12th International Symposium on Turbulence and Shear Flow Phenomena (TSFP12)
Osaka, Japan (Online), July 19-22, 2022

THE QUASI-SPECTRAL VISCOSITY (QSV) CLOSURE TO SUBFILTER SCALE FLUXES: UNIFYING SHOCK CAPTURING AND LARGE-EDDY SIMULATION

Victor C. B. Sousa
School of Mechanical Engineering
Purdue University
West Lafayette, IN, 47907
vsousa@purdue.edu

Carlo Scalo
School of Mechanical Engineering
School of Aeronautics and Astronautics
Purdue University
West Lafayette, IN, 47907
scalo@purdue.edu

ABSTRACT

The Quasi-Spectral Viscosity (QSV) method is a closure for a high-order finite-difference discretization of the filtered compressible Navier-Stokes equations capable of unifying dynamic subfilter scale (SFS) modeling and shock capturing under a single mathematical framework. It introduces a physical-space implementation of a spectral-like SFS dissipation term by leveraging residuals of filter operations, achieving two goals: (1) estimating the energy of the resolved solution near the grid cutoff; (2) imposing a plateau-cusp shape to the spectral distribution of the added dissipation. The QSV approach was tested to showcase its capability to act interchangeably as: a shock capturing method, in a shock/sinusoidal wall interactions problem; or as a SFS closure, in a subsonic Taylor Green Vortex (TGV). QSV performs well compared to previous eddy-viscosity closures and shock capturing methods in such test cases. In a supersonic TGV flow, a case which exhibits shock/turbulence interactions, QSV alone outperforms the simple superposition of separate numerical treatments for SFS turbulence and shocks. QSV's combined capability of simulating shocks and turbulence independently, as well as simultaneously, effectively achieves the unification of shock capturing and Large-Eddy Simulation.

INTRODUCTION

Although hydrodynamic turbulence and shock formation have been treated separately in previous literature, both are characterized by an energy cascade from large to small scales due to nonlinear interactions (Gupta & Scalo, 2018). Therefore, it is proposed to treat the numerical simulation of both phenomena in a similar fashion, as the consequence of solving the filtered compressible Navier-Stokes equations.

The Quasi-Spectral Viscosity (QSV) method (Sousa & Scalo, 2022*b*) builds upon previous development in turbulence modeling by Kraichnan (1976) and Chollet & Lesieur (1981) and, in shock capturing by Tadmor (1989). The first introduced the concept of a wavenumber-dependent eddy viscosity with a plateau at large scales and a cusp near the resolution limit to model the energy transfer across a filter cutoff while the second proposed to inform the magnitude of such Spectral Eddy Viscosity (SEV) from the kinetic energy at the cutoff, $\sqrt{E(k_c)/k_c}$. This renders the model dynamic, being only active when small scales are present in the flow.

On the other hand, Tadmor (1989) studied the use of Fourier-based discretization methods to solve the inviscid Burgers' equations and concluded that, if no regularization term was introduced, the covergence of the numerical solution was not guaranteed. He then proposed to introduce a wavenumber-dependent viscosity term concentrated on small scales, named Spectrally Vanishing Viscosity (SVV), that would prevent oscillations and lead to convergence to the unique entropy solution. The idea was shown to be successful by mathematical proofs and numerical experiments.

The current manuscript analyzes the SEV and SVV frameworks and highlights their similarities bridging a gap between methods initially developed for different purposes. Such analysis is carried out exploring the common features that exist in both incompressible Navier-Stokes equations and the Burgers' equation. This understanding allows for the specific design of a unified method for shock capturing and subfilter turbulence modeling. The QSV method is then developed in the context of high-order finite-difference implementations and it is tested in situations of shock dominated problems, in predominantly hydrodynamic flows and also in shock/turbulence interaction cases. For example, QSV is able to simulate a Taylor Green Vortex with both sub and supersonic initial perturbations using identical structures.

