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TOWARDS AN IMPROVED UNDERSTANDING AND MODELLING OF SENSIBLE TURBULENT HEAT FLUXES OVER MELTING GLACIERS

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ABSTRACT

This paper addresses the demonstrated need for improved modeling of sensible turbulent heat fluxes over melting glaciers, critical for predicting and mitigating the effects of climate change. Through a series of wind-tunnel experiments wherein hot-wire anemometry and cold-wire thermometry were used to make simultaneous measurements of velocity and temperature (respectively) at various heights and free-stream velocities in the turbulent boundary layer over melting ice. The combined statistics of velocity and temperature were then evaluated with respect to their coherence spectra, which measures the normalized contribution from different scales to the sensible turbulent heat flux, as well as their normalized Probability Density Functions (PDFs), which are used to explain the relationship between velocity and temperature fluctuations.

Using the measured statistics, the accuracy of the prevailing existing method (the bulk aerodynamic method) for estimating sensible turbulent heat fluxes over melting glaciers in field studies was evaluated using the directly-measured sensible turbulent heat flux. The average RMS error for this method was found to be approximately 11.2%. A novel method for estimating the sensible turbulent heat flux was then proposed, which used statistical quantities already attainable with most common field measurement equipment. The new method (herein called the statistical heat flux method) was found to offer much better accuracy than the existing bulk method, with a significantly improved RMS error of only 4.9% for the experiments in this study.

BACKGROUND & CONTEXT

The world is already observing the effects of climate change, which are becoming increasingly severe. To effectively mitigate these effects, accurate and robust models are critical for understanding and predicting climate change. This is especially important in polar regions, where exceptionally high warming rates are accelerating glacier melt (Rantanen *et al.*, 2022). Modelling and predicting glacier melt is typically done through a balance of the complex energy sources and sinks which govern the melting process. One of the most common models for predicting glacier melt involves the use of a surface energy balance, which is given by (Hock, 2005):

$$0 = Q_N + Q_H + Q_L + Q_G + Q_R + Q_M, \qquad (1)$$

It accounts for the net heat fluxes from solar radiation (Q_N), sensible heat fluxes from convection (Q_H), latent heat fluxes from evaporation and condensation at the surface (Q_L), heat flux to the ground via conduction (Q_G), and heat fluxes caused by rainfall (Q_R). Since they are both transported by turbulent velocity fluctuations in the air above the surface, the sensible and latent heat fluxes are together referred to as “turbulent heat fluxes.” The sum of the aforementioned components in the surface energy balance determines the total amount of energy available for melting the glacier’s ice (Q_M). Quantifying each of these heat flux components is crucial to understanding and predicting glacier melt. However, some of these components are more challenging to model than others. Of these, the so-called turbulent heat fluxes (Q_H and Q_L) are particularly difficult to model, and the sensible heat fluxes tend to be much larger than latent heat fluxes (Suter *et al.*, 2004). However, the models for latent heat fluxes are similar to those for sensible heat fluxes, such that improvements to sensible heat flux modelling will likely also improve latent heat flux modelling. Thus, there is a demonstrated need for improved modelling capability of these turbulent heat fluxes (Hock, 2005). The present work is therefore motivated by the need for an improved understanding of the sensible turbulent heat fluxes which occur in the atmospheric boundary layer over a melting glacier. By way of a series of wind tunnel experiments, in which simultaneous measurements of longitudinal velocity, vertical velocity, and temperature are measured over a melting ice surface, this paper attempts to answer the following questions:

1. How do the combined statistics of velocity and temperature measured over a melting ice surface change with surface height and free-stream velocity?
2. How do current methods for estimating sensible heat fluxes over glaciers compare with actual sensible heat

fluxes measured in these experiments?

