SMART TARGETING PARTICLES FOR EFFICIENT POLYMER DRAG REDUCTION

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ABSTRACT

The effect of polymer drag reduction by targeted injection is studied in comparison to that of a uniform concentration (or polymer ocean) in a turbulent channel flow. Direct Numerical Simulations are performed using a pseudo-spectral method to solve the coupled equations of a viscoelastic fluid using the FENE-P model. Light and heavy particles are used to carry the polymer in some cases, and polymer is artificially injected into specific flow regions in the other cases. To study drag reduction, the global mass flux through the channel is computed over time and compared to a turbulent channel with no polymer. There is no significant difference between any targeted method and a polymer ocean because of the short duration of polymer release. Future work includes implementing a more gradual polymer addition without sacrificing computational cost, numerical stability, or physical accuracy.

INTRODUCTION Background

Turbulent drag reduction is a common goal for many different applications, such as military vehicles and commercial transport. For many such situations, it is necessary to understand how turbulence produces drag in order to mitigate it as much as possible. It is well-known that wallbounded turbulence has characteristic "coherent structures" (Kim *et al.*, 1987; Fiedler, 1988) that contribute to Reynolds stresses, which in turn increase the skin-friction drag (Bernard *et al.*, 1993). Therefore, it may be prudent to target and disrupt these coherent structures using polymer additives. Since Toms (1948) demonstrated that dilute polymer solutions can be highly effective at reducing drag, the field has seen significant advances in understanding, with Lumley (1969, 1973) synthesizing many experimental results to characterize polymer drag reduction and propose a mechanism and Virk (1975) proposing a maximum drag reduction asymptote. More recently, Mortimer & Fairweather (2022) used a hybrid direct numerical simulation approach to study the relationship between polymer extension and turbulent boundary layer features. They found that the extension of the polymer is greatest in the buffer layer due to having the largest streamwise velocity gradients. In general, most of the dominant terms in the flow contributing to polymer extension contribute to the streamwise extension, oriented with the flow. Serafini et al. (2024) also used sophisticated direct numerical simulations to show how polymer solutions affect the kinetic energy budget in a wall-bounded turbulent flow using Lagrangian modeling for polymer molecules compared with the more common FENE-P model. For excellent recent reviews, see the works of Xi (2019) on polymer additive drag reduction and Dubief et al. (2023) on general elasto-inertial turbulence.

While polymer drag reduction has been successful in closed systems such as pipe flow, it is not practical for many external flows, for example around the hull of a ship. Therefore, it is necessary to understand how to efficiently use polymer additives to target vital regions of drag-producing structures. One potential method is to introduce polymer from solid particles, which can move differently than fluid particles, notably crossing streamlines. Wang & Maxey (1993) showed that bubbles tend to accumulate significantly in the vortices produced by homogeneous isotropic turbulence, which means that light particles may naturally target coherent structures in complex flows. Other types of particles may also prove useful to target drag-producing structures and help elucidate fundamental physics of turbulence. The findings of Mortimer & Fairweather (2022), for example lead naturally to a potential target- perhaps the buffer layer or generally high-strain regions- for some targeting particle carrying a polymer payload.

According to the elastic theory of polymer drag reduction (Tabor & de Gennes, 1986), kinetic energy from vortex structures may be transferred to elastic potential energy of the polymer, but that elastic potential energy must be released back into the flow, and this energy transfer can produce new, different structures. The energy is transferred back-and-forth while the polymer exists in the flow until it equilibrates and we get a steady-state drag reduction where we observe the mass flux to fluctuate close to a mean value.

Goal of Current Study

It has been previously shown by Suryanarayanan et al. (2018) that localized polymer is just as effective as uniformly distributed polymer at disrupting a hairpin vortex, and Kelly et al. (2021) demonstrated the potential of a vortex to capture a light particle. The current study combines these results to show the effectiveness of a targeted approach to polymer drag reduction. We simulate turbulent channel flow using direct numerical simulations (DNS) with Lagrangian particle tracking to model particles which carry and release a polymer payload locally. We demonstrate the efficacy compared to a uniformly distributed polymer solution to determine whether this is a more effective polymer drag reduction technique. This study seeks to answer the questions of whether this targeting approach is viable, and how it can be done most effectively, thus providing significant insight into the mechanisms of turbulent drag. To further understand the physics involved in polymer drag reduction, other cases are additionally tested in which polymer is directly injected into particular flow regions rather than relying on particle motion.

