THREE-DIMENSIONAL EFFECTS IN TURBULENT SHEAR LAYERS

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ABSTRACT

The effects of three-dimensionality in shear layers, while expected to be very common in reality, has received relatively little attention in the literature. The current study is focused on using Direct Numerical Simulations (DNS) to investigate the qualitative and quantitative effects by studying the temporal evolution of shearing two misaligned turbulent boundary layers. A skewed shear layer with 25° misalignment between its freestreams is considered and analyzed in a moving reference frame aligned with the flow's mean shear direction. While the mean-shear-aligned velocity profile appears to evolve self-similarly for both planar and skewed shear layers, the latter also has an initial non-zero mean flow component in the direction orthogonal to mean shear that evolves like a self-similar planar jet. The Reynolds stress profiles initially have a large mismatch, but over time tend to overlap over one another, suggesting that any three-dimensional effects are transient and that a skewed shear layer should be similar to a planar shear layer when viewed in the right coordinate frame in its long-term evolution.

INTRODUCTION

Turbulent shear layers are often encountered in engineering applications and are one of the building blocks of turbulent flows. While most canonical studies have often been restricted to planar shear layers with zero crossflow, more realistic applications involve three-dimensional effects. The current project is motivated by the observation that three-dimensionality in turbulent boundary layers leads to a reduction in the Reynolds shear stress (Eaton, 1995; Lozano-Durán *et al.*, 2020). The leading hypotheses for explaining the effect of three-dimensionality in boundary layers is that skewing causes misalignment between turbulent eddies which then decreases their ability to interact. Since the dominant turbulent structures in shear layers differ from that in boundary layers, it is possible that shear layers respond differently to three-dimensional effects. The current project is focused on using both numerical simulations and experiments to develop a quantitative as well as qualitative understanding of these effects in turbulent shear layers. However, this article will only elaborate on the numerical simulations aspect of the project.

Prior computational studies on the topic include the work of Lu & Lele (1993), who performed a linear stability analysis for an inviscid instability of a skewed compressible shear layer. The instability amplification rate was found to be larger for skewed shear layers. More recently, Meldi *et al.* (2020) and Boukharfane *et al.* (2021) performed DNS for spatially evolving skewed shear layers with an initial hyperbolic tangent mean profile perturbed with random perturbations in space and time for incompressible and compressible shear layers, respectively. The former also involved a senstivity analysis using two parameters: the skew angle between the two freestreams θ, and the mean shear parameter $\alpha = ||\vec{U}_1 - \vec{U}_2||/||\vec{U}_1 + \vec{U}_2||$, with $\vec{U}_1$ and $\vec{U}_2$ as the two freestream velocity vectors. While α was primarily noted for its effect on the shear layer growth, large values of θ also contributed towards the development of the mixing region. The initial conditions in these studies however lacked any physical turbulence structures, and one could argue that these flows are essentially two-dimensional, if viewed from a reference frame aligned with the flow's mean shear direciton.

In the current work, we generate three-dimensional turbulent shear layers by shearing two fully-developed turbulent boundary layers at their interface. One of the initial works on this approach is by Hackett & Cox (1970) who studied the shear between two boundary layers flowing perpendicular to each other at their interface using theoretical and experimental methods. The three-dimensional shear layers showed self-similar growth downstream and $\approx 40\%$ increase in the Reynolds shear stress (normalized with the square of velocity difference) as compared to a planar shear layer. Fric (1996) performed similar experiments, although with a smaller skew angle of 39° between the two boundary layers. They noted an increase in mixing with skewing, although this could be due to a simultaneous increase in the velocity difference ΔU between the two streams. More importantly, largest differences were noted in the early transient. Later, Azim & Islam (2003) studied planar and skewed shear layers over a range of freestream velocity ratios of the two boundary layers. They similarly concluded that while both types of shear layers attained self-similarity in terms of the mean velocity and Reynolds stress profiles, the skewed shear layers grew faster and attained self-similarity earlier as compared to planar shear layes at the same velocity ratio. These observations are supported by an earlier review article by Fiedler *et al.* (1998), who concluded that while three-dimensional shear layers are similar to their planar counterparts in their spread and structural development, the three-dimensionality destroys the otherwise two-dimensional structures, leading to increased mixing. The literature review hence suggests that the early transient encountered when the boundary layers start shearing at their interface until they achieve self-similar behavior might be more relevant for the analysis of three-dimensional effects.