3. Is there a better method for modelling sensible turbulent heat fluxes over melting ice?

EXPERIMENTAL APPARATUS & PROCEDURES

The experiments in this work were performed in an open circuit wind tunnel in the Aerodynamics Laboratory at McGill University, which is described in detail in (Harrison, 2022). To best emulate highly turbulent atmospheric flow – which has turbulent intensities on the order of 10% (Litt *et al.*, 2015) – a wind tunnel with an active grid (Makita & Sassa, 1991) was used. A specialized test section was designed and built for these experiments to facilitate the inclusion of a large ice patch, used as a model for a melting glacier. Measurements of velocity and temperature were made using hot-wire anemometry and cold-wire thermometry, respectively. In the experiments performed in this work, three free-stream velocities (approximately 1.1, 2.0, and 3.2 m/s) and two measurement heights (corresponding to y/δ of 0.13 and 0.33) were chosen, resulting in six different cases in the test envelope. These free-stream velocities were chosen to maximize the stratification as quantified using the Richardson number, while also being representative of the speeds typically observed in glacier flows (Mott *et al.*, 2016). A schematic of the wind tunnel, which illustrates the experimental setup, is shown in figure 1.

COMBINED STATISTICS OF VELOCITY AND TEMPERATURE OVER MELTING ICE

To assess the joint statistics of the (vertical) velocity and temperature, coherence spectra and joint probability density functions (PDFs) were computed. A detailed analysis of the single- and multi-component statistics is performed in Harrison & Mydlarski (2023). In all cases, the coherence between v and θ increases with free-stream velocity (i.e. Reynolds number), as increased turbulent activity increases the turbulent heat transfer. Peak coherence was observed at approximately $\kappa_1 \eta = 10^{-3}$ for the highest free-stream velocity at all heights, which was not observed in the coherence spectra of u and v. This peak suggests a preferential scale for the exchange of heat in the vertical direction that is not the largest scale of the flow, which has been previously observed in near-neutral turbulent boundary layers over cooled water (Mestayer, 1982).

The overall magnitude of the coherence (and therefore the correlation coefficient, $\rho_{v\theta}$, was not found to scale predictably with height. This result is consistent with those of Arya (1975) and Ohya *et al.* (1997), who found that profiles of $\rho_{v\theta}$ exhibited a distinct peak at varying wall-normal distances. The joint PDFs of the vertical velocity and temperature fluctuations were found to vary primarily with height and showed little dependence on the free-stream velocity. As expected given the PDFs of v (which deviated from Gaussian behaviour in the tails) and θ (which were increasingly non-Gaussian with increasing height), none of the joint PDFs are joint-Gaussian, although the tendency to joint-Gaussian PDFs increases with height, apart from a long tail associated with rare temperature fluctuations. This is shown in figure 2. The largest temperature fluctuations were also typically associated with smaller velocity fluctuations (i.e. the lowest point of the contour is near the y-intercept of $v = 0$). Closer to the surface (figure 2 a), larger velocity fluctuations are associated with smaller temperature fluctuations (i.e. near the x-axis of $\theta = 0$). This may be thought of as better mixing of heat energy due to increased shear and therefore increased turbulent kinetic energy which helps to dissipate heat energy. As the height is increased (figure 2 b), larger negative velocity fluctuations are still associated with small temperature fluctuations, but larger positive velocity fluctuations are also associated with larger positive temperature fluctuations. In other words, upward motion brings warmer air parcels, as buoyant and turbulent motions are aligned, perhaps due to entrainment of warm air from outside of the boundary layer (Lortie, 2022).

EVALUATION OF EXISTING SENSIBLE HEAT FLUX MODELS

The directly-measured sensible turbulent heat fluxes over melting ice were used to evaluate the accuracy of one of the most common models used in field studies of glaciology. This method is typically referred to as the bulk aerodynamic method, or the bulk method for short. Based on the work of (Prandtl, 1935), the flux of (momentum or) a passive scalar in a wall-bounded flow can be estimated using the mean gradient and an "eddy diffusivity." Assuming a constant (0°C) surface temperature (Θ_S), and integration of Prandtl's mixing length model equation (see Tennekes & Lumley, 1972), the sensible heat flux can estimated as:

