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DISSIPATION ELEMENT ANALYSIS FOR PHYSICS BASED TAILORED NUMERICAL GRID IN STATISTICAL TURBULENCE MODELS

Fettah Aldudak, Holger Foysi
Department of Mechanical Engineering, University of Siegen
Paul-Bonatz-Straße 9-11, 57076 Siegen, Germany
fettah.aldudak@uni-siegen.de, holger.foysi@uni-siegen.de

Abstract

In the realm of turbulent flow simulations, the quality and resolution of the numerical grid are paramount for achieving accurate results. Direct Numerical Simulations (DNS) make use of the Kolmogorov length scale η to determine grid size. However, statistical turbulence models such as Reynolds Averaged Navier Stokes (RANS), Detached Eddy Simulation (DES) or Large Eddy Simulation (LES) pose a challenge in determining an appropriate resolution criterion for e.g. automatic grid refinement. Given that DNS are often impractical for most real-world flow conditions, statistical methods are typically employed. Optimally, locally adapted, often inhomogeneous grids are used to accurately account for individual and complex flow regimes.

In our previous work (Aldudak & Foysi (2022)), we proposed the application of the Dissipation Element (DE) scale ℓ_{DE} and its relation to the Taylor length scale λ as a novel guide for automatic grid resolution. The applicability of this method had been successfully demonstrated for practical cases, encompassing a range of increasingly complex flow regimes. Based on high resolution DNS data of lower Reynolds number turbulent channel flow, it was shown that the relationship between ℓ_{DE} and λ exhibited only minor variations across the majority of the channel width. In this work, we incorporate turbulent channel flow with significantly higher Reynolds number. These flows suggest, that the ratio of Dissipation Elements and Taylor length exhibits a weak variation with increasing Reynolds number across most of the wall-normal distance. This can be used to explore how varying the thus proposed grid resolution criterion (ℓ_{DE}/λ ranging from 1 to 6) impacts the accuracy of numerical simulations and the resulting grid size. Therefore, we perform simulations using the purely statistical RANS method. Additionally, we utilize a traditional mesh, characterized by refinement regions composed of several standard boxes, as is typically done. The outcomes derived from this conventional grid are then compared to those from two of our automatically refined tailored grids.

This analysis is expected to provide valuable insights into the trade-off between simulation accuracy and computational expense, guiding the selection of an optimal resolution criterion for numerical simulations of complex turbulent flows.

Introduction

Creating high-quality numerical grids is a fundamental step in the simulation process, often demanding a considerable amount of time. To achieve grid-independent numerical solutions, typically multiple iterations of grid generation and subsequent flow simulations are performed until the results show only negligible changes. Reductions in this effort directly translate into decreased temporal and computational costs.

The presented method is designed to expedite the attainment of the desired grid while also enabling local resolution adjustments. This is crucial for efficiency and solution accuracy, as it not only ensures refinement where necessary, but also in physically active areas. When performing DNS, resolution is usually chosen by making use of the Kolmogorov length η. In Aldudak & Foysi (2022), our aim was to identify a physically-based length scale that can guide grid resolution adjustments for statistical turbulence models. The intention was to find an universally applicable length scale, directly derived from flow physics, making user interference minimal. As such, adjusting the resolution based on this global value should result in a unique mesh tailored to the specific flow under consideration. In Aldudak & Foysi (2022) we suggested the Dissipation Element length (DE) as a "unique, physically-based length scale" and to be a basis for this procedure. The intermediate spatial structures associated with turbulent scalar fields, are identified by linking pairs of zero-gradient extremal points. Gradient trajectories of finite length, originating from every point in the scalar field and moving in the directions of increasing and decreasing scalar gradients, should invariably reach a minimum and a maximum point. A DE is characterized by the union of all points along the trajectories that belong to the same pair of extremal points. Consequently, DEs are directly derived from the turbulent flow field and are therefore irregularly shaped, but filling the entire space. This implies that a turbulent scalar field should qualify to be decomposed into these elements. Quantities such as velocity components, turbulent kinetic energy k or its dissipation rate ε can be selected for this decomposition, for example.