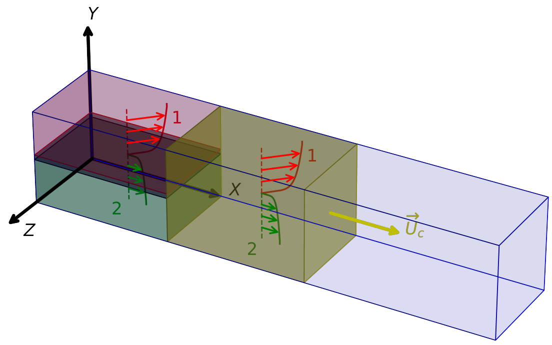

Figure 1. Schematic of the computational domains for the boundary layer and shear layer simulations. The blue box denotes the lab frame resembling an equivalent wind tunnel with a splitter plate on the left boundary. The red and green boxes above and below the splitter plate, respectively, denote the simulation domain for the top and bottom boundary layers 1 and 2. These boxes are stacked vertically to generate the shear layer initial condition, denoted by the yellow box, which convects downstream with the mean convection velocity $\vec{U}_c = (\vec{U}_1 + \vec{U}_2)/2$.

METHOD

The simulation setup is shown in fig. 1. The blue box denotes the lab frame where boundary layers could be generated on either side of the splitter plate (on the left boundary) with flow from left to right. The coordinate axes are defined such that x points along the downstream wind tunnel direction, y is normal to the splitter plate, and z is along the span. The boundary layers on the top and bottom surfaces of the splitter plate, labeled as 1 and 2, have mean freestream velocities $\vec{U}_1$ and $\vec{U}_2$, and skew angles θ_1 and θ_2 with respect to x-axis, respectively. The shearing of these boundary layers at their interface past the splitter plate trailing edge generates turbulent shear layers. We recreate an ideal version of this setup for our numerical experiments, where we first simulate temporally evolving boundary layers 1 and 2 in a box with periodic boundaries in the x and z directions, denoted by the red and green boxes in fig. 1, respectively. Once fully developed, an instantaneous volume snapshot of boundary layer 1 is stacked vertically on top of another volume snapshot of boundary layer 2 to create an idealized flow state just after the splitter plate trailing edge. This is denoted using the yellow box in fig. 1 and this flow state is used as an initial condition for the shear layer simulations in a similar double-periodic box, albeit twice as tall as that of the boundary layer simulations. The current setup thus assumes a zero-thickness splitter plate and hence does not perfectly model the merging of the two streams right after the plate's trailing edge, as one would observe in reality. However, it gives us great flexibility towards picking initial conditions that could help us better understand the underlying three-dimensional effects, as compared to spatially evolving simulations with splitter plate of finite thickness.

Considering that the long-term evolution of shear layers is expected be along the mean shear direction $\vec{U}_1 - \vec{U}_2$, both planar and skewed cases are analyzed in the mean shear frame instead of the lab frame. For this, we first define the mean convection velocity $\vec{U}_c = (\vec{U}_1 + \vec{U}_2)/2$ (denoted with an yellow arrow in fig. 1) and rotate the flow relative to $\vec{U}_c$ such that

$$\hat{u}(y)\vec{m} + \hat{w}(y)\vec{n} = \vec{U}(y) - \vec{U}_c(y) \qquad (1)$$

where, $\vec{m}$ and $\vec{n}$ are unit vectors defined in the $x-z$ plane with $\vec{m} \parallel (\vec{U}_1 - \vec{U}_2)$ and $\vec{m} \perp \vec{n}$. For skewed shear layers, there would be a non-zero mean flow component along $\vec{n}$ near the interface, which is expected to decay down to zero over the shear layer's long term evolution. Hence, our hypothesis is that in the mean shear frame, any three-dimensional effects are expected to be transient in nature and the skewed shear layer should evolve like a planar shear layer over its long-term evolution.