$$Q_H = \rho C_p C_H \langle U \rangle \left(\langle \Theta \rangle - \Theta_S \right), \tag{2}$$

where C_H is the so-called bulk exchange coefficient, from the Monin-Obukhov profile equations. Since the bulk method is derived from the gradient transport model, it is technically only valid for near-neutral stability conditions (Tennekes & Lumley, 1972). C_H is parameterized by is the aerodynamic roughness length, which is subject to high levels of uncertainty (Chambers *et al.*, 2020), making it difficult to model accurately (Hock, 2005). To overcome this, typical parameterizations of C_H use the bulk Richardson number instead (Webb, 1970):

$$C_H = \frac{\kappa}{\ln\left(\frac{y}{y_0}\right) ln\left(\frac{y}{y_{0_\theta}}\right)} (1 - 5.2 Ri_b)^2, \tag{3}$$

where the Richardson number is calculated as:

$$Ri_b = \frac{gy}{\langle T \rangle} \frac{\langle \Theta \rangle - \Theta_s}{\langle U \rangle^2}, \tag{4}$$

and where the roughness parameters y_0 and y_{0_θ} are typically on the order of 10^{-3} m for a real glacier (Fitzpatrick *et al.*, 2017). Using values representative of real, full-scale glaciers was not appropriate given the smaller scales of the present work. Similarly, using the RMS roughness value of the ice surface (approximately 0.4 mm) was not appropriate, since the surface roughness does not share the same definition as the aerodynamic roughness. Therefore, to compare the results of the bulk method to the true values measured in the present work, the parameters y_0 and y_{0_θ} had to be estimated using other methods. For experiments performed at the same height, y, y_0, and y_{0_θ} are all constant, and equation (3) reduces to:

$$C_H = K(1 - 5.2 Ri_b)^2, \tag{5}$$

where K is a constant.

In the present work, a thermocouple embedded just below the surface of the ice was used to approximate the true surface temperature Θ_S throughout the course of the experiment. Using this value, as well as analyzing the evolution in time of the measured statistics of velocity and temperature, it was possible to identify the period of time in which the surface temperature was most likely to be 0°C. For further details on this approximation, readers may refer to Harrison (2022). Using the measured values of $\langle U\rangle$, $\langle\Theta\rangle$, and $\langle v\theta\rangle$, as well as by assuming a 0°C surface temperature, the true bulk exchange coefficient C_H could then be calculated using equation 2. Having found the true bulk exchange coefficient for each of the experiments in the present work, the accuracy of equation 5 (and thereby equation 3) could then be tested by solving for the constant K using least-squares regression. The results are presented in figure 3, which shows the directly calculated bulk exchange coefficient, as well as the curve fits found using equation 5. Overall, there is better agreement at $y = 25$ mm than $y = 10$ mm, when the heat flux is lower in magnitude.

It is important to note that using the curve-fit constant K to estimate C_H and subsequently $\langle v\theta\rangle$ will provide better accuracy than using 3, since there are fewer degrees of freedom in which measurement and/or curve-fitting errors can cascade. However, using this method is not possible unless one has knowledge of $\langle v\theta\rangle$ to begin with, such that the method used to estimate C_H in the present work can be seen as an upper-limit on the accuracy of equation 3 for the given application. Using these optimal estimations of C_H, the bulk method was finally used to model the sensible heat flux. It was found to have a root-mean-square error of approximately 11.2% on average for the experiments of the present work. This value will prove useful for benchmarking the development of future models for modelling sensible turbulent heat fluxes over melting ice.

