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RESOLVENT-BASED ESTIMATION AND CONTROL OF WAKES AND JETS

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ABSTRACT

We present a suite of resolvent-based estimation and control tools and their application to a series of shear flows, namely laminar and turbulent wakes behind an airfoil and a turbulent jet. The resolvent-based tools provide several advantages over standard methods. When equivalent assumptions are made, the resolvent-based estimator and controller reproduce the Kalman filter and LQG controller, respectively, but at substantially lower computational cost, making them applicable to the large-scale problems that arise in fluid mechanics. Unlike these standard methods, the resolvent-based approach can naturally accommodate forcing terms (nonlinear terms from Navier-Stokes) with colored-in-time statistics, which significantly improves the accuracy of the estimates and effectiveness of the control. In this abstract, we present estimation and control results for a laminar wake and estimation results for a turbulent wake and a turbulent jet.

BACKGROUND

Flow estimation and control provide a path forward toward achieving many important engineering objectives. For example, sparse measurements on the surface of an aircraft can be used to determine the state of the surrounding flow and subsequently used as input to active control efforts to manipulate the flow via suitable actuation to achieve objectives such as reducing drag, increasing lift, delaying transition, preventing separation, and so on.

State estimation and control are classical topics in dynamical systems and control theory, but standard methods have several disadvantages when applied to fluid mechanics and turbulent flows in particular. First, standard techniques assume a linear system forced by white noise. When these methods are applied to fluid mechanics, the white noise forcing is used to represent the nonlinear fluctuations within the Navier-Stokes equations. However, these nonlinear fluctuations are not white for real flows, and properly accounting for the 'colored' nature of these forcing statistics is critical for obtaining accurate models. Second, fluids problems, and especially turbulent flows, are high dimensional in the sense that they require a large number of variables (after spatial discretization) to represent the state of the flow, often on the order of tens or hundreds of millions. Standard estimation methods scale poorly and become very expensive for large systems, which hinders their application to realistic flows.

Leveraging the well-known existence of organized motions – coherent structures – provides a principled approach to overcome these limitations. If the footprint of a coherent structure can be detected from its contribution to a probe measurement, then we gain knowledge of its impact on the flow over its entire spatial extent. Similarly, the underlying physics responsible for coherent structures can be leveraged to modify their properties and the engineering quantities of interest to which they contribute. Resolvent analysis, which captures the dominant energy amplification mechanisms within a flow, has been shown to provide an excellent model for these coherent structures (McKeon & Sharma, 2010; Towne *et al.*, 2018) and provides the basis for the tools described next.

RESOLVENT-BASED ESTIMATION & CONTROL

Over the last few years we (along with a team of collaborators) have developed a suite of resolvent-based estimation and control tools that overcome the aforementioned limitations of standard methods (Towne *et al.*, 2020; Martini *et al.*, 2020, 2022; Jung *et al.*, 2023; Jung & Towne, 2024). The

