The coefficient θ helps to set the region of influence of the shock on invoked eddies. The shock-wave influences the flow-scales larger than itself, hence, we disallow the eddies that are of length scale smaller than the turbulent shock thickness, which is expressed as a multiple of the Kolmogorov length scale (Lacombe *et al.*, 2021).

$$E_{STI} = \int_{x_o}^{x_o+l} a_{STI} K(x)\rho(x) dx \tag{5}$$

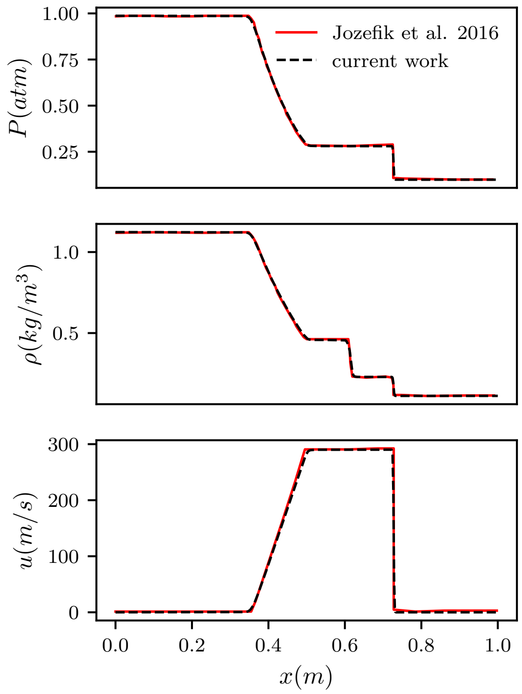

Figure 2. Propagation of shock-front and rarefaction wave, compared with solution of sod-shock tube problem (Riemann, 1860; Sod, 1978; Jozefik *et al.*, 2016).

RESULTS

Stationary shock to study STI

The developed compressible solver in ODT is tested in terms of the production of shock and rarefaction waves. The simulated profiles of density, ρ, pressure, P, and velocity, u, closely match with the results from Jozefik *et al.* (2016). An ODT domain is initialized with a discontinuity separating the two regions with states: (i) left: $\rho = 1.23\ kg/m^3$ and $P = 100\ kPa$, and (ii) right: $\rho = 0.123\ kg/m^3$ and $P = 10\ kPa$. The case is compared with results for gas molecular mass of 28.0115 $kg/kmol$, specific heat ratio γ of 1.4 and initial gas temperature of 300 K.

For generation of a stationary shock, an ODT case is setup where the required upstream and downstream conditions are set. The shock is subjected to the incoming turbulence from the left boundary. The downstream pressure needs minor adjustment to hold the shock stationary when incoming turbulence disturbs the set pre- and post-shock conditions.

Characteristics of incoming turbulence

Forced HIT is generated in a periodic domain of length 2π at a constant fluid viscosity of $4.0 \times 10^{-4} m^2/s$ at $Re_\lambda \sim 200$. The targeted M_t is achieved by modifying the sound speed via changes in the gas molecular mass. The Kolmogorov length scale $\eta = (\nu^3/\langle\varepsilon\rangle)^{1/4}$, which is set based on the dissipation rate $\langle\varepsilon\rangle$, is used to define the minimum permitted cell size; $dxmin = 3\eta$.

A set of HIT is generated at M_t varying from 0.042 to 0.261. All cases are run for time duration much longer than the integral time-scale, and, thereafter, the statistics are computed. We present the turbulent kinetic energy spectra attained for varying M_t, see figure 4. The obtained turbulent Mach number M_t, integral length scale L_ε, Kolmogorov length scale η, velocity fluctuation magnitude u', sound speed a, and Taylor-scale Reynolds number Re_λ are tabulated in table 1. The

Table 1. List of cases of HIT in the present study with the domain of length 2π.

M_t	L_ε	$\eta\ (10^{-3})$	u'	a	Re_λ
0.042	1.696	3.14	1.334	31.913	190
0.125	1.738	2.76	1.451	11.653	173
0.223	1.730	2.39	1.659	7.453	169
0.261	1.731	2.48	1.384	5.312	128

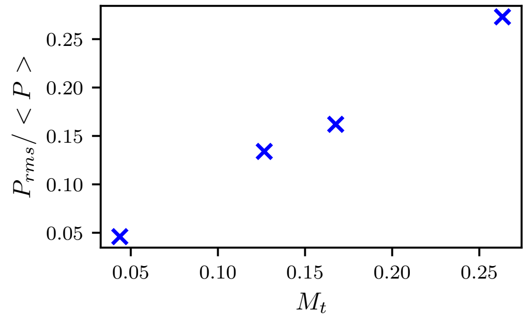

Figure 3. Pressure fluctuations with respect to the mean pressure at different M_t.