A baseline planar shear layer is simulated such that $||\vec{U}_1||/||\vec{U}_2|| = 1.5$ and $\theta_1 = \theta_2 = 0°$. The initial shear layer Reynolds number $Re_{SL,t=0} = \rho_e \Delta U \delta_o/\mu_e = 5000$, where $\Delta U = ||\vec{U}_1 - \vec{U}_2||$ and δ_o is the initial shear layer thickness, computed as the sum of the two boundary layer thicknesses and subscript 'e' denotes freestream properties. For the skewed shear layer simulation, in order to maintain the same $Re_{SL,t=0}$, we choose $||\vec{U}_1||/||\vec{U}_2|| \approx 1.17$, $\theta_1 = 25°$ and $\theta_2 = 0°$. Both simulations are performed in a domain of size $L_x/\delta_0 = L_z/\delta_0 = 25$ and $L_y/\delta_o = 100$, with $y = 0$ marking the location of the interface between the two streams. The initial grid size (which is decided by the precursor boundary layer simulations) is $N_x \times N_y \times N_z = 1200 \times 730 \times 1200$ with uniform grids in x and z and a stretched grid along y on either side of the interface. However, as discussed ahead, once the interface between the top and bottom streams smoothens out, the solution is interpolated on a relatively coarser grid along y with grid size $N_x \times N_y \times N_z = 1200 \times 648 \times 1200$. Lastly, both the temporally evolving boundary layer and shear layer simulations are performed using the "Hybrid" code, a finite-difference code with low numerical dissipation that has been used in several previous studies.

RESULTS

Figure 2 shows the temporal evolution of the profiles of mean velocity components $\hat{u}/\Delta U$ and $\hat{w}/\Delta U$, plotted as functions of $\hat{y}/\delta_{tke}$ and y/δ_{tke}, respectively, with time $t^* = t\Delta U/\delta_o$. The former has its transverse coordinate shifted as $\hat{y}(t^*) = y - y(\hat{u} = 0, t^*)$, with both plots scaled using δ_{tke}, a measure of shear layer thickness based on thresholding the turbulent kinetic energy ($\geq 5\%$ of the maxima at the time instance) on either side of the shear layer. The initial conditions at $t^* = 0$ are shown in the inset plots. Viscous diffusion effects quickly dissipate the initial sharp cusp in both profiles. This is followed by the evolution of $\hat{u}/\Delta U$ to reach a fully-developed self-similar state, with the skewed shear layer making this transition faster, expectedly, due to its smaller initial velocity deficit. For the $\hat{w}/\Delta U$ profile in skewed shear layers, the monotonic decay and profile shape resembles to a planar jet-like velocity profile. Hence in fig. 3 we rescale the $\hat{w}$ profile with pla-