Governing Equations

The simulations used in the current study solve four sets of equations corresponding to the fluid, the particles, the movement of the polymer, and the polymer state. The fluid is evolved using the conservation of momentum for a general 3D incompressible viscoelastic fluid, given by

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + v_0 \left[\beta \frac{\partial^2 u_i}{\partial x_j \partial x_j} + (1 - \beta) \frac{\partial \tau_{p,ij}}{\partial x_j} \right]_{(1)}$$

where u_i is the fluid velocity, ρ is the mass density of the solvent, p is the pressure, $\beta = v_s/v_0$ is the ratio of kinematic viscosities of the solution to the solvent, $\tau_{p,ij}$ is the polymer stress tensor.

The polymer concentration $\gamma(x_i, t)$ is a scalar governed by an advection-diffusion equation with a source:

$$\frac{\mathrm{D}\gamma}{\mathrm{D}t} = \alpha \frac{\partial^2 \gamma}{\partial x_j \partial x_j} + S(x_i, t) \tag{2}$$

where α is the material diffusivity of the polymer in the solvent, and *S* is a source term. Polymer injection is handled by $S(x_i, t)$, where x_i is either a particle location or a specified point in the flow. The concentration is used to calculate the viscosity ratio,

$$\beta = \exp(-A_p \gamma) \tag{3}$$

where A_p is an empirical constant derived from the work of Nsom & Latrache (2018).

The polymer stress is governed by its molecular configuration, which is described collectively by the conformation tensor, C_{ij} , which is based on the FENE-P model (Wedgewood & Bird, 1988). The conformation tensor evolves according to the following equation:

$$\frac{\mathrm{D}C_{ij}}{\mathrm{D}t} = C_{ik}\frac{\partial u_j}{\partial x_k} + C_{kj}\frac{\partial u_i}{\partial x_k} - \frac{1}{\lambda}[f(C_{kk})C_{ij} - \delta_{ij}] + \alpha_p \frac{\partial^2 C_{ij}}{\partial x_j \partial x_j}$$

where λ is the polymer relaxation time, $f(C_{kk})$ is the Peterlin function, and α_p is a numerical diffusion constant, added to stabilize the simulation. Equations (1) and (4) are well-known and validated.

The particles are simulated as point particles, governed by the Maxey-Riley equations (Maxey & Riley, 1983). Because we are considering the microscale (on the order of Kolmogorov scale), the Basset history term may safely be ignored, and we neglect gravity. These particles have no direct effect on the flow, other than to release polymer at their locations which may alter the flow locally.

Numerical Method

To solve the above equations, we use a pseudo-spectral code based on Kim et al. (1987) and Handler et al. (1989). The code runs in a rectangular domain which is solved in Fourier space in x and z and real space in y with Chebyshev-collocated points. The x- and z-directions have periodic boundary conditions, and the y-direction has no-slip boundary conditions on both sides, defining the top and bottom of the channel. The particles are integrated in physical space using an RK4 integration scheme and trilinear interpolation between grid points. We use a constant pressure gradient to sustain a turbulent channel flow with a Reynolds number of $Re_{\tau} = 180$ in a domain of size $4\pi\delta \times 2\delta \times \pi\delta$, where δ is the channel half-height, with a grid resolution of $256 \times 129 \times 128$ in x, y, and z, respectively. Particles are placed in the turbulent flow and allowed to move for a certain time before releasing polymer into the domain at a fixed rate, modeled as a 3D Gaussian distribution centered on the particle position. The polymer is released until the globally-averaged β is equal to that of the polymer ocean case. While the polymer is in the domain, polymer stresses are calculated based on C_{ii} and β , which is a function of local polymer concentration, and used in the last term of Eq. (1).

In certain cases, polymer was placed in particular regions of the flow based on local conditions. In each case, certain quantities (e.g., the Q-criterion or strain rate magnitude) were calculated at each point, sorted, then the polymer was placed in the cells with the largest values. This was updated each time step, and a limit of local $\beta = 0.99$ was enforced to ensure the polymer was reasonably spread out throughout the addition step.

Although the code is set up to run with non-scaled variables, we chose them such that certain non-dimensional numbers are aligned with previous studies in the literature. The most important of these for polymeric flows is the Weissenberg number, Wi, defined as the ratio of polymer relaxation time to some fluid time scale. In accordance with Min *et al.* (2003), we chose the following definition

$$Wi \equiv \frac{\lambda u_{\tau}^2}{v_0} \tag{5}$$

where u_{τ} is the friction velocity of the channel flow before polymer is added, and λ is the same polymer relaxation time as used in Eq. (4). According to this definition, Min *et al.* (2003) showed that Wi = 1 is a critical value, below which no drag reduction occurs, and Virk's maximum drag reduction asymptote occurs around Wi > 40. For our simulation, we chose $\lambda = 0.1$ s to give Wi = 32.4, a material diffusivity coefficient such that the material Schmidt number is $Sc \equiv v_0/\alpha = 10$, and a numerical diffusivity coefficient such that the artificial Schmidt number, $Sc_p \equiv v_0/\alpha_p$ is 0.1. Sureshkumar & Beris (1995a) showed that $Sc_p \sim 0.2$ is the threshold for numerical stability for a spectral scheme.