FOUNDATIONS FOR THE UNIFICATION OF SHOCK CAPTURING AND LES

The objective of this section is to highlight similarities and establish a connection between eddy viscosity models, the

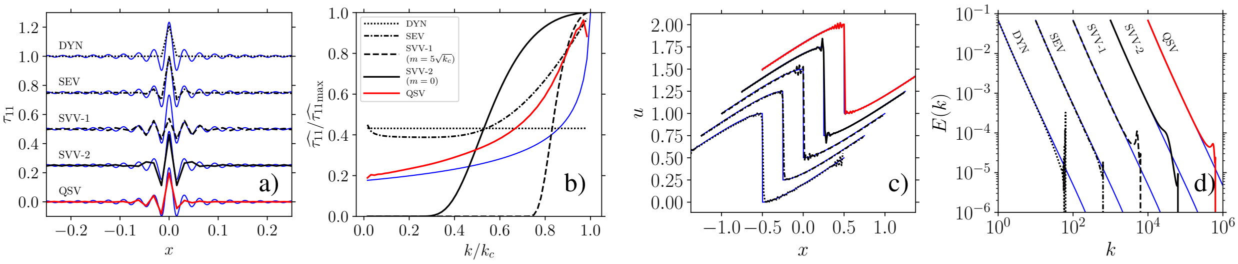

Figure 1. Results from *a priori* and *a posteriori* analysis of the filtered Burgers' equation at time $t = 1$ with $N = 128$ grid points. Exact SFS stress (blue) compared against SFS stresses predicted by various models (see legend), in the physical (a) and spectral (b) domain. Exact filtered solution (blue) versus *a posteriori* solution by various models (see legend) in the physical space (c) and their respective energy spectrum (d).

discontinuity regularization method based on artificial addition of a spectrally vanishing viscosity (SVV) (Tadmor, 1989) and the implementation of the Quasi-Spectral Viscosity (QSV) method.

To begin, it is pointed out that it is possible to solve only for the large scales present in the Navier-Stokes equations if the original system is filtered by an operation that commutes with the derivatives with an associated filter width ($\overline{\Delta}$). That procedure forms the mathematical framework of Large-Eddy Simulations (LES) allowing, for example, the simulation of turbulence in relatively coarse grids through the solution of the filtered incompressible Navier-Stokes, constituted by the divergence free condition $\frac{\partial \overline{u}_i}{\partial x_i} = 0$, and by the conservation of momentum equation,

$$\frac{\partial \overline{u}_i}{\partial t} + \frac{\partial \overline{u}_i \overline{u}_j}{\partial x_j} = -\frac{\partial \overline{\mathscr{P}}}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \tag{1}$$

where $\mathscr{P} = p/\rho_{\text{ref}} U_{\text{ref}}^2$ and $\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$ is the subfilter scale (SFS) stress tensor, a remainder of the filtering operation applied to the nonlinear governing equations. Since the SFS term depends on the unresolved scales in the flow, it must be modeled. The same procedure can be used to derive the filtered version of the inviscid Burgers' equation,

$$\frac{\partial u_1}{\partial t} + \frac{1}{2}\frac{\partial u_1 u_1}{\partial x_1} = 0, \quad \frac{\partial \overline{u}_1}{\partial t} + \frac{1}{2}\frac{\partial \overline{u}_1 \overline{u}_1}{\partial x_1} = -\frac{1}{2}\frac{\partial \tau_{11}}{\partial x_1}. \tag{2}$$

Because of its simplicity, its parallel with the LES framework and its nonlinearity that can lead to the formation of shock discontinuities in finite time, the Burgers' equation is used as a way to showcase the similarities between the Dynamic Smagorinsky (DYN) (Smagorinsky, 1963; Germano *et al.*, 1991), the Spectral Eddy Viscosity (SEV), the SVV and QSV models.