SENSIBLE HEAT FLUX MODELLING

Given the demonstrated error of the bulk method, and the associated difficulty of successfully parameterizing the bulk exchange coefficient, a new approach is proposed to better model the sensible turbulent heat flux over melting ice. This new approach makes use of statistics of the velocity and temperature fields, namely the i) root-mean-square (RMS) longitudinal velocity fluctuation u_{rms} (since it is easier to measure in the field than vertical velocity), and ii) RMS temperature fluctuation θ_{rms}, to estimate the sensible turbulent heat flux ($\langle v\theta\rangle$, i.e. the covariance of the vertical velocity and temperature). This is defined as the statistical heat flux model, in which:

$$\langle v\theta\rangle = \sigma u_{rms}\theta_{rms}, \tag{6}$$

where σ is an empirical coefficient. Figure 4 shows the strong linear relation between these two quantities for the $y/\delta = 0.33$ height. The slope, σ, was found to vary between different experiments, likely due to the varying levels of stratification as well as turbulent kinetic energy in the flow. The exact value of the scaling coefficient (σ) can be determined analytically using the equation for the correlation coefficient of v and θ:

$$\rho_{v\theta} \equiv \frac{\langle v\theta\rangle}{v_{rms}\theta_{rms}}. \tag{7}$$

Using the anisotropy ratio ($\phi \equiv v_{rms}/u_{rms}$), v_{rms} can be replaced with u_{rms} in equation (7) such that:

$$\rho_{v\theta} = \frac{\langle v\theta\rangle}{\phi u_{rms}\theta_{rms}}. \tag{8}$$

By re-arranging equation (8) and combining it with equation (6), it can be seen that:

$$\sigma = \rho_{v\theta}\phi. \tag{9}$$

Therefore, knowledge of the correlation coefficient ($\rho_{v\theta}$) and anisotropy ratio (ϕ) can facilitate direct calculation of the turbulent heat flux ($\langle v\theta\rangle$) using the statistical heat flux ($u_{rms}\theta_{rms}$). This method to calculate the turbulent heat flux requires no assumptions about the surface temperature, stability conditions, or surface roughness. Unfortunately, profiles of $\rho_{v\theta}$ for a stably stratified boundary layer show complex shapes which are difficult to predict analytically (as shown by Harrison & Mydlarski, 2023; Arya, 1975; Ohya *et al.*, 1997), especially with the limited number of measurement heights used in the present work. Furthermore, while the anisotropy was found to be strongly related to the turbulent Richardson number Ri_t (see Harrison, 2022), its calculation requires knowledge of the vertical RMS velocity, which is not a parameter typically measurable in glacier field studies (Hock, 2005; Chambers *et al.*, 2020; Fitzpatrick *et al.*, 2017).

It was therefore necessary to find an empirical correlation between the coefficient σ and a non-dimensional quantity which could be reasonably measured by an automated weather station over a melting glacier. Given the observed dominance of the turbulent heat flux by the turbulent velocity field shown in Harrison & Mydlarski (2023), it is reasonable to expect that this quantity should be related to velocity statistics. To this end, a turbulent Reynolds number (Re_y) with u_{rms} as its velocity scale and the height of the measurement (y) as its length scale was defined:

$$Re_y \equiv \frac{u_{rms}y}{\nu}. \tag{10}$$

This Reynolds number is similar to the turbulent Reynolds number Re_ℓ (see Tennekes & Lumley, 1972), whereby the integral length scale is replaced by the height y. This definition bears similarity to the definitions of the Reynolds number used in stratified boundary layer turbulence, which use the free-stream Reynolds number ($Re_\delta = \frac{U_\infty\delta}{\nu}$) to characterize the flow (Arya, 1975; Ohya *et al.*, 1997; Williams *et al.*, 2017).

To justify the choice of y as the characteristic length scale, consider Prandtl's mixing length model, often used in atmospheric science, where it has been found to be valid approximation despite its criticisms in fundamental studies of turbulence (Chan & Sofia, 1987). The mixing length model, which is also commonly used in glacier melt studies (Hock, 2005), states that the integral length scale (ℓ) will be proportional to y (Tennekes & Lumley, 1972). Therefore, Re_y employs the foundation of Prandtl's mixing length model to replace the integral length scale with the height y in the classical turbulent Reynolds number. Since the purpose of this substitution is to make this non-dimensional quantity (Re_y) an easily measurable one in glacier field studies, in which Prandtl's mixing length model is reasonably valid, the choice of y as a characteristic length scale is sensible.

