MULTI-FIDELITY APPROACH FOR TRANSITIONAL BOUNDARY-LAYER

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ABSTRACT

A multi-fidelity computational approach is proposed for transitional boundary layer flow in order to obtain both high fidelity and high efficiency in flow simulation. Because we can categorize the transitional flow into three regions, i.e., laminar flow with growing instabilities, transition region, and turbulent flow, three models are judiciously selected. A boundary layer stability method, here nonlinear parabolized stability equations (NPSE), is chosen for the laminar region incorporating nonlinear interactions between the instabilities. Large-eddy simulation (LES) is applied for the transition region including the beginning of the turbulent flow. Reynolds-averaged Navier-Stokes (RANS) simulation is used for the downstream turbulent regime. Since the NPSE-coupled LES method has been validated for transitional boundary layers including both incompressible (Kim *et al.*, 2018, 2019, 2020, 2021) and compressible (Lim *et al.*, 2021) flows, appropriate treatments for the interface between LES and RANS are mainly investigated in this study. The current multi-fidelity approach is tested for a transitional boundary layer on a flat plate. Computational fidelity and cost are compared with previous high-fidelity computations. We successfully demonstrate that the proposed multi-fidelity approach provides both high fidelity and high efficiency in transitional flow computations.

Introduction

Turbulent transition in boundary layer flow is a critical flow phenomena which is directly related to vehicle drag and heat transfer. Nonetheless, an efficient and accurate prediction of a whole transition process is challenging. A main difficulty arises from three distinct stages involved in a complete transition process: laminar flow, transition region, and turbulent flow. Historically, flow modeling approaches have been developed mainly for each individual region. Consequently it is imperative to suggest a unified approach which provides both high fidelity and computational efficiency for a whole transitional flow, which is the main goal of this study.

The NPSE-coupled LES method has been validated for transitional boundary layers including both incompressible (Kim *et al.*, 2018, 2019, 2020, 2021) and compressible (Lim *et al.*, 2021) flows. The NPSE method can incorporate nonlinear interactions between instabilities up to a weakly nonlinear region where the mean flow is still laminar. LES with major instabilities assigned at the LES inlet from NPSE can carry the simulation from the nonlinear region up to fully turbulent flow. Our previous study (Kim *et al.*, 2020) shows that appropriate sub-grid-scale models are dormant in pre-transition region where all the instabilities are resolved in a given LES grid and quickly active in generating the eddy viscosity in the late-nonlinear region to the turbulent flow where the given spatial resolution is not fine enough to capture all the turbulence spectrum. Since the NPSE-coupled LES method already demonstrates DNS-like fidelity with a roughly 10% of the DNS computational cost for transitional wall-bounded flows (Kim *et al.*, 2019; Lim *et al.*, 2021), the current study focuses on the LES-RANS coupling method to maintain the DNS-like fidelity and a further reduction in the computational cost.

The several multi-fidelity method for a turbulent study have been suggested. The most famous approach is the detached eddy simulation (DES) while it is not proper for the transitional flow due to modeling the boundary-layer using RANS. To overcome this limitation, Bader et. al suggested the $k-\omega$ delayed DES method (Bader *et al.*, 2022). It used the operator developed in Vreman sub-grid-scale model (Vreman, 2004) instead of Smagorinsky operator. It predicts transition very roughly with under-resolved vortical structures. On the other hands, the zonal RANS-LES method, i.e. embedded-LES, have been suggested and used (Quéméré & Sagaut, 2002; Anupindi & Sandberg, 2017; Woodruff, 2019; Roidl *et al.*, 2012). For the zonal method, two interfaces exist: RANS-to-LES interface and LES-to-RANS interface. The interface from the RANS to LES have been widely investigated. The small eddies which are not captured in RANS model are required for the LES so that the inlet boundary conditions which generate small-scale fluctuations are studied. For example, the synthetic eddy method was widely used to assign more physical fluctuations rather than white noises. The interface from the LES to RANS is straightforward because a variables for RANS are reconstructed from the LES by applying the Reynolds average. Additionally, these hybrid RANS-LES method have been generally applied in separated flow. Therefore, the detail consideration was not conducted in order to investigate sensitivity of this simple reconstruction method. Only a few studies focused on the interface from the LES to RANS (Nolin *et al.*, 2005; Schluter *et al.*, 2002). It should be noted that the most study using the zonal method is applied in turbulent flow.