The Burgers' equation is solved in a periodic domain $x_1 \in [-1, 1]$, starting with with initial conditions $u_1(x, t = 0) = 1 + \frac{1}{2}\sin(\pi x)$, until $t = 1$ using a pseudo-spectral Fourier method for the spatial derivatives and a 4th order Runge-Kutta scheme for the time integration. The results obtained using the different eddy viscosity closures are then compared with the analytical solution based on the method of characteristics in the physical and spectral domain. In the current work, the procedure of comparing simulated results to a reference solution will be referred to as *a posteriori* analysis. In addition, an *a priori* analysis is also conducted. It is defined as an operation that filters the exact solution and its nonlinear terms at a certain time instant by a sharp spectral transfer function down to the same resolution as the numerical simulations performed. This allows the comparison between the exact SFS stress, $\overline{u_i u_j} - \overline{u}_i \overline{u}_j$, and the output of each different model when the exact filtered solution $\overline{u}_i$ is used as input. The intent of this operation is to exploit the access to a reference solution to calculate τ_{ij} explicitly, the term that needs to be modeled to sustain a filtered solution of a given equation, and assess each model's ability to generate similar effects while only having access to the information present on the large flow scales. The both *a priori* and *a posteriori* results are shown in physical and spectral space in figure 1.

It can be observed that the exact energy flux to subfilter scales spans across all resolved wavenumbers and it peaks near the cutoff (k_c), similar to Kraichnan (1976); Chollet & Lesieur (1981)'s theoretical findings for turbulent flows. Interestingly, previous methods for either turbulence modeling or shock capturing are able to stabilize the flow with different degrees of efficacy. The DYN model leads to a local response in physical space and, therefore, a flat broadband spectral response, what causes spurious high-wavenumber build up due to the insufficient damping of the resolved scales near the cutoff. Other models do a better job by ramping up the dissipation at the small scales but either over (SEV) or underdamp (SVV) the large scales present in the flow.

Despite the differences in the development, purpose and use between the models, the results in figure 1 indicate that they can all be encompassed under a generalized format of subfilter scale flux model consisting of a magnitude pre-factor composed by a length scale (ℓ) and velocity a velocity scale (υ_c) and a kernel (K) which is convolved with the strain rate tensor (S_{ij}) and is responsible for the dissipation wavenumber dependence, as in

$$\tau_{ij} = \ell \upsilon_c \left(K * S_{ij}\right). \tag{3}$$

Additional evidences for the generalization are provided in Sousa & Scalo (2022*b*).

This demonstrates that the same principles used by large-eddy simulations of hydrodynamic turbulence, focusing on simulating only the large, resolved or filtered scales, can be used for discontinuity capturing as well. Ultimately, this inspired the Quasi-Spectral Viscosity (QSV) closure, designed specifically to unify the LES and shock capturing methodologies in high-order finite difference implementations, which can be generically written as,

$$-\tau = \sqrt{2\overline{\Delta} E_{k_c}}\left(\left(1 - \widetilde{G}_{\text{qsv}}\right) * \overline{S}\right). \tag{4}$$

There are two steps for implementing the method: first, estimating the cutoff energy, E_{k_c}, and second, introducing a *plateau-cusp* behavior as a function of wavenumber, done via the $1-\widetilde{G}_{\mathrm{qsv}}$ kernel. For these to be implemented in a high-order finite difference setting, it was necessary for them to be performed only using spatial operators and that was achieved by leveraging the residuals of Padé ((Lele, 1992) and Fejér (Vandeven, 1991) filtering operations. The details behind the estimation of E_{k_c} and the construction QSV wavenumber modulation kernel can be found in Sousa & Scalo (2022*b*).

Although the current manuscript is focused on applications based on structured finite difference solvers, the contents of this section may serve as the foundation for the application of an unified approach to shock capturing and SFS turbulence modeling to different platforms, such as unstructured block-spectral solvers. In fact, the shock capturing capability of solving for filtered governing equations was shown for high-order flux-reconstruction settings in Sousa & Scalo (2022*a*). A projection onto the Legendre basis functions was used in that work to allow for the estimation at the energy near the cutoff and for the modulation of the dissipation at different scales. An extension for turbulent and shock/turbulence interaction problems in this context is the next step.

