STOKES-NUMBER DEPENDENCE OF INERTIAL PARTICLE CLUSTERING IN TURBULENCE INERTIAL SUBRANGE

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ABSTRACT

The dynamics of inertial heavy particles in high Reynolds number turbulence, especially particle clustering, are important fundamental processes, e.g., for raindrop formation in atmospheric flows. To examine the clustering behavior in the turbulence inertial subrange, three-dimensional direct numerical simulations of particle-laden homogeneous isotropic turbulence is performed at resolution 4096^3 for high Reynolds number $Re_\lambda = 648$ and with 3.2×10^9 particles for several Stokes numbers. The results show that the slope of the particle number density spectra at scales in the turbulence inertial subrange is dependent on the Stokes number. It is also observed that the number density spectra in the inertial subrange obey a function of the scale-dependent Stokes number.

INTRODUCTION

Turbulent flows laden with inertial particles are frequently found in natural or industrial flows, for instance, atmospheric flows with cloud droplets and aerosols, protoplanetary disks with dust particles, and spray combustion. Inertial particles display nonuniform distribution, i.e, clustering, in turbulence due to deviation of particle motion from the fluid particle trajectory. The clustering mechanism has been studied extensively (see, e.g., Brandt & Coletti, 2021; Bec *et al.*, 2024). For the case of dilute and heavy particles, such as cloud droplets and aerosols, one of the important parameters to characterize the clustering is the Stokes number St, which is a measure of particle inertia and is defined as the ratio of the particle relaxation time τ_p to the Kolmogorov time τ_η. For small inertia, i.e., $St \ll 1$, the particles are swept out due to the centrifugal effect from turbulent vortices and concentrate in low vorticity regions, which is referred to as the preferential concentration (Maxey, 1987; Squires & Eaton, 1991).

In the atmospheric science, modeling of the clustering behavior in high Reynolds number turbulence is important for improving cloud microphysics models because the clustering of cloud droplets can increase collision and coalescence frequency in raindrop formation process. To estimate the clustering behavior in such high Reynolds number turbulence, it is useful to clarify the characteristics of particle clustering in inertial subrange of turbulence. In our previous work (Matsuda *et al.*, 2022), we reported that for $Re_\lambda > 300$, the particle number density spectra exhibit two well pronounced bumps. The bump at near dissipation scales is attributed to the large enstrophy of such small eddies, whereas the bump at larger scale is attributed to the clustering in the turbulence inertial range. This result can raise the question whether the particle clustering shows a scale similarity in the turbulence inertial subrange.

When we consider particle clustering at scales in the inertial subrange, it is expected that the clustering behavior is strongly affected by the flow at similar scales. Therefore, to discuss the clustering in the inertial subrange, previous studies introduced the scale-dependent Stokes number (Falkovich *et al.*, 2003; Bec *et al.*, 2007; Bragg *et al.*, 2015; Ariki *et al.*, 2018), which is defined as $St_r = \tau_p/\tau_r$, where τ_r is the flow time scale at the scale r. According to dimensional analysis following Kolmogorov's idea (Kolmogorov, 1941) (K41), the flow time scale at the scale r in the inertial subrange is given by $\tau_r = \varepsilon^{-1/3} r^{2/3}$, which means $St_r = \tau_p \varepsilon^{1/3} r^{-2/3}$. Here, ε is the mean energy dissipation rate. Therefore, it has been expected that the preferential concentration mechanism becomes dominant at sufficiently large scales depending on the Stokes number. Based on this expectation, a universal scaling for the number density fluctuation in the inertial subrange has been proposed so far as a function of St_r (Bragg *et al.*, 2015; Ariki *et al.*, 2018). However, such St_r dependence has not been confirmed even by experiments at high Reynolds numbers (Saw *et al.*, 2012; Petersen *et al.*, 2019) and remains an open question (Bec *et al.*, 2024).

Thus, in this work, we aim to clarify the Stokes-number dependence of the number density spectrum, particularly in the inertial subrange using three-dimensional direct numerical simulation (DNS) of particle-laden homogeneous isotropic turbulence at high Reynolds number.