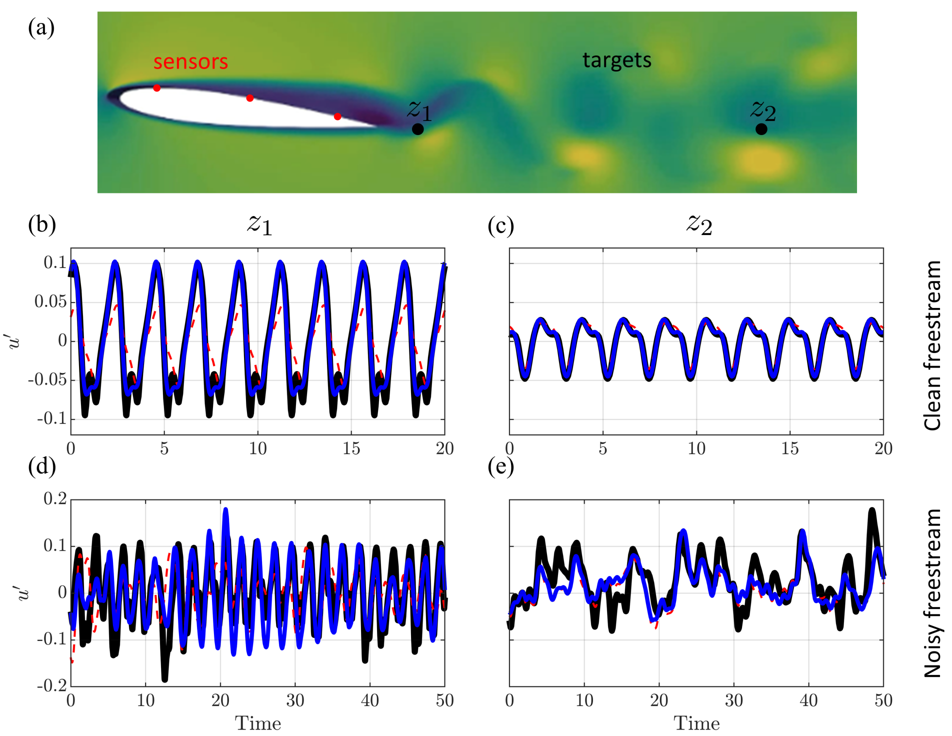

Figure 1. Estimation results for the laminar airfoil: (a) sensor and target placement; (b,c) estimation in noiseless environment; (d,e) estimation in noisy environment. Colors: (black) DNS data; (pink) previous method; (blue) our method.

resolvent-based framework provides a hierarchy of methods for different tasks, including space-time statistical modeling, flow-field reconstruction in the time domain, real-time causal estimation, and control. Causality (when desired) is optimally enforced using a Weiner-Hopf formalism, ensuring that the current estimate and control signal depend only on current and previous measurements. When equivalent assumptions are made, the resolvent-based estimator and controller reproduce the Kalman filter and LQG controller, respectively, but at substantially lower computational cost thanks to the development of an efficient time-marching algorithm for computing the kernels. Unlike those standard methods, the resolvent-based approach can naturally accommodate forcing terms (nonlinear terms from Navier-Stokes) with colored-in-time statistics, which significantly improves the accuracy of the estimates and performance of the controller. These methods were initially validated for a turbulent channel (Towne *et al.*, 2020; Amaral *et al.*, 2021), a transitional boundary layer (Martini *et al.*, 2020), and the flow over a backward facing step (Martini *et al.*, 2022).

We have implemented the resolvent-based estimation and control tools within a high-fidelity solver (Jung *et al.*, 2023), namely the compressible, unstructured finite-volume solver CharLES (Bres *et al.*, 2017). All routines are fully parallelized, allowing us to apply our tools to any flow that can be simulated in CharLES. Additionally, we formulated a data-driven implementation of the resolvent-based tools, which expands their applicability to experimental settings and simulations that lack adjoint capabilities (Martini *et al.*, 2022). Finally, we have developed an approach to efficiently explore sensor and actuator placement by avoiding redundant calculations and obtaining an a priori estimate of the control performance without actually applying the controller to the flow.

In our presentation, we will give a brief overview of these capabilities followed by a deeper discussion of there application to the two types of shear flows described next: laminar and turbulent airfoil wakes and turbulent jets.

RESULTS: WAKES

First, we use the resolvent-based tools to estimate and control fluctuations in the wake of a NACA 0012 airfoil at laminar and turbulent conditions, corresponding to chord-based Reynolds numbers of 5000 (Jung *et al.*, 2023) and 23000 (Towne *et al.*, 2023; Jung & Towne, 2024), respectively. In both cases, a small number of sensors and actuators are placed on the surface of the airfoil, and the goal is to estimate and suppress fluctuations downstream in the wake.

Figure 1 shows sample estimation results for the laminar airfoil (Jung *et al.*, 2023). We use the three shear-stress sensors shown in Figure 1(a) to estimate the streamwise velocity at the indicated downstream target positions. The resolvent-based estimator (blue line) outperforms a standard method (red dashed line) in matching the DNS data (black line) at both targets, both when the airfoil is immersed in a clean (b-c) and noisy (d-e) freestream. The noisy environment is generated by forcing the flow well upstream of the airfoil. This kicks the flow out of its natural limit cycle, leading to a chaotic flow that is more challenging to estimate.