QSV CLOSURE APPLIED TO THE FILTERED COMPRESSIBLE NAVIER STOKES EQUATIONS

In this section, the compressible implementation of large scale simulations focusing on the unification of the treatment between shocks and turbulent eddies will be discussed. First the governing equations will be derived from the compressible Navier-Stokes relations using filtering operations and the unclosed terms as well as their closure will be presented.

Governing equations

The compressible Navier-Stokes system of equations can also be filtered by an operation that commutes with the derivation similarly to its incompressible counterpart (1). After the initial filtering, it is chosen to implicitly solve density related nonlinear terms by defining the Favre filter operation as, $\check{f}=\frac{\overline{\rho f}}{\overline{\rho}}$, and solve for the Favre filtered quantities ($\check{f}$), ultimately simplifying the final LES system for compressible flows. Previous compressible LES implementations based on the Favre-filtered equations were successfully performed with different closure models by, for example, Vreman *et al.* (1995) and Nagarajan *et al.* (2003). In this work, we follow the path of the latter, and derive the Favre-filtered Navier-Stokes relations introducing a pressure correction based on the subfilter contribution to the velocity advection,

$$\frac{\partial\overline{\rho}}{\partial t}+\frac{\partial\overline{\rho}\check{u}_j}{\partial x_j}=0, \tag{5}$$

$$\frac{\partial\overline{\rho}\check{u}_i}{\partial t}+\frac{\partial\overline{\rho}\check{u}_i\check{u}_j}{\partial x_j}=-\frac{\partial\overline{p}}{\partial x_i}+\frac{\partial\mu\check{\sigma}_{ij}}{\partial x_j}-\frac{\partial\overline{\rho}\tau_{ij}}{\partial x_j}, \tag{6}$$

$$\frac{\partial\overline{E}}{\partial t}+\frac{\partial(\overline{E}+\overline{p})\check{u}_j}{\partial x_j}=\frac{\partial}{\partial x_j}\left(k\frac{\partial\check{T}}{\partial x_j}\right)+\frac{\partial\mu\check{\sigma}_{ij}\check{u}_i}{\partial x_j}-\frac{\partial\overline{\rho}C_p q_j}{\partial x_j}, \tag{7}$$

$$\frac{\overline{p}}{\gamma-1}=\overline{E}-\frac{1}{2}\overline{\rho}\check{u}_i\check{u}_i-\frac{1}{2}\overline{\rho}\tau_{ii}. \tag{8}$$

The non negligible terms that contribute to the energy flux from large to small scales are, the SFS stress tensor, $\tau_{ij}=\widetilde{u_i u_j}-\check{u}_i\check{u}_j$, and the SFS temperature flux, $q_j=\widetilde{Tu_j}-\check{T}\check{u}_j$.

SFS modeling via the QSV method

A complete model for performing compressible large scale simulations that could include both shocks and turbulent events is then proposed as,

$$\tau_{ij}=-C_{\tau_{ij}}\frac{1}{2}\left(\mathscr{D}_{ij}\frac{\partial\ddot{u}_i}{\partial x_j}+\mathscr{D}_{ji}\frac{\partial\ddot{u}_j}{\partial x_i}\right), \tag{9}$$

$$q_j=-C_q\mathscr{D}_{jj}\frac{\partial\ddot{T}}{\partial x_j}, \tag{10}$$

where $\mathscr{D}_{ij}=\upsilon_i(\check{u}_j)\ell_j$ is the dissipation magnitude tensor, being $\ell_j=\overline{\Delta}_j$ is the subfilter length scale and

$$\upsilon_i(\check{u}_j)=\sqrt{\frac{2E^i_{k_c}(\check{u}_j)}{\overline{\Delta}_i}} \tag{11}$$

being the subfilter velocity scale. Note that the cutoff energy estimation operation, explained in detail in Sousa & Scalo (2022*b*), is carried out in the i-th spatial direction but on the j-th filtered velocity component. Additionally, the double dot superscript in (9) and (10) indicates the filter modulated quantities, defined as

$$\frac{\partial\ddot{u}_i}{\partial x_j}=\frac{\partial\check{u}_i}{\partial x_j}*\left(1-\widetilde{G}_{\mathrm{qsv}}\right), \tag{12}$$