Having justified the use of the turbulent mixing Reynolds number, it was used to plot the scaling coefficient (σ) for the statistical heat flux method, as shown in Figure 5. A consistent power law correlation is observed between these two quantities, with a best-fit exponent of 0.34. The predicted values of σ using the power-law fit were then used to estimate the true value of the turbulent heat flux above melting ice, similar to the procedure for the bulk method. The overall error was found to be 4.9%, less than half of that for the bulk method. For further comparison, the true heat flux values are presented alongside the predictions of the statistical and bulk methods in Table 1.

To further reinforce the utility of the newly-defined statistical heat flux method, the applications were extended to cover the full length of the experiments, which included the measurement of $\langle v\theta \rangle$ over sub-zero degree ice, as well as cold water on top of a thick layer of ice (i.e. throughout the melt process). The results are shown in figure 6, where the statistical method vastly outperforms the bulk method, especially as the magnitude of the heat flux increases.

CONCLUSION

This paper highlighted the results of a novel wind tunnel experiment in which two components of velocity and temperature were simultaneously measured over melting ice with the purpose of emulating the turbulent atmospheric flow over a melting glacier in a controlled laboratory environment. This paper was motivated by a demonstrated need for improved methods for modelling the sensible turbulent heat fluxes over melting ice for the purpose of improving glacier melt predictions, and sought to answer the following questions:

1. How do the combined statistics of velocity and temperature measured over a melting ice surface change with surface height and free-stream velocity?
2. How do current methods for estimating sensible heat fluxes over glaciers compare with actual sensible heat fluxes measured in these experiments?
3. Is there a better method for modelling sensible turbulent heat fluxes over melting ice?

These questions were responded to as follows:

1. Combined statistics of velocity and temperature show a preferential scale for the turbulent exchange of heat (Harrison & Mydlarski, 2023) which is not the largest scale of the flow. Joint PDFs of vertical velocity and temperature show changing behaviour with increasing height, though as expected when the shear is highest there is greater mixing of heat energy and subsequently smaller temperature fluctuations.
2. The most commonly used method for estimating sensible turbulent heat fluxes is highly dependent on a bulk exchange coefficient whose estimation has several degrees of freedom and as such, a high potential for induced error. Through evaluation of this method in a controlled laboratory study, for the current application a best-case implementation of the bulk method yields an 11.2% error for modelling sensible turbulent heat fluxes.
3. A novel method was presented which does not rely on the same assumptions as the bulk method, making it more generally applicable for estimating the sensible turbulent heat flux (even for other use cases) while providing less than half of the relative error (4.9% on average).

With the stated objectives of this paper accomplished through the work depicted herein, the authors recommend that the statistical heat flux method should be applied in a wide variety of other scenarios so as to refine the parameterization of σ, and ultimately improve the capability of estimating the sensible heat flux $\langle v\theta \rangle$ using large-scale statistical quantities of longitudinal velocity and temperature.

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y (mm)	U_∞ (m/s)	$\langle v\theta \rangle$ (m/s K)	$\langle v\theta \rangle_{stat}$ (m/s K)	Relative Error	$\langle v\theta \rangle_{bulk}$ (m/s K)	Relative Error
10	1.1	-0.0112	-0.0124	10.74 %	-0.0121	8.32 %
	2.0	-0.0275	-0.0267	-2.94 %	-0.0242	-12.03 %
	3.2	-0.0341	-0.0325	-4.81 %	-0.0365	7.00 %
25	1.1	-0.0048	-0.0049	1.51 %	-0.0050	4.72 %
	2.0	-0.0130	-0.0124	-4.72 %	-0.0108	-16.63 %
	3.2	-0.0163	-0.0170	4.64 %	-0.0193	18.42 %
RMS Error			-	4.9 %	-	11.2 %

Table 1: True heat flux versus statistical and bulk method predictions with relative and RMS error.