DIRECT NUMERICAL SIMULATION

We consider a statistically homogeneous velocity field $\boldsymbol{u}(\boldsymbol{x},t)$ of an incompressible fluid. The governing equations are the incompressible Navier–Stokes equations, i.e., the momentum and continuity equations, respectively:

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\boldsymbol{u} + \boldsymbol{f}, \quad (1)$$

$$\nabla\cdot\boldsymbol{u} = 0, \quad (2)$$

where $p(\boldsymbol{x},t)$ is the pressure, $\boldsymbol{f}(\boldsymbol{x},t)$ is an external solenoidal forcing, ν is the kinematic viscosity, and ρ is the fluid density. The particles are modeled as point particles. We assume that the particle size is sufficiently smaller than the Kolmogorov scale $\eta = (\nu^3/\varepsilon)^{1/4}$ and the particle density ρ_p is sufficiently larger than ρ. Then, Lagrangian motion of inertial heavy particles is described by the following equations:

$$\frac{d\boldsymbol{x}_p}{dt} = \boldsymbol{v}_p, \quad (3)$$

$$\frac{d\boldsymbol{v}_p}{dt} = -\frac{\boldsymbol{v}_p - \boldsymbol{u}(\boldsymbol{x}_p)}{\tau_p}, \quad (4)$$

where $\boldsymbol{x}_p$ and $\boldsymbol{v}_p$ are the position and velocity of a Lagrangian particle, and τ_p is the relaxation time of particle motion. The particles are one-way coupled with the fluid flow because we consider that the particles are sufficiently dilute to neglect the reaction of the particles to the fluid flows. Gravity is neglected to examine only the inertial effect.

We use the same DNS code described in, e.g., Onishi *et al.* (2011) and Matsuda *et al.* (2021). Equation (1) coupled with Eq. (2) is solved in a cubic computational domain with length 2π. Periodic boundary conditions are imposed. The equations are discretized on Cartesian staggered grids. Fourth-order central-difference schemes are used for the advection and viscous terms. A second-order Runge–Kutta scheme is used for time integration. The velocity and pressure are coupled by the Highly Simplified Marker and Cell (HSMAC) method (Hirt & Cook, 1972). We applied random forcing for turbulence in accordance with the velocity forcing proposed by Yoshida & Arimitsu (2007). The forcing is applied to the large-scale flow, where the wavenumber $|\boldsymbol{k}|$ is smaller than 2.5, and changes randomly, having a correlation time of T_f. We set $T_f = 1$ to be close to the large-eddy turnover time. Individual particles are tracked by the Lagrangian method. The initial particle distribution is random and homogeneous. The time evolution of $\boldsymbol{x}_p$ and $\boldsymbol{v}_p$ are computed by the second-order Runge–Kutta scheme.

The important parameters in this study are the Stokes number defined above and the Taylor-microscale Reynolds number $Re_\lambda \equiv u'\lambda/\nu$, where u' is the root-mean-square velocity fluctuation, and $\lambda = (15\nu u'^2/\varepsilon)^{1/2}$ is the Taylor microscale. The DNS is performed for $Re_\lambda = 648$ using 4096^3 grid points. The Stokes number St is 0.1, 0.2, 0.5 and 1.0. The number of particles for each Stokes number is $N_p = 3.2\times 10^9$. The initial particle distribution is random and homogeneous. The flow velocity and particle distribution data are sampled after the time integration of $10T_f$, and three-dimensional data are saved for 10 time instances at interval of T_f.

RESULTS AND DISCUSSIONS

When the particle clustering is dominated by the preferential concentration mechanism, the clustering formation, i.e., the divergence of particle velocity, is proportional to the second invariant of velocity gradient tensor (Maxey, 1987),