$$\frac{\partial\ddot{T}}{\partial x_j}=\frac{\partial\check{T}}{\partial x_j}*\left(1-\widetilde{G}_{\mathrm{qsv}}\right), \tag{13}$$

where the modulation step is performed in the three directions and $\widetilde{G}_{\mathrm{qsv}}$ is a weighted average between Padé- and Fejér-filtered quantities (Sousa & Scalo, 2022*b*). It is possible that the cutoff energy estimation procedure will lead to large variations in space when highly localized flow features, such as shocks, do not align with the grid or the grid itself is deformed. In those cases, a smoothing procedure, i.e. a gaussian filter (Cook, 2007), can be performed on the $\upsilon_i(\check{u}_j)$ term.

The values of the pre-factors, C_q and $C_{\tau_{ij}}$ are given by:

$$C_{\tau_{ij}}=\begin{cases}1.0, & \text{if } i=j,\\ 0.6, & \text{if } i\neq j,\end{cases}\quad\text{and}\quad C_q=0.8. \tag{14}$$

These values are obtained by carrying out a similar *a priori* analyses as the one shown in figure 1. The magnitude of these constants are informed, though, through the application of such procedure to test cases such as the 1D Riemann shock tube problem (Sod, 1978) and the Taylor Green Vortex (TGV). The details of such analyses are gathered in Sousa & Scalo (2022*b*). Ultimately, the suggested constants (all in the order of unitary value) work well for a broad set of test cases as shown below. However, if different numerical schemes or filters are used, adjustments might be necessary.

DEMONSTRATING QSV'S SHOCK CAPTURING CAPABILITY

It is chosen to simulate a challenging test case for a shock capturing model to assess how the QSV model behaves and compare it against the established Localized Artificial Diffusivity (LAD) (Cook, 2007; Kawai *et al.*, 2010). Inspiration is

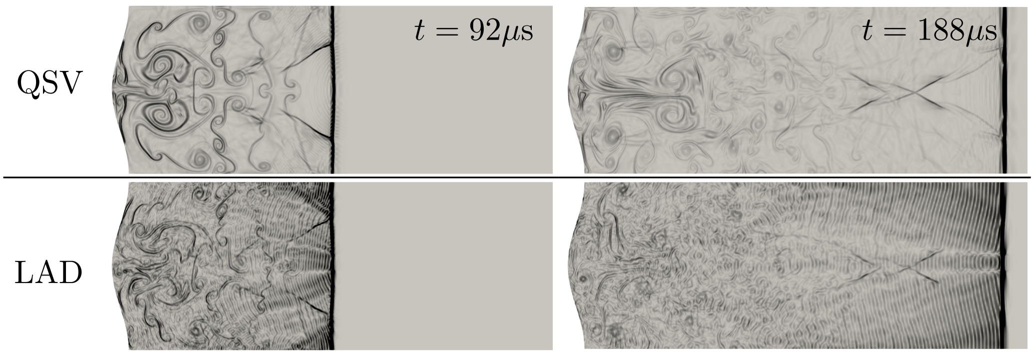

Figure 2. Numerical Schlieren comparison between QSV- and LAD-based simulations of a $M = 5.0$ shock in a fluid with $\gamma = 1.15$ after it is reflected from a sinusoidal wall performed with $N_x = 1024$ and $N_y = 256$ grid points
.

taken from experiments reported by Denet *et al.* (2015) and numerical simulations by Lodato *et al.* (2016), where a planar shock wave impinges on a sinusoidal wall with 1.0 mm amplitude and wavelength of 2.0 cm. These tests were designed to confirm the theoretical results by Clavin (2013) that predicted the formation of a lasting pattern of triple points after a shock reflection off a smooth sinusoidally-perturbed wall.