The DE concept was pioneered by Peters & Wang (2006), analyzing passive scalar fields of homogeneous shear turbulence and their small-scale statistics by using DNS. An important result was that the mean DE length ℓ_{DE} is of the order of the Taylor scale, defined as $\lambda = (10\nu k/\varepsilon)^{1/2}$ in isotropic turbulence, where ν is the kinematic viscosity, k is the turbulent kinetic energy, and ε is the dissipation rate. However, an accurate derivation of DE length scales is not feasible when using methods like RANS, therefore a relationship between the DE and Taylor length scale is required to allow determining the needed parameter for grid refinement in RANS simulations.

In a series of studies by Aldudak & Oberlack (2009), Aldudak & Oberlack (2011), Aldudak & Oberlack (2012) and Aldudak (2012), the DE methodology has been explored using direct numerical simulations of turbulent channel flow. These works examined the direct and indirect influence of rigid walls on flow geometry and flow structures, revealing a clear dependency of the DE length on the wall-normal direction. It was observed that ℓ_{DE} increases linearly starting from the logarithmic region in the wall-normal direction y, before reaching a plateau at the channel center h, explained by the decreasing shear. The linear behavior is mainly detectable in those flow regions, where the turbulent production of the turbulent kinetic energy (TKE) k and dissipation ε are of similar magnitude, at high enough Reynolds numbers (Hinze, 1975). This region was determined between $y^+ = u_\tau y/\nu = 30$ and $y/h = 0.7$ (Aldudak, 2012) and agreed well with results of Lee & Moser (2015), where the balance of production and dissipation over the wall-normal distance of the TKE was determined in a turbulent channel flow DNS for $Re_\tau = u_\tau h/\nu = 5200$. h denotes the channel half width and $u_\tau = \sqrt{\tau/\rho}$ the friction velocity, with wall shear stress τ and density ρ. The region of relative balance corresponds well with the above-mentioned range where the mean DE length scale increases linearly. Moreover, it was found that DE length scale decreases with increasing Reynolds number Re, i.e. $\ell_{DE} = f(y, Re)$, similar to the Taylor length λ. Considering the above-mentioned space-filling characteristics of the DEs, the number of structures generated decays with the distance from the wall and rises with the Reynolds number in a wall-bounded turbulent flow.

In Aldudak (2012) the relation between DE and typical classical length scales were explored. Due to the above mentioned pioneering work, particular focus was put on the intermediate Taylor microscale λ. An important result was, that the Dissipation Element length is proportional to λ in incompressible turbulent channel flows across almost the full channel height ($\ell_{DE} \quad \lambda$). Although both turbulent length scales clearly depend on the wall-normal distance y, their quotient remains widely insensitive to y over a large part of the channel height. This finding directly suggests that the DE length scale can be reliably estimated by making use of the Taylor microscale, which can be determined without problems in RANS simulations, too. This behavior can be used to find an expected value for the proportionality factor by plotting the ratio ℓ_{DE}/λ as a function of wall-normal distance. In the investigated range of Reynolds numbers, the quotient ℓ_{DE}/λ fell between 2 and 3, which is in line with other work investigating shear flows, for example Wang & Peters (2006).

In the present work, the Dissipation Element study has been extended to the higher Reynolds number $Re_\tau = 2003$ (Hoyas & Jiménez (2006)). The DE analysis is done using a code originally written by Peters & Wang (2006). Furthermore, typical benchmarks of varying degree of complexity are discussed by employing the proposed criterion for grid refinement.

Results

In the OpenFOAM framework (Weller et al. (1998)), a solver was developed to automatically identify and refine numerical cells during the simulation process if their size didn't follow a prescribed inequality of the form

$$\mathsf{D} < \ell_{DE} \quad a \ \lambda, \tag{1}$$

where a was confined to the interval $[1, 6]$. As a starting point, a very coarse mesh with uniform grid cell size is created to initialise the simulations. The length of the cubic cell, D, is determined by the cubic root of the individual cell volume. As soon as the simulation reaches an intermediate steady state, the algorithm further refines all cells that exceed the resolution criterion. Conversely, cells smaller than half the resolution criterion are coarsened. Following each refinement step, the previous solution is linearly interpolated from the old onto the new mesh using the new solver. This serves as a new initial condition for the simulation until another intermediate steady state is achieved. This process of refinement and interpolation is repeated until all cells meet the resolution criterion and a steady state is reached. The algorithm proved successful in producing high-quality grids with flow-physics guided local refinement regions. Note that the criterion is the only user-defined specification to guide the automatic refinement.