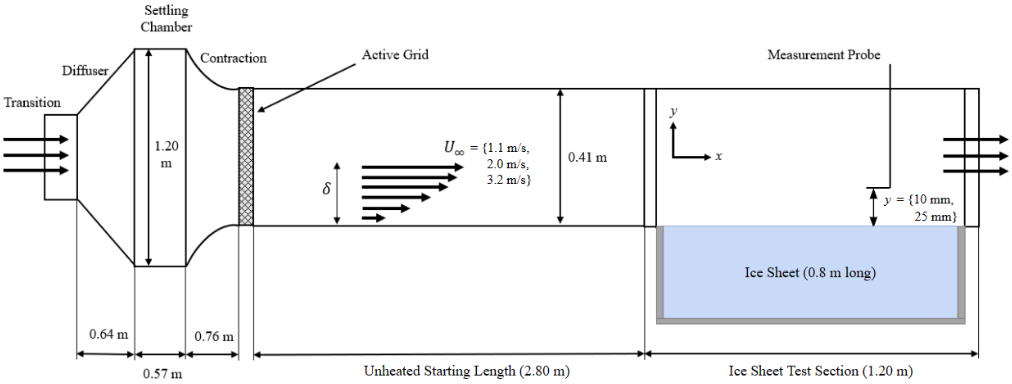
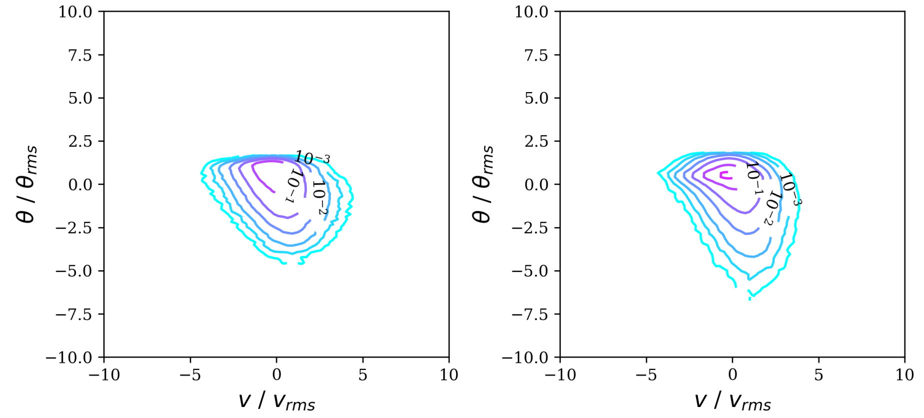


Figure 1: Schematic of the wind tunnel and experimental apparatus used for this work



(a) $y/\delta = 0.13$

(b) $y/\delta = 0.33$

Figure 2: Joint PDFs of the vertical velocity and temperature fluctuations for $U_\infty = 2.0$ m/s. Figure used with permission from Harrison & Mydlarski (2023).

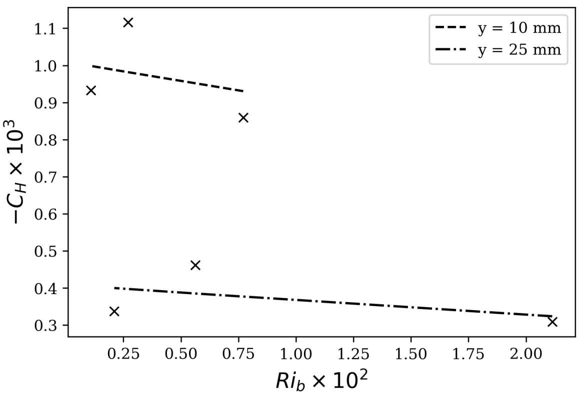


Figure 3: Bulk exchange coefficient fitted using equation (5).