Following Tonicello *et al.* (2020), a computational domain encompassing one wavelength and 10 cm in the wall normal direction is used to simulate a $M = 5.0$ shock propagating in a fluid with a lower specific heat ratio of $\gamma = 1.15$, which increases the strength of the reflected shock and approaches the Newtonian limit (Lodato *et al.*, 2017). Moreover, the shock is initialized 7.5 cm away from the wall and the Sutherland's law for air was used to model dynamic viscosity.

The simulations are solved with periodic conditions at the top and bottom boundaries together with a no slip adiabatic wall on the left. Moreover, this test case confirms the capability of the QSV model to solve the filtered equations in distorted grids. The grid-transformations necessary for such calculations is described in Sousa & Scalo (2022*b*).

Figure 2 shows a comparison between the resolution capability of QSV- and LAD-based simulations for the same test case. The QSV approach is able to accurately simulate the flow dynamics, e.g. the presence of triple points at the intersection of the incident shock (IS), the Mach stem (MS), the reflected shock and the slip line (SL) and the formation of strong counter-rotating vortex pairs after the collision and subsequent detachement of slip lines, as well as preserve symmetry. On the other hand, when the LAD approach is used to solve the current test case, a high level of spurious oscillations was observed. Additionally, an early symmetry breaking behavior is present. Similar post shock oscillations were reported to be a numerical challenge during in the analysis performed in Lodato *et al.* (2017).

DEMONSTRATING QSV'S SUBFILTER SCALE (SFS) TURBULENCE MODELING CAPABILITY

The complete QSV closure relations (9) - (10) for the filtered compressible Navier-Stokes are now tested by assessing their capability of modeling the energy flux from large to small scales in a three-dimensional turbulence problem. The evolution of a Taylor-Green vortex (TGV) is studied via *a priori* and *a posteriori* analyses.

The latter consists in comparing the result of a coarse simulation started from the initial conditions against a reference solution, in this case a Direct Numerical Simulation (DNS) and is shown hereafter. The prior was the source of valuable information regarding the coefficients used in the QSV closure and it is gathered in Sousa & Scalo (2022*b*).

The subsonic TGV test case is initialized in a cubic domain $\Omega \in [-\pi L, \pi L]^3$ with a Reynolds number $Re = \rho_0 V_0 L/\mu_0$ equal to 5000 and a Mach number $M_t = 0.1$. This choice avoids compressibility effects with the objective to focus only on QSV's performance when applied to hydrodynamic turbulence.

An *a posteriori* study is performed with 96^3 grid points, comparing the QSV model, the Smagorinsky model (SMAG), the Dynamic procedure (DYN) and the Coherent vorticity Preserving (CvP) method against the DNS sharp-spectrally filtered down to the LES grid resolution. Details on the implementation of the latter method, such as constants and test filter strength, can be found in Chapelier *et al.* (2018).

The state of the turbulence of the different TGV simulations is monitored by analyzing the evolution of volume-averaged kinetic energy, $E_{\text{kin}} = \langle \frac{1}{2}\rho u^2 \rangle$, and its dissipation rate, defined as, $\varepsilon_{\text{kin}} = -\frac{\partial E_{\text{kin}}}{\partial t}$, with results gathered in figure 3. It can be observed that simulating a $M_t = 0.1$ TGV flow with 96^3 points without any turbulence model leads to a numerically unstable run. In the absence of a model, ε_{kin} becomes positive around $tV_0/L \approx 10$, indicating a spurious generation of kinetic energy, ultimately leading to diverging numerical results. Nonetheless, the results without the addition of extra dissipation to account for the energy flux to subfilter scales can be used to assess the performance of SFS models in the early stages of the simulation, when only large scales exist and the models should be inactive.