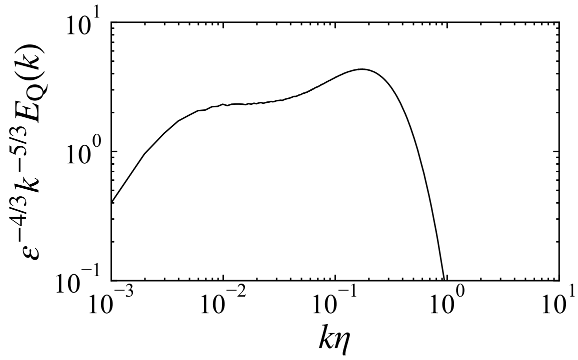


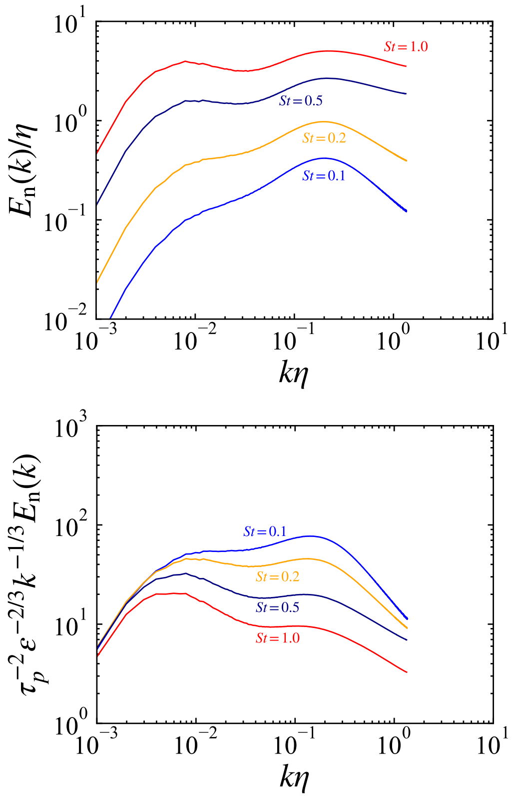
Figure 1. Spectrum of the second invariant of the velocity gradient tensor Q for $Re_\lambda = 648$ as a function of $k\eta$.

which is defined by $Q = (\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij})/2$ where $\Omega_{ij} = (D_{ij} - D_{ji})/2$ and $S_{ij} = (D_{ij} + D_{ji})/2$ with $D_{ij} = \partial u_i/\partial x_j$. Thus, we first assess the scale similarity of Q in the inertial subrange. The Fourier spectrum of Q is defined as $E_Q(k) = \sum_k |\widehat{Q}(\boldsymbol{k})|^2$, where $\widehat{\cdot}$ denotes the Fourier transform, and $\sum_k = (4\pi k^2/N_k)\sum_{k-1/2\le|\boldsymbol{k}|<k+1/2}$ with the number N_k of wavenumber vectors in the range $k-1/2 \le |\boldsymbol{k}| < k+1/2$. According to the dimensional analysis following K41, the spectrum $E_Q(k)$ has a $k^{5/3}$ scaling in the turbulence inertial subrange at sufficiently high Reynolds number (Monin & Yaglom, 1975). Figure 1 shows the spectra $E_Q(k)$, as a function of $k\eta$. This figure confirms that, the compensated spectrum for $0.008 \lesssim k\eta \lesssim 0.03$ is almost flat, which implies that $E_Q(k)$ well obeys the $k^{5/3}$ scaling in this range.

We examine the number density spectrum $E_n(k) = \sum_k |\widehat{n}(\boldsymbol{k})|^2$ in the inertial subrange. To compute the spectrum, the number density field $n(\boldsymbol{x})$ on equidistant grid points is obtained by using the histogram method, and $n(\boldsymbol{x})$ is normalized so that $\langle n\rangle = 1$, where $\langle\cdot\rangle$ means the ensemble average. Figure 2(a) shows the number density spectra $E_n(k)$ for different Stokes numbers. We can observe that, in $0.008 \lesssim k\eta \lesssim 0.03$, the slope of $E_n(k)$ is clearly dependent on the Stokes number. Based on the expectation that the preferential concentration mechanism is dominant for $St_r \ll 1$, Ariki *et al.* (2018) predicted $E_n(k) \propto \tau_p^2\varepsilon^{2/3}k^{1/3}$ for a scale $k = r^{-1}$ that satisfies $r \gg \Lambda$, where $\Lambda = \tau_p^{3/2}\varepsilon^{1/2}$. Figure 2(b) shows the spectra $E_n(k)$ compensated by the prediction by Ariki *et al.* (2018). The spectrum for $St = 0.1$ is almost flat in $0.008 \lesssim k\eta \lesssim 0.03$, but the spectra for the other St show different slopes in that wavenumber range. These results indicate that the particle response to the Q distribution is dependent on the Stokes number, and thus the clustering mechanism can be different from the classical preferential concentration mechanism.