Figure 2 shows sample estimation results for the turbulent airfoil (Jung & Towne, 2024). This time, we use the six shear-stress sensors shown in Figure 2(a) to estimate the streamwise velocity at the indicated downstream target positions. In this case, we used the data-driven implementation of the resolvent-based estimator. Since the turbulent flow is three-dimensional, we consider multiple options for handling the spanwise coordinate. In Figure 2(b-c), we exclusively estimate the spanwise-averaged fluctuations, while in Figure 2(d-e) we estimate the fluctuations in the mid-span plane. The latter task is more challenging, since the three-dimensional turbulent motions lead to

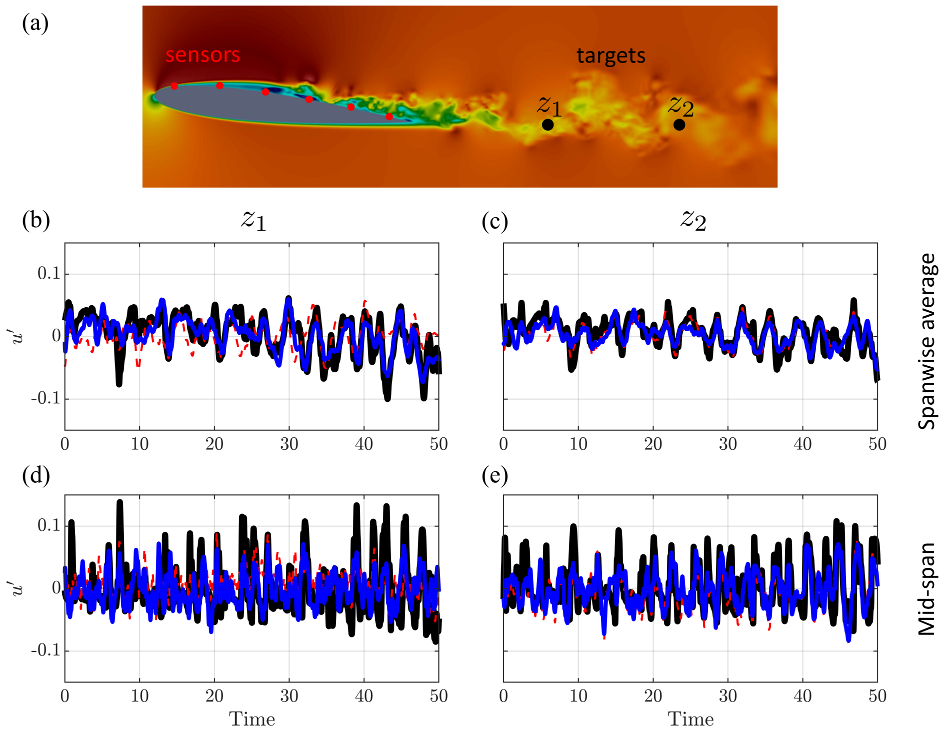

Figure 2. Estimation results for the turbulent airfoil: (a) sensor and target placement; (b) one sensor; (c) two sensors. Colors: (black) LES data; (pink) previous method; (blue) our method.

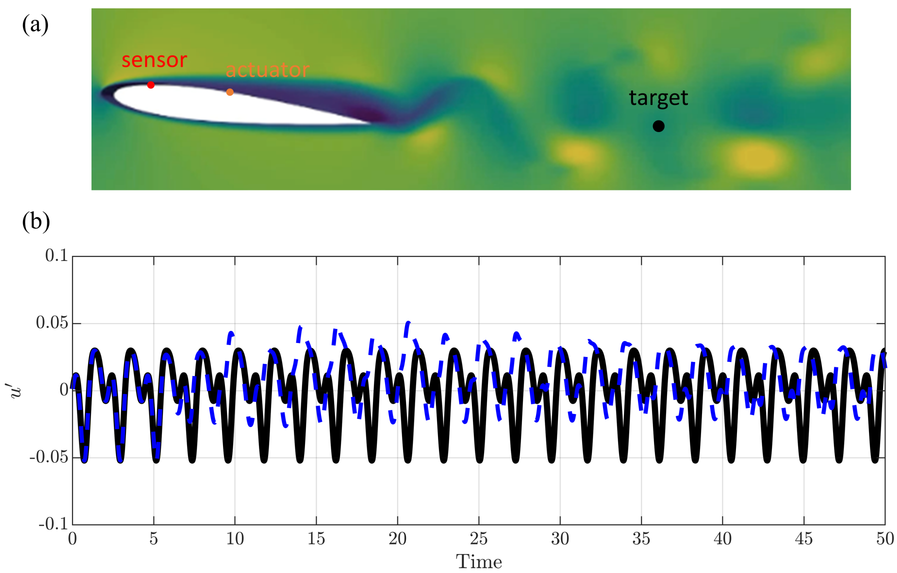

Figure 3. Control results for the laminar airfoil. Colors: (black) uncontrolled; (blue) controlled.

fluctuations that are observed at the targets in the mid-span plane but cannot be sensed in that same plane. We are currently exploring whether additional out-of-plane sensors can improve the estimation accuracy. Nevertheless, it is clear for all cases that the resolvent-based estimator outperforms the standard approach.