A visualization of the proportionality relation between the ℓ_{DE} and λ for the streamwise velocity fluctuation u at three different friction Reynolds numbers Re_τ over the wall-normal distance y/h is shown in figure 1. The lower two Reynolds number simulations were already used in Aldudak (2012). The $Re_\tau = 2003$ case is based on data of Hoyas & Jiménez (2006). The data was analyzed and the dissipation length determined using the algorithm described in Wang & Peters (2006). It can be observed that, for a particular Re, the ratio ℓ_{DE}/λ remains largely constant across most of the channel width, excluding the region close to the wall. This suggests that the DE length scale can be reliably approximated by the Taylor scale in these regions, allowing the easier to determine λ to be used for grid resolution adjustments in statistical turbulence models. Overall, ℓ_{DE}/λ varies between 1 to 3, which provides a practical guideline for setting the intended grid resolution criteria. The ratio exhibits a slightly greater degree of wall-normal variation at the highest Reynolds number ($Re_\tau = 2003$) compared to the observed variation at lower Reynolds numbers. In the core region of the channel ($y/h > 0.7$), the curves representing the ratio at different Reynolds numbers show smaller differences compared to the near wall region, associated with the decrease in shear experienced in this region. A noteworthy observation from the image is the decrease of the ratio ℓ_{DE}/λ with an increase in Reynolds number. Whether this trend will persist or rather shift towards a plateau at higher Reynolds numbers is a compelling question which we cannot reliably answer and which is currently under investigation.

$\mathsf{D}/\lambda <$	1	2	3	4	5	6
mean $h\mathsf{D}/\lambda i$	0.79	1.35	1.84	2.50	3.20	3.61
Cells [10^6]	113.13	20.08	2.70	0.76	0.27	0.15

Table 1: *Grid resolution criteria along with their mean ratios (ratio of cell length* D *and Taylor scale λ) and corresponding total cell numbers of the automatically tailored grid for the surface-mounted cube case.*

For assessing the influence of our proposed grid resolution criterion on the accuracy of numerical simulations and the resulting grid size, a series of numerical computations of different flow cases has been conducted.

First, the tailored grid approach was applied to the surface mounted cube (Martinuzzi & Tropea (1993)) using RANS, where no-slip boundary condition were applied to the horizontal top, bottom walls and the cube, respectively. The Reynolds number, based on the cube height H, was set at $Re_H = 40000$. The spatial dimensions in the streamwise, spanwise and wall-normal directions were $19.5H \quad 9H \quad 2H$. For more details refer to Martinuzzi & Tropea (1993), where the flow was experimentally investigated thoroughly. Figure 2(a) exemplarily depicts final tailored grids obtained by our algorithm for four relevant resolution criteria after starting with a very coarse initial grid. All grids were refined to satisfy the respective resolution criterion as listed in table 1. While their resolutions differ, important areas are identified and refined well. For each grid, the refinement level is highest near the walls and around the cube, as well as in regions with large gradients, transitioning to less refined regions using consecutive refinement levels.

As expected, as the ratio ℓ_{DE}/λ decreases, the cell size increases drastically. Table 1 depicting the cell size, indicates an almost exponential growth in cell size as the ratio decreases. This underscores the importance of carefully selecting ℓ_{DE}/λ to strike a balance between simulation accuracy and computational cost. Furthermore, the resulting mean ratios averaged over the entire simulation domain are also depicted. The proposed resolution criterion of $\mathsf{D}/\lambda < 3$ for every cell (Aldudak & Foysi (2022)) resulted in an overall mean D/λ of the automatically tailored grid of approximately 1.8 as presented in table 1. Here, D represents the grid spacing for cubic hexahedral cells and is calculated using the cubic root of the individual cell volume. This result aligns well with the findings depicted in figure 1 for higher Reynolds numbers.