To examine St_r dependence of $E_n(k)$, the spectra is normalized by Λ, since $k\Lambda = St_r^{3/2}$ for $k = r^{-1}$, and displayed in figure 3 as a function of $k\Lambda$. The spectra for the wavenumber range $0.008 \lesssim k\eta \lesssim 0.03$ are indicated by the solid lines. We can observe that the spectra for different Stokes numbers form a single curve, which can be given by a function of St_r. Therefore, the inertial particle clustering at the inertial scales is strongly affected by the flows at the similar scales, but the slope of the spectrum changes significantly depending on St_r.

The spectra that obey the $k^{1/3}$ scaling could be observed for $k\Lambda \ll 10^{-3}$ (i.e., $St_r \ll 10^{-2}$), whereas a different regime is observed for $k\Lambda \gtrsim 10^{-3}$. In our manuscript (Matsuda *et al.*, 2024), we examined the mechanism behind the St_r dependence of the spectra using a balance equation for the particle number



(a)

(b)

Figure 2. (a) Particle number density spectra for $St = 0.1$, 0.2, 0.5 and 1.0 at $Re_\lambda = 648$ as a function of $k\eta$. (b) The same spectra compensated by the prediction by Ariki *et al.* (2018).

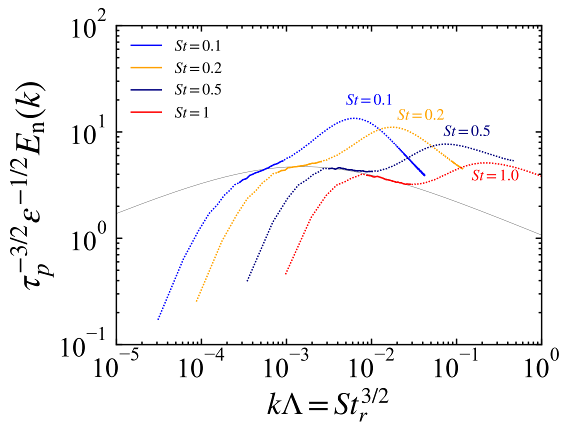


Figure 3. Particle number density spectra for $St = 0.1$, 0.2, 0.5 and 1.0 at $Re_\lambda = 648$ as a function of $k\Lambda$. For each St, the solid line indicates the spectrum in the inertial subrange ($0.008 \lesssim k\eta \lesssim 0.03$), and the dotted line is used for other wavenumbers. The gray solid line is a fitting curve representing St_r dependence of the inertial-scale clustering.

density spectrum. The results suggest that the change in the slope can be explained by the modulation of the preferential concentration mechanism due to the conservation of n.

We note that the prediction of Kolmogorov-like power law for $E_n(k)$ is equivalent to $h(r) \propto St_r^2 = \tau_p^2 \varepsilon^{4/3} r^{-4/3}$ in physical space (Ariki *et al.*, 2018), where $h(r)$ is the pair-correlation function (PCF) $h(r) = \langle \theta(\boldsymbol{x}+\boldsymbol{r})\theta(\boldsymbol{x})\rangle$ with $\theta(\boldsymbol{x}) = \{n(\boldsymbol{x}) - \langle n \rangle\}/\langle n \rangle$. The PCF satisfies $h(r) = g(r) - 1$ for the radial distribution function (RDF) $g(r) = \langle n(\boldsymbol{x}+\boldsymbol{r})n(\boldsymbol{x})\rangle/\langle n \rangle^2$. When $St_r \ll 1$, the above prediction also corresponds to the prediction for the RDF, $\log g(r) \propto St_r^2$ (Bragg *et al.*, 2015; Bec *et al.*, 2024), which is also based on the preferential concentration mechanism. On the basis of our findings, we conjecture that a regime different from these predictions could also be observed for the PCF and RDF in the inertial subrange for sufficiently high Reynolds number.

CONCLUDING REMARKS

Stokes-number dependence of inertial heavy particle clustering has been examined by using three-dimensional DNS of particle-laden homogeneous isotropic turbulence at a high Reynolds number $Re_\lambda = 648$ and with 3.2×10^9 particles. The results show that the slope of the particle number density spectra in the turbulence inertial subrange is dependent on the Stokes numbers, implying that the clustering mechanism can be different from the classical preferential concentration mechanism. It is also observed that the number density spectra in the inertial subrange well obey a function of the scale-dependent Stokes number, but the slope of the spectrum changes significantly depending on the scale-dependent Stokes number.

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