Figure 3 shows sample control results for the laminar airfoil. As shown in Figure 3(a), here, we use one shear-stress sensor and one actuator that injects momentum into the flow, and the goal is to minimize the streamwise velocity fluctuation at the target. After a brief transient when the controller is turned on, the largest negative velocity fluctuations at the target are reduced by a factor of two, as shown in Figure 3(b). We expect improved performance when multiple sensors and actuators are employed, as we are currently performing these calculations.

RESULTS: JETS

Second, we apply our resolvent-based tools to a Mach 0.9 turbulent jet, with the objective of estimating (and later suppressing) velocity fluctuations in the jet. These fluctuations take the form of wavepackets that are known to be the primary source of noise emitted by the jet. In Figure 4(a), two

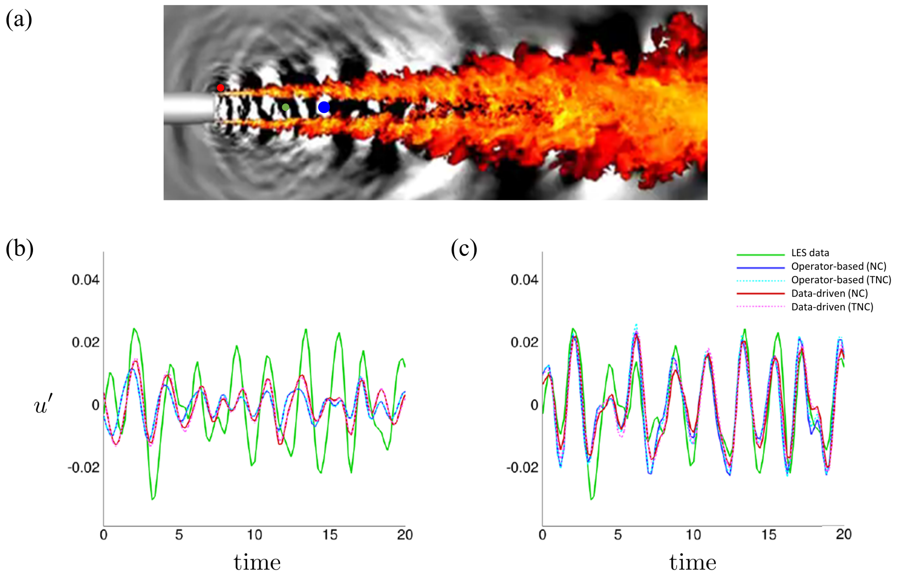

Figure 4. Estimation results for the turbulent jet: (a) sensor and target placement; (b) estimation results using the near-nozzle (red) sensor; (c) estimation results using the downstream (green) sensor. Line colors: (green) LES data; (blue) non-causal; (red) causal.

different sensor locations are considered, one near the nozzle lip (red circle) and one further downstream in the jet (green circle), with the goal of estimating the streamwise velocity fluctuations at the location of the blue circle. Four different resolvent-based estimators are employed: non-causal and causal operator-based and data driven kernels. Unsurprisingly, the downstream sensor is more effective; the error for the near-nozzle case can be reduced by increasing the number of sensors. We have also studied the impact of sensor placement (results not shown), finding that the optimal radial position for a near-nozzle sensor is just outside of the shear layer. We are currently repeating these analyses for a supersonic jet, with the goal of assessing the ability of our tools to estimate and control the noise emitted by the jet using near-nozzle actuation.

CONCLUSIONS AND FUTURE WORK

This paper provides a brief overview of a suite of resolvent-based estimation and control tools and their application to laminar and turbulent airfoil wakes and jets. Our results demonstrate the utility of the resolvent-based tools, specifically their ability, relative to standard methods, to improve accuracy, reduce cost, and handle the large systems typically encountered in fluid mechanics and especially turbulent shear flows. In the future, we will explore and formally optimize sensor and actuator placement by using the quickly commutable performance estimates discussed earlier.

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