To further evaluate the accuracy of our simulations, we compared our results for different resolution criteria, ranging from 1 to 5, with experimental data from Martinuzzi & Tropea

(1993). This comparison aims to identify which resolution criterion offers the best cost-performance ratio. For this, the grids were used to perform steady-state *k-ω-SST* (Langtry & Menter, 2009) RANS simulations.

As shown in figure 2 (b), our simulations yield accurate velocity results across all tested resolution criteria except for $\mathrm{D}/\lambda < 5$ and higher, where the average resolutions are clearly above the required criterion as highlighted in table 1. The degree of agreement with the experimental results increases as the grid resolution is enhanced. It can be seen that the coarsest grid depicted, corresponding to the resolution criterion $\mathrm{D}/\lambda < 5$, exhibits the strongest deviations from the experimental results. In contrast, all other tailored grids with higher resolutions yield improved results exhibiting minimal variation among each other. These resolution criteria, which have a common characteristic of a mean resolution ratio less than 3, can be considered appropriate. This underscores that our proposed grid resolution criterion, $\mathrm{D}/\lambda < 3$, can effectively guide grid refinement in conjunction with statistical turbulence models, thereby enhancing both the efficiency and accuracy of turbulent flow simulations. The method automatically refines the mesh in areas where it is reasonable, such as near walls, wake regions or areas with large gradients with minimal user input. Obviously, this results in an overall grid that corresponds to the flow characteristics. Note that the transition between different levels of refinement is smooth.

In order to further substantiate the efficacy and universal applicability of our method, we apply the same parameter variation to a more demanding turbulent test case, namely the generic Ahmed body configuration with a 25° slanted back face (Ahmed et al. (1984)). Despite its simple shape, the Ahmed body demonstrates complex flow characteristics related to car aerodynamics, including a pair of counter-rotating vortices and a pronounced separation and recirculation area. The Reynolds number based on the length of the body is $Re = 2.8 \times 10^6$.

In this case, we compare the results from two tailored grids ($\mathrm{D}/\lambda < 3$ and 4) with those of a conventional numerical grid. The corresponding tailored grids contain about 48 million and 19 million cells, respectively, while the regular mesh consists of 29 million cells. Regular meshes are usually generated by refining user-defined areas by hand, where significant physical effects are expected and are thus dependent on the experience of the researcher at hand. The resolution of the present regular grid matches that of the tailored grid, which contains 19 million cells. This is ensured by enclosing and refining all cells of the same resolution level, which are identified by our algorithm, using generic boxes. In addition to the question of whether the proposed criterion is appropriate, the proposed method is tested against the regular grid in terms of efficiency and accuracy. Upon completion, refinement regions of varying intensity according to the specified resolution criterion with finer cells near walls as well as in separation zones are generated (figure 3)(a). Interestingly, the grid refinement process takes into account the characteristic longitudinal counter-rotating vortices that occur in the body wake as depicted in figure 3 (a)(right). Here, the finest grid cell level is shown which correlates with the above-mentioned vortices, closely mimicking the character of the underlying flow physics. Steady-state *k-ω-SST* RANS simulations have been performed to acquire velocity profiles at several positions along the center line of the bluff body wake. Similarly to our findings from the surface-mounted cube simulations, the results obtained for the turbulent flow around the Ahmed body exhibit good agreement with experimental data from Lienhart et al. (2002) for all meshes (3 (b)).

Despite the significant difference in cell count due to different resolutions criteria, both tailored grids produce practically identical results, demonstrating that the coarser grid ($\mathrm{D}/\lambda < 4$), which achieved an average ratio $\langle \mathrm{D}/\lambda \rangle \approx 2.9$, was able to sufficiently resolve the turbulent flow (figure 3(b). The finer mesh, on the other hand, reached mean a $\langle \mathrm{D}/\lambda \rangle$ of 2.3. The practical absence of differences between the two resolution criteria ($\mathrm{D}/\lambda < 3$ and 4) indicates that ensuring a mean resolution criterion of less than 3 seems to be reasonable for obtaining accurate results in the case of RANS simulations.