In figure 3, a zoomed region focused on the period $tV_0/L = 0-5$ shows that all the models considered, apart from the plain Smagorinsky model, are able to mitigate the addition of excess dissipation in the early stages of the flow, following both filtered DNS and no-model results closely. The over attenuation induced by the plain Smagorinsky model persists as the flow develops and ends up leading to a smaller dissipation peak, in comparison with the other models considered. Furthermore, QSV's results, when compared to ones obtained via DYN or CvP, are closer to the filtered DNS results from $tV_0/L \approx 6$ onwards, introducing a slight over dampening in the prior period. Similar levels for the peak in ε_{kin} are recovered for these three models but, only the QSV-based results recover the dissipation plateau existent in the DNS results af-

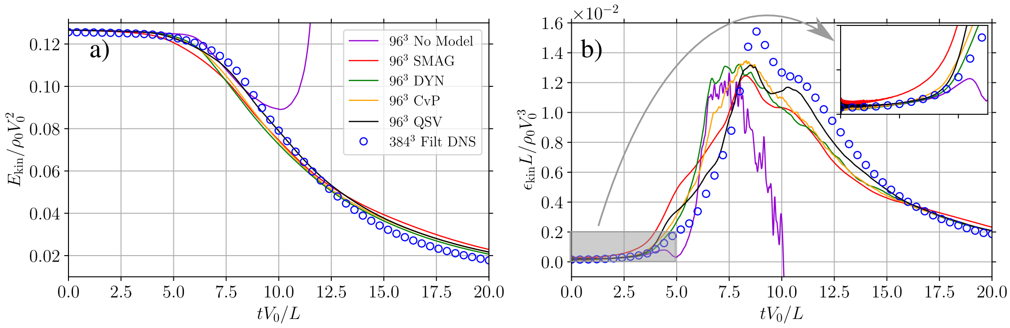

Figure 3. Evolution of volume-averaged kinetic energy (a) and dissipation (b) of the LES of the Taylor-Green Vortex with initial amplitude of $M_t = 0.1$. Different subfilter models are shown and compared against sharp spectral filtered DNS data up to the same grid resolution.

ter its peak.

QSV AS A UNIFIED APPROACH FOR TURBULENCE MODELING AND SHOCK CAPTURING

After having established the capability of the QSV model to act separately as a shock capturing and a SFS turbulence modeling closure, we now test a case exhibiting shock-wave turbulence interaction such as a supersonic TGV flow with initial Mach number $M_t = 1.2$. At this level, the initial perturbations rapidly induce wave steepening and shocks in the solution before the initial vortex breaks down into turbulence and the hydrodynamics start to govern the flow. The objective of this test case is to assess how the different models cope with the presence of both shocks and turbulence in the simultaneously.

To study the evolution of a supersonic TGV, an exact compressible energy norm (Myers, 1991) that accounts for energy stored in both hydrodynamic and thermodynamic fields is used. As in the subsonic TGV case, the exact compressible norm can be volume-averaged,

$$E_{\mathrm{c}} = \langle \rho(h-h_0) + \frac{1}{2}\rho \mathbf{u}^2 - \rho T_0(s-s_0) - (p-p_0) \rangle, \quad (15)$$

and its dissipation rate can be determined by a time derivative as $\varepsilon_{\mathrm{c}} = -\frac{\partial E_{\mathrm{c}}}{\partial t}$.

Again, an *a posteriori* study is performed with 96^3 grid points, comparing the QSV model and various eddy viscosity methods when augmented by the addition of the LAD shock capturing approach against a QSV-based $M_t = 1.2$ TGV simulation with 384^3 grid points, used here as a reference solution since the presence of a shock in the flow field renders a DNS is not strictly possible. In Sousa & Scalo (2022*b*), though, it is shown that the solution is essentially converged at that resolution level. Additionally, if the current test case is simulated without the inclusion of a shock capturing approach, it leads to numerical instabilities regardless of the turbulence model considered (Sousa & Scalo, 2022*b*).

The results from such *a posteriori* analysis are gathered in figure 4. The base formulation of the LAD model (Kawai *et al.*, 2010) induces the addition of artificial dynamic viscosity (μ_{art}), which ultimately contributes to the stabilization of the run performed with LAD as the only active model. The results from the LAD model are the most inaccurate and become unstable if μ_{art} is deactivated and only the artificial bulk viscosity and artificial conductivity components are active.