In this study, we suggest a multi-fidelity approach which combines three methods: (1) stability analysis, particularly nonlinear parabolized stability equations (NPSE), for laminar flow with instabilities, (2) large-eddy simulation (LES) for the highly-nonlinear transition region and the beginning of turbulent flow, and (3) Reynolds-averaged Navier-Stokes (RANS) model for the subsequent fully-turbulent region. The PSE-LES interface treatment was successfully validated in the authors' previous studies (Kim *et al.*, 2019; Lim *et al.*, 2021). Thus, the current study focuses on the discussion of the RANS-LES interface which have not been gained enough attention in the community. It is expected that the suggested method is able to provide DNS-like fidelity with a fraction of DNS/LES computations for a whole transitional flow.

Computational Methods

The multi-fidelity approach is suggested in the current study by combining NPSE, LES, and RANS as represented in figure 1. NPSE is conducted to predict growth of instability modes in 2-D laminar boundary-layer. NPSE is able to consider nonlinear interactions and growth of instabilities accurately with low-computational cost. Because LES is able to accurately resolve turbulent transition with moderate cost, it is applied from nonlinear region up to the subsequent early turbulent flow as shown in figure 1. RANS simulation is carried out for turbulent boundary-layer following LES for further reducing the computational cost while maintaining reasonable accuracy.

The total variable is decomposed into the mean flow part, $\bar{\phi}$, and the sub-grid-scale, residual part, ϕ', for the LES method.

$$\check{\phi} = \bar{\phi} + \phi' \tag{1}$$

For the RANS approach, the total variable is decomposed into the Reynolds-averaged part, Φ, and the turbulent fluctuation, ϕ.

$$\check{\phi} = \Phi + \phi \tag{2}$$

NPSE

The nonlinear parabolized stability equations (PSE) is a powerful tool to predict growth and interactions of instability modes both in linear and nonlinear stages. It takes into account of non-parallel effects and complex modal interactions. In the current study, PSE is applied to accurately capture the instability modes and secondary instability in laminar boundary-layer with low-cost. The nonlinear PSE code used in the current study have been successfully validated in incompressible boundary-layer (Park & Park, 2013; Kim *et al.*, 2019, 2020; Park *et al.*, 2019; Kim *et al.*, 2021) and compressible boundary-layer (Park & Park, 2016; Lim *et al.*, 2021).

LES

The late transition regions and turbulent boundary-layer have matured wave spectrum. In this situation, PSE should consider a lot of instability modes so that the computational cost of PSE extensively increases; it will be a DNS-like computational cost. Thus, large-eddy simulation (LES) is conducted instead of PSE to capture the late transition regions and subsequent early turbulent flow. The authors have successfully conducted turbulent transition simulation using PSE-LES combining method (Kim *et al.*, 2019; Lim *et al.*, 2021). The detail methods for the current LES and combining method are documented in the reference (Kim *et al.*, 2019) including the grid information and numerical schemes.

RANS

Reynolds-averaged Navier-Stokes (RANS) method is useful for the turbulent boundary-layer. It is well known that RANS have been successfully validated for the simple flow like non-separated flow or mild pressure-gradient condition. In the current case, the flow is well attached, and the current simulation is a canonical boundary-layer over a flat plate so that RANS guarantees the acceptable accuracy with low-cost. Therefore, RANS is applied to the turbulent boundary-layer after LES in order to reduce computational cost.

The governing equations for the RANS approach are written as equation 3 and equation 4.

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{3}$$

$$\frac{\partial U_j}{\partial t} + \frac{