While the tailored grid approach verifies each individual cell for its fulfillment of the refinement criterion and refines or coarsens it as needed, the regular grid generation relies on generic bodies such as boxes to identify refinement areas. This inevitably leads to significantly more grid cells, as many cells are captured by the refinement which are not in regions of strong shear or physically active. In the present study, where the regular grid satisfies $\mathrm{D}/\lambda < 4$, this amounts to an increase of more than 50% in the number of cells.

Numerical meshes produced with the tailored grid approach yield practically the same results as the conventional mesh, underscoring the robustness of the method. These findings suggest that using our approach, it is possible to achieve the same accuracy in the results with considerably fewer cells than with a commonly used conventional mesh and reduces user input. This is a significant advantage, as it allows for more efficient use of computational resources. In addition, the grid generation task is greatly accelerated and expensive grid independence studies are avoided since the ready-for-use grid is obtained directly.

Conclusion

In the present work, a recently proposed tailored grid approach by Aldudak & Foysi (2022) was further examined, where Dissipation Element (DE) length scales, ℓ_{DE}, serve as guidline for constructing a grid resolution criterion. The connection between DE and the well-known Taylor length scale λ, which is typically accessible to statistical simulation techniques, is known from previous studies. In order to obtain useful estimates for the dissipation length to Taylor length dependence, we made use of turbulent channel flow simulations, where both length scales can be calculated from the DNS data accurately. The analysis revealed that the ratio of DE to the Taylor length scale, D/λ, starting from the logarithmic region, exhibits only minor variations across the majority of the channel height. New data from Hoyas & Jiménez (2006) was analyzed, too, to add a higher Reynolds number case. It showed a further decreased ratio ℓ_{DE}/λ as expected, but no significant different behavior. Based on this investigation, the DE length is proposed to be determined via the Taylor scale to automatically refine the grid whenever the grid cells exceed a predetermined threshold. This threshold is user-definable, but the channel flow simulations suggest choosing a value around 3 for the mean ratio $\langle \mathrm{D}/\lambda \rangle$ to strike a balance between accuracy and computational expense.

Benchmark cases have been examined using the suggested meshing technique. It was observed in all instances that a well-structured numerical grid with a smooth transition was built by applying the resolution criterion, without the need for any additional external intervention. The produced numerical grids are demonstrated to closely adhere to the physical characteristics of the underlying turbulent flow, following relevant flow features accurately, thereby avoiding unnecessarily refined cells and are capable of replicating the validation results with acceptable accuracy. A thorough comparison between different resolution thresholds as well as to classical meshes was conducted. It is shown that the tailored grid method produces meshes with significantly fewer cells than regular grids of similar resolution while maintaining the same accuracy. Through this approach, costly grid independence studies can be minimized, a contrast to traditional grid generation methodologies.

Our study demonstrates that careful selection of the grid resolution criterion leads to accurate and computationally efficient turbulent flow simulations by effectively guiding the grid refinement. We believe that our proposed criterion, based on the relationship between Dissipation Element length and Taylor length scale, provides a promising approach for automatic grid refinement in statistical turbulence models. This approach can effectively enhance both the efficiency and accuracy of turbulent flow simulations. A potential area for future research is the application of the tailored grid approach to scale-resolving simulations, such as DES and LES.

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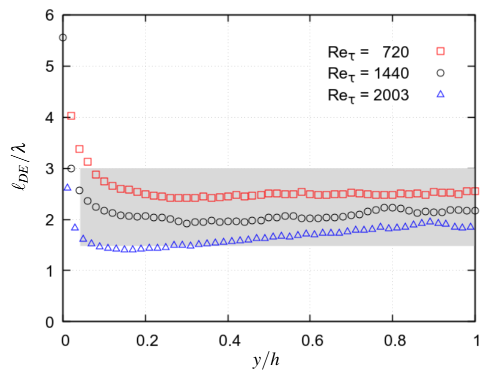

Figure 1: Proportionality relation between DE and classical Taylor length scales for the streamwise velocity fluctuation u at different Re_τ over wall-normal distance y/h. Gray area starts at the log-layer and marks the region of validity for the refinement condition. Data for $Re_\tau = 2003$ obtained from Hoyas & Jiménez (2006).