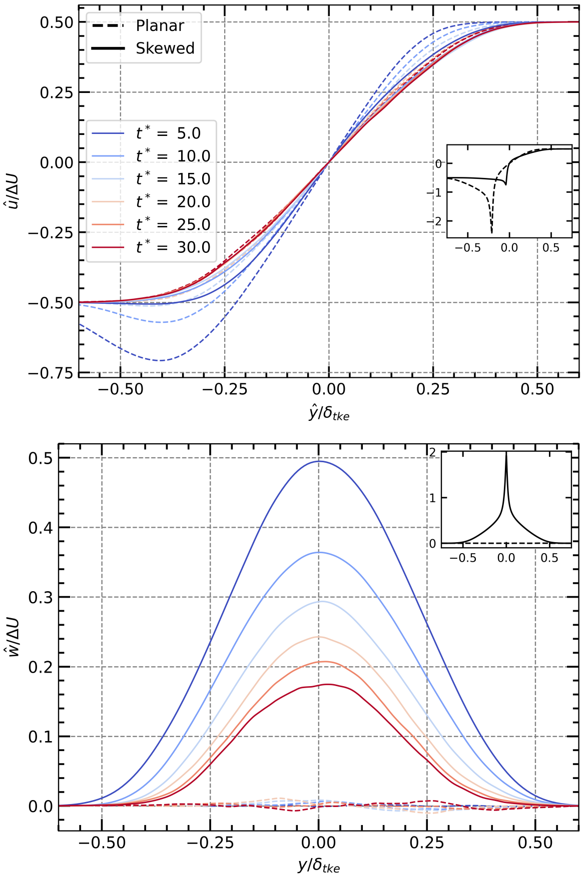

Figure 2. Temporal evolution of the mean velocity profiles $\hat{u}/\Delta U$ (top) and $\hat{w}/\Delta U$ (bottom), as functions of $(\hat{y}/\delta_{tke})$ and (y/δ_{tke}), respectively, in the rotated $\vec{m}-\vec{n}$ frame for the planar and skewed shear layers. Inset plots show the initial condition for each case at $t^*=0$.

nar jet scaling and plot $\hat{w}/\hat{w}_o$ as a function of $(y/y_{1/2})$, where $\hat{w}_o(t^*) \equiv \hat{w}(y=0,t^*)$ is the centerline velocity and $y_{1/2}(t^*)$ is the jet half-width $(0.5\hat{w}(t^*) \equiv \hat{w}(y_{1/2}(t^*),t^*))$. This collapses the profiles and agrees very well with the approximate self-similar solution in Pope (2000) over $y/y_{1/2} \in [-1,1]$. We hence infer that $\hat{w}_o(t^*)$ decays as some function $f(1/t^*)$.

Next, fig. 4 shows the temporal evolution of the three normal Reynolds stress components $\hat{R}_{uu}/\Delta U^2$ (mean-shear-aligned), $\hat{R}_{vv}/\Delta U^2$ (transverse), $\hat{R}_{ww}/\Delta U^2$ (orthogonal to mean shear), and the shear stress component $-\hat{R}_{uv}/\Delta U^2$ in the rotated $\vec{m}-\vec{n}$ frame, with the inset plots denoting the respective initial conditions. Despite a large initial mismatch, the $\hat{R}_{uu}/\Delta U^2$ profiles for both planar and skewed shear layer evolve rapidly and overlap over one another, pointing towards reaching a self-similar state. For other components, the evolution of profiles for the planar shear layer is relatively slower, but appears to be reaching a state where it could possibly overlap with the profiles for the skewed shear layer. These observations support our initial hypothesis that indeed, in the mean shear frame, the three-dimensional effects for a skewed shear layer are transient and that it evolves similar to a planar shear layer over its long-term evolution. Additionally, the uneven shapes of profiles at later times are attributed to imperfect averaging due to the shear layer growing temporally in a computational domain of fixed size.

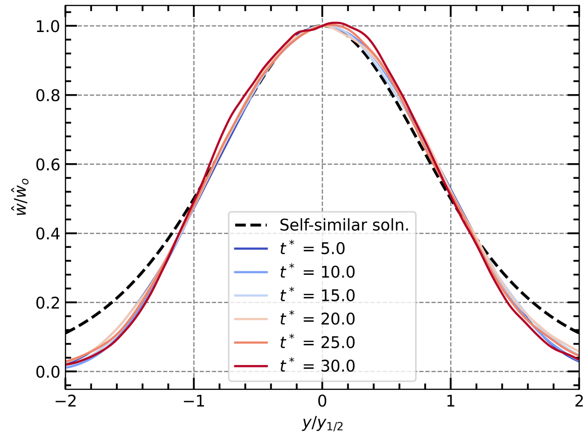

Figure 3. Temporal evolution of the $\hat{w}$ profile for the skewed shear layer, scaled as a planar jet —$\hat{w}_o$ is the jet centerline velocity and $y_{1/2}$ is the jet half-width. Profiles are overlaid on top of an approximate self-similar solution for planar jets (dashed line) from Pope (2000).