RESULTS/DISCUSSION

Two different types of targeting are compared in this study. The first is targeting via advected particles that gradually emit polymer into the flow. The second is an artificial injection of polymer at specified regions of the flow based on the Q-criterion and the strain rate. In both cases, targeting results are compared directly to a polymer ocean with an equivalent amount of total polymer, and these cases are all normalized by a "clean" run of Newtonian turbulence.

Advected Particles

For the advected particles, light $(\rho_p/\rho_f = 0)$ and heavy $(\rho_p/\rho_f = 10)$ particles are simulated and compared to the polymer ocean case, where ρ_p and ρ_f are the densities of the particles and fluid, respectively. For each density, the particles were initially seeded either near the wall or in the center of the channel. In all cases, a total of 1024 particles were simulated, and they were allowed to move for 1 second in simulation time, which is enough time for them to be sufficiently correlated with the vortex structures (Kelly *et al.*, 2023), before releasing polymer at a fixed rate of 10 PPM per particle per second into the domain until the total amount of polymer in the channel was equal to the polymer ocean case. The early stage of polymer release is shown in Fig. 1



Figure 1. Early-time polymer release of the near-wall bubbles case. The red dots are the bubbles, and the green blobs are iso-surfaces of $\beta = 0.999$. The vortex structures shown are iso-surfaces of the Q-criterion equal to 2000 s⁻², colored by streamwise velocity. Only a portion of the channel ($0 \le x \le 4$) is shown for better visualization.

To determine drag reduction for a constant-pressuregradient channel, we calculated the spatially-average mass flux at each time step, which gives us a single value to quantify how much material is moving through the channel at any given time. For a reduction in drag, we expect to see a corresponding increase in the mass flux, so we take this as sufficient demonstration of polymer drag reduction. Fig. 2 shows the time trace of the mass flux as it approaches a steady state. All cases are normalized by the time-averaged mass flux of the no-polymer case, so the values shown are the relative percent increase in mass flux due to the addition of polymer to the channel. Notice that all values start around 0.5%, which just means that the initial state is a little higher than the average mass flux in the clean turbulence channel. At early times, there is a slight dip in mass flux across all polymer cases which reflects the change in viscosity of the fluid before the elastic interactions of the polymer and flow structures start increasing mass flux.



Figure 2. Mass flux relative to average mass flux of Newtonian turbulence for advected particles and polymer ocean. The *y*-axis is the percent difference in mass flux, corresponding to a drag reduction percentage.

Targeting

We simulated other cases using instantaneous polymer injection into the channel to understand how polymer drag reduction changes based on where the polymer is placed relative to certain physical characteristics of the flow. This decouples complex particle motion from targeted polymer drag reduction. In this study, we targeted two specific features of the flow: vortical structures and regions of high strain. For the former case, we identified vortical structures as regions in the flow with high Q-criterion values. For both cases, the regions were identified and targeted by the following procedure at every time step:

- 1. Calculate the field in physical space
- 2. Identify all local maxima in the field
- 3. Sort the local maxima by field value
- Insert polymer at a fixed rate in the top 100 local maxima, skipping regions where polymer concentration has already reached a specified maximum

Since polymer is only injected at 100 locations, we inject at a rate of 100 PPM per point per second for comparable polymer addition to the particle cases. A snapshot of polymer injection targeting regions of high Q-criterion is shown in Fig. 3, before any drag reduction is observed (roughly 0.01 seconds).



Figure 3. Early-time polymer injection of high Q-criterion targeting. Green blobs are iso-surfaces of $\beta = 0.999$, vortex structures are iso-surfaces of Q-criterion equal to 2000 s^{-2} , colored by streamwise velocity. Only a portion of the channel $(0 \le x \le 4)$ is shown for better visualization.

Other than the targeted positioning, everything else is the same as the advected particle cases. The relative mass flux until steady state is shown in Fig. 4.



Figure 4. Mass flux relative to average mass flux of Newtonian turbulence for targeted injection and polymer ocean. The *y*-axis is the percent difference in mass flux, corresponding to a drag reduction percentage.

Despite an ultra-dilute polymer concentration with an average $\beta = 0.999$, there is still noticeable mass flux increase (and therefore drag reduction) for all polymer cases. We found that mass flux curves of the polymer ocean case may vary within 1-2% depending on the initial state of turbulence when, and perhaps where, polymer is added to the channel. This variability is around the maximum variations we see across the different targeting scenarios examined from the polymer ocean case, indicating that the targeting methods presented in this study added no significant benefit to drag reduction compared to the distributed polymer ocean model. However, this does not mean that a targeting approach cannot be useful. The main factors in the efficacy of these drag reduction methods are the various inter-connected time scales. The most relevant time scales to the problem appear to be the time for polymer injection or release, t_{rls} (i.e., when the polymer release/injection stops), the time is takes the polymer to diffuse into uniform concentration, t_{diff} , and the time it takes for polymer to disrupt a vortex structure, t_{dsrpt} .