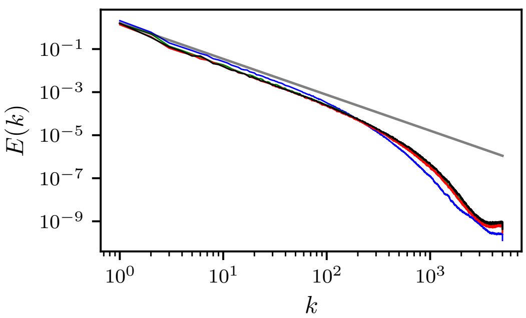

Figure 4. TKE spectra for HIT at varying M_t.

generated turbulence exhibits a monotonic increase in the ratio of root-mean square pressure oscillation to mean pressure; see figure 3. With regard to the turbulent kinetic energy spectra, the cases with different M_t show consistent characteristics as shown in figure 4.

Several snapshots of the generated turbulence are stitched together and are input into the ODT domain from left boundary, leading to the interaction of the generated turbulence with a shock wave.

Interaction of shock with turbulence

To study the interaction of turbulence with a shock wave, the shock is held stationary at $x = 0$ in a ODT domain of length 12.5. The upstream and downstream conditions of the shock are set based on the desired upstream Mach number, while taking the mean pressure and temperature of the incoming turbulence into consideration. The kinematic viscosity of the fluid in the STI domain is identical to the generated HIT. A 2.5 units

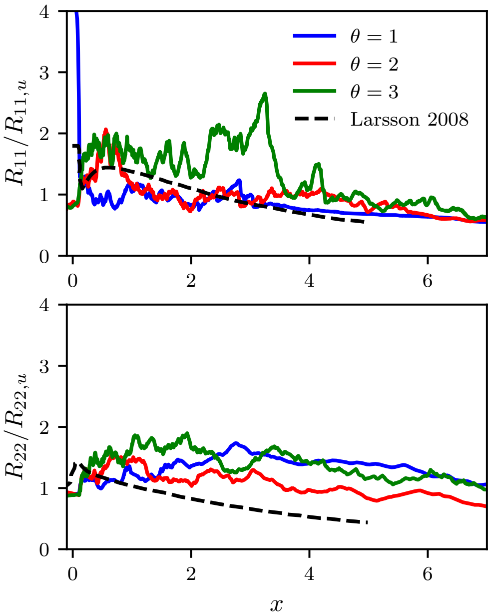

Figure 5. Variation of rms velocities with change in θ at $(M, M_t, Re_\lambda) = (1.5, 0.223, 169)$ and $C_{STI} = 1.0$, compared with results from Larsson (2008) at $(M, M_t, Re_\lambda) = (1.5, 0.221, 40)$.

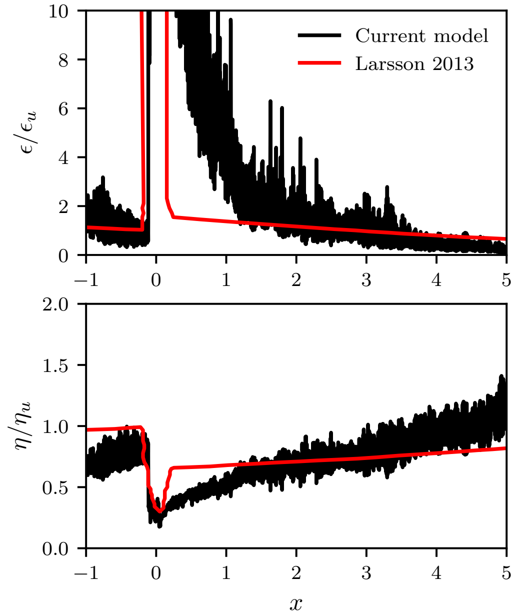

Figure 6. Variation of dissipation rate and Kolmogorov length-scale post-shock at $(M, M_t, Re_\lambda) = (1.5, 0.223, 169)$ and $C_{STI} = 0.5$, compared with results from Larsson *et al.* (2013) at $(M, M_t, Re_\lambda) = (1.5, 0.14, 70)$.

thick sponge layer is introduced at the right boundary to eliminate reflections from the boundary.