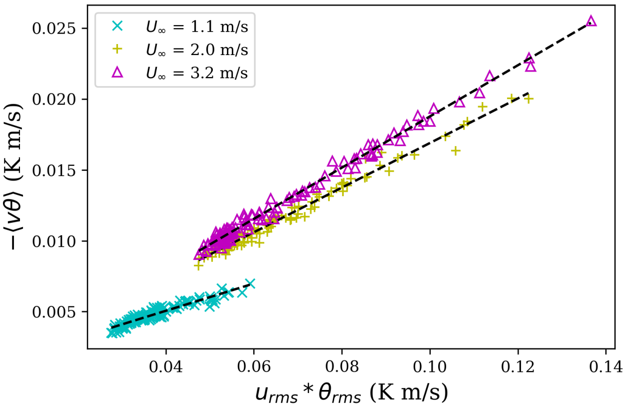


Figure 4: Statistical heat flux versus sensible turbulent heat flux for $y = 25$ mm. Dashed lines show a least-squares linear regression.

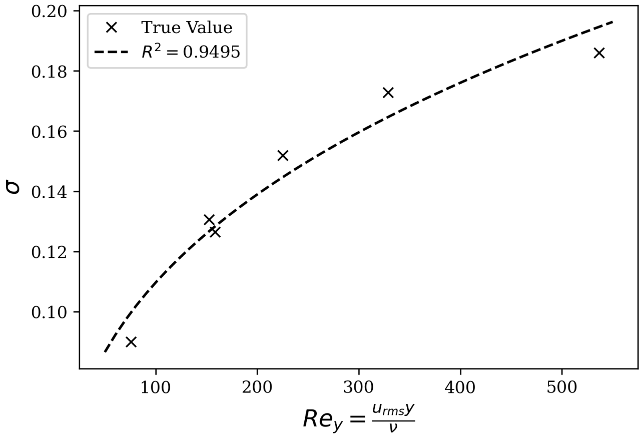
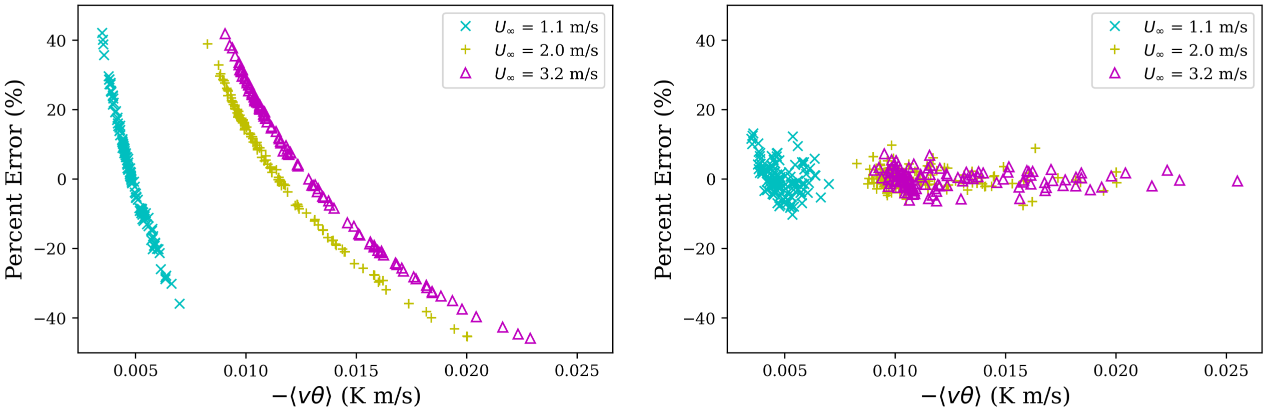


Figure 5: Statistical heat flux scaling coefficient σ plotted as a function of the new turbulent Reynolds number Re_y. The best-fit power law was $\sigma = 0.0228 Re_y^{0.34}$



(a) Bulk method

(b) Statistical heat flux method

Figure 6: Percent error as a function of the sensible turbulent heat flux for both estimation methods.