Despite achieving numerical stability, all the LAD-aided runs overestimate the dissipation rate around $tV_0/L \approx 6.5$, during the presence of the largest shock discontinuities in the flow, and underestimate the dissipation rate peak, in comparison with the reference results. In comparison with the LAD-aided turbulence models, the QSV-obtained curve remains closest to the reference results throughout the whole evolution of the flow, introducing less dissipation in the shock-dominated period and predicting better the magnitude of the dissipation rate peak. Additionally, it can be observed that, in the initial stages of the flow, when only large scales are present QSV is the model that is closest to the reference results, showcasing its ability to dynamically modulate the added SFS dissipation magnitude.

These results ultimately support the claim that the proposed QSV model is a genuine unified approach for turbulence modeling and shock capturing. On top of being able to perform each task separately, it outperforms the simple addition of separate turbulence and shock capturing models in flow setups where both hydrodynamic turbulence and shock discontinuities are happening simultaneously.

CONCLUSION

A novel technique named the Quasi-Spectral Viscosity model (QSV), was introduced. It is designed to simulate accurately the large scales present in both shock and turbulence dominated flows by exploiting the residual of filter transfer functions to estimate both the amplitude of fluctuations near the grid cutoff and modulate the viscosity magnitude for different wavenumbers. This feature allows for an implementation using only spatial operators, applicable to finite-difference solvers.

The QSV mathematical framework is based on an extension of LES closures and a parallel between these and spectral vanishing viscosity (SVV) based models. The 1D Burgers' problem is used to showcase the connection between previous LES models and how they can be understood as a way of solving shock dominated solutions. Moving forward, the QSV model was shown to perform well in both low-speed and highly compressible flow setups. For example, the same QSV framework can be used to solve a Taylor-Green Vortex with both sub and supersonic initial conditions. Moreover, the QSV model is flexible, being able to be applied in curved and stretched domains by using grid transformations. The collection of satisfactory results across intrinsically different flow setups supports the claim that the QSV method can simultaneously capture shocks and act as a subfilter turbulent closure.

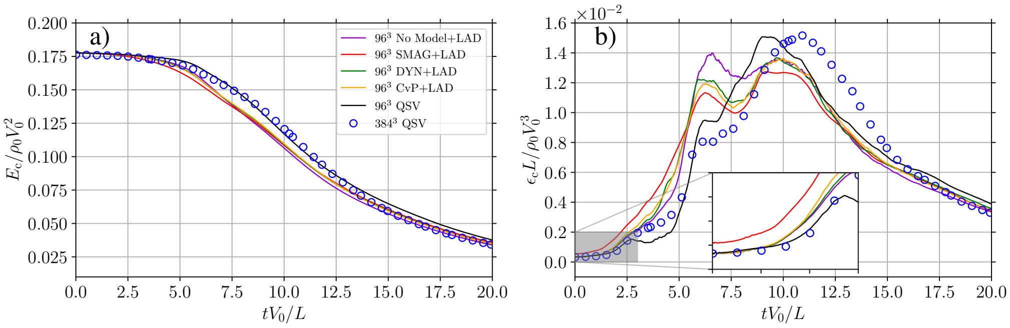

Figure 4. Evolution of the volume-averaged exact compressible energy norm (a) and its dissipation rate (b) in a $M_t = 1.2$ TGV simulation. Results obtained by different sub-filter models using 96^3 grid points are compared against a reference solution computed using QSV model with 384^3 points.

As a final remark, although the current implementation is aimed at finite-difference solvers, it is possible to extend the QSV approach to unstructured solvers based on spectral numerics. Due to the opportunity of projecting the solution of each element onto a hierarchical set of orthogonal basis functions, a spectrally based implementation would be able to easily gage the magnitude the energy near the cutoff and would be able to modulate freely the amplitude of the viscosity kernel for different wavenumbers. This fact renders the use of global filtering operations unnecessary and could lead to simpler and more flexible implementations. An initial implementation of a shock capturing method that exploits the Legendre basis in discontinuous flux reconstruction settings was introduced in Sousa & Scalo (2022*a*). Its extension to turbulence and shock/turbulence interaction modeling will be explored in future research.

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