ODT eddies downstream of the shock lead to increased compressional straining and energy is added to the eddy regions using the STI model elaborated earlier. The C parameter of ODT is fixed at 10.0 to have consistent representation of eddy events post-shock. For a turbulent Mach number of $M_t = 0.223$, the velocity fluctuation magnitude increases downstream of the shock leading to the expected TKE jump. The location of the peak TKE depends on the region of influence of the shock, which is set by the θ parameter in this study. The impact of varying θ is evident from figure 5, where an increase in θ from 1.0 to 3.0 shifts the position of peak post-shock velocity oscillation magnitude. $\theta = 2.0$ gives a clear TKE peak close to the location $x = 1$, which is close to the TKE peak in the work by Larsson *et al.* (2013) at $x/L_\epsilon = 1.0$, hence the value $\theta = 2.0$ is fixed for other simulations carried out. It must be noted that the results obtained in this work is at a higher Reynolds number than the values reported by Larsson (2008); Larsson *et al.* (2013).

A surge in TKE post-shock is known to be accompanied by a jump in dissipation rate at the shock location with a gradual fall post-shock, see (Larsson *et al.*, 2013). The obtained dissipation rate from this model follows this trend, however, with much higher dissipation rate post-shock, see figure 6. The Kolmogorov length scale exhibits a drop at the shock location consistent with the findings of Larsson *et al.* (2013).

The extent of the amplification of TKE is an outcome of the STI model with the coefficient C_{STI}. Increase of C_{STI} shows a clear increment in post-shock velocity fluctuation magnitudes in figure 7. C_{STI} is an important factor which can impact the TKE jump prediction from ODT. A small increase in the value of C_{STI} causes a very high TKE jump and thus, it must be set precisely based on the STI model considerations. An increase of upstream Mach number causes multi-fold increase in a_{STI} from equation 5, and this effect can be simulated by an increase in C_{STI}. From this analysis, it is evident that an increase in upstream Mach number is equivalent to an increase in C_{STI}, accompanied by increased post-shock TKE jump magnitude in ODT simulations.

Incoming turbulence with lower turbulent Mach number ($M_t = 0.125$) is impinged onto a stationary shock, to study the performance of the developed model. When M_t is reduced from $M_t = 0.223$ to 0.125, a large increase in the TKE jump magnitude post-shock is observed. The jump is obtained with the STI model parameters (C_{STI}, θ) consistent with the simulations mentioned above at $M_t = 0.223$. The trends of TKE jump obtained with changing M_t are same as reported elsewhere, however, detailed study of the STI model is warranted to ensure the accurate amplification magnitude.

CONCLUSION

The interaction of shocks with turbulence in ODT has been previously studied, in a limited perspective, with respect to Richtmyer-Meshkov instability. This work builds on the compressible ODT to handle homogeneous isotropic turbulence in one-dimensional sense and thereby study its propagation through a normal shock wave. Homogeneous isotropic turbulence is generated in ODT and input into the STI domain wherein the shock is kept stationary with a balance of conditions upstream and downstream. The instantaneous profiles show attenuation of turbulent fluctuations across the shock, and their statistical variations are computed. Results at high Reynolds number ranging from 128 to 190 show the potential of ODT to provide a modelling strategy for the canonical shock-turbulence interaction problem. This work attempts a structural view of the amplification mechanism of turbulent ki-

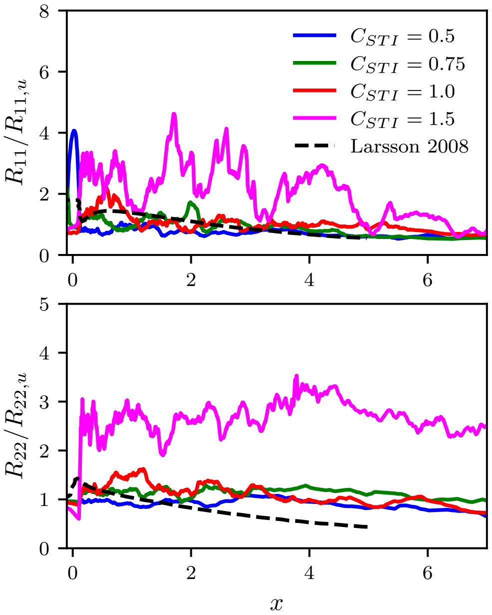

Figure 7. Variation of rms velocities with change in C_{STI} at $(M, M_t, Re_\lambda) = (1.5, 0.223, 169)$ and $\theta = 2$, compared with results from Larsson (2008) at $(M, M_t, Re_\lambda) = (1.5, 0.221, 40)$.

netic energy across the shock. Presence of extremely small eddies just downstream of the shock can explain the post-shock TKE jump and increased dissipation rate at the shock accompanied by a sharp drop in Kolmogorov scales. Increased compressional straining caused by eddies is implemented using the potential energy formulation in ODT eddy modelling framework and favorable trends in TKE post-shock are observed. Parametric studies reveal that the structural approach can provide an insight into the shock-turbulence phenomenon with a well-designed control of the eddy generation process.

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