Figure 5. Early simulation times with relevant time scales.

In Fig. 5, early times of the simulation are shown (the first 5 seconds of the approximately 30 seconds it takes to reach steady state mass flux). The first line (solid black, virtually coincident with the vertical axis on the left) is at $t_{rls} = 0.01$ s, when the globally-averaged β is 0.999 and polymer release stops. This extremely short release time is akin to having the particles "explode" and release their load of polymer abruptly. The second line (dashed black) is the approximate time it takes to disrupt the vortex structures, $t_{dsrpt} \approx 1$ s. This was determined by measuring the swirl criterion- nearly identical to positive Q-criterion but is always non-negative- integrated throughout the channel. t_{dsrpt} corresponds to the time when the swirl criterion stops decreasing significantly and levels out with moderate fluctuations. The last line (dot-dashed black) corresponds to the time it takes for an initially targeted polymer to disperse and diffuse into a regime nearly identical to that of a polymer ocean, around 2.5 seconds. All of these time scales are particular to our setup and will vary with the release rate, polymer relaxation time, λ , and material Schmidt number, Sc, as well as flow time scales. We see that t_{rls} is much smaller than the other two time scales (an order of magnitude smaller than the next smallest, λ), so we end up getting only different initial locations of the polymer rather than an extended targeting injection. Even the largest time scale, t_{diff} is much smaller than the time for the flow to reach a steady state.

Fig. 6 shows enstrophy at early times ($t < t_{dsrpt}$) demonstrating the difference between targeted polymer and a polymer ocean. We see that particles released near the walls reduce enstrophy more effectively for most of the time in this window, and the targeted injections are close and end up more effective by the end of the time in this window. Notably, both particle cases released in the center of the channel have significantly less disruption of enstrophy at early times, which is possibly due to the fact that there are fewer vortex structures away from the wall to disrupt.

Fig. 7 shows a relation between the swirl criterion and polymer concentration (via β) which is a measure of how effectively polymer targets vortex structures. We compute the



Figure 6. Global enstrophy at early times for each polymer case



Figure 7. Spatially integrated relation between vortex structures and polymer concentration.

relation as the normalized volume integral:

$$\frac{1}{\lambda^*(1-\bar{\beta})}\int_V \lambda_{ci}(1-\beta)\,dV\tag{6}$$

where λ_{ci} is the swirl criterion. We choose this variable again because it has the desirable feature of always being nonnegative. The normalizing variables are integrated variables, with λ^* being the total swirl in the domain and $\bar{\beta}$ being the average β in the domain. This scaling is chosen such that the polymer ocean case is always one for all time, and the targeting cases are interpreted relative to the polymer ocean. As expected, we see the high-Q targeting case being highly effective by this measure at early times, but it decreases quickly, presumably a result of gradual vortex disruption and dispersion of the polymer. The near-wall particle cases are slightly more effective at targeting the structures, but they also tend to one as time progresses. Unsurprisingly, particles in the center of the channel show less effective targeting as they tend to be in regions with fewer vortex structures since they are far away from the walls, and emitting polymer from randomly selected tracers shows a slightly worse targeting relative to polymer ocean. We expect that maintaining a value greater than one over longer periods of time may improve drag reduction (and conversely, maintaining a value less than one should show reduced drag reduction).

It may then be possible to improve the targeted performance by extending the release time significantly. If we can continually or periodically add tiny amounts polymer to the flow in order to target specific regions, we could extend t_{rls} such that the polymer targets structures, disrupts them, then targets different structures, disrupts those, and so on until reaching a global limit.

FUTURE WORK

The original idea was that keeping the concentration extremely dilute would reveal significant differences between a polymer ocean and a targeted injection of polymer. This was not the case since the time over which the polymer is released is too small to continuously target structures in the flow. Therefore, we must change our approach to extend the release time and continually target structures over a longer duration. However, we have seen that if polymer is released too slowly, there is virtually no effect on the mass flux until the total amount of polymer reaches a certain threshold. In order to release a small amount over long periods of time and see a significant effect, it may be necessary to make the comparison β smaller than what has been shown in the present study (perhaps $\beta \approx 0.9$) in order to have a slow release with enough local polymer to disrupt vortex structures. Recent attempts to implement this strategy have failed due to numerical stability issues. We are currently working on a way to accurately resolve these issues with reasonable computational expense and without sacrificing the realistic effectiveness of the polymer.

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