CONCLUSIONS AND FUTURE WORK

We simulate temporally-evolving shear layers, a skewed case with 25° misalignment between its two freestreams, along with a reference planar shear layer. Once fully-developed, the mean-shear-aligned velocity profiles for both cases overlap, indicating self-similar evolution. Additionally, only for skewed shear layers, there is a non-zero mean velocity component orthogonal to the mean shear direction near the interface, which decays monotonically and collapses to a self-similar profile when treated as a planar jet. Furthermore, the normal Reynolds stress tensor component along the mean shear direction appears to be the fastest to evolve and reach the same self-similar state for both the planar and skewed shear layers. Other components take longer to evolve, highlighting that the three-dimensional effects, while present, are only transient and that the skewed shear layer should also evolve as a planar shear layer over its long-term evolution.

Possible avenues of ongoing and future work involve analyzing the transient before the flow reaches a self-similar state to highlight qualitative and quantitative differences between planar and skewed shear layers. Furthermore, we plan on investigating the mismatch in the rate of evolution of different Reynolds stress components, with one approach related to creating a range of fictitious ‘what-if?’ numerical experiments to isolate the effects of skew in mean flow and turbulent fluctuations. Different skew angles could also be considered for varying three-dimensionality effects. Lastly, the uneven variation of profiles at later times, a possible artifact of imperfect averaging, will be addressed by performing these simulations over larger domain sizes.

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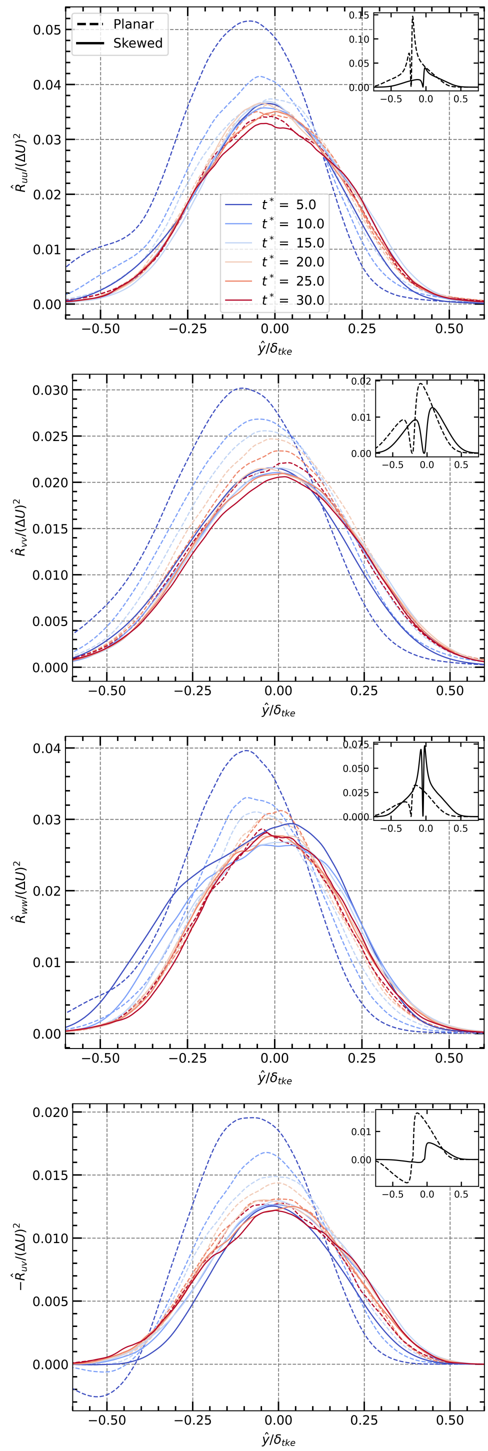

Figure 4. Temporal evolution of Reynolds stress components in the rotated $\vec{m}-\vec{n}$ frame, ordered as (from top to bottom): $\hat{R}_{uu}/\Delta U^2$ (mean-shear-aligned), $\hat{R}_{vv}/\Delta U^2$ (transverse), $\hat{R}_{ww}/\Delta U^2$ (orthogonal to mean shear) and shear stress component $-\hat{R}_{uv}/\Delta U^2$. The inset plots denote the initial condition at $t^*=0$ for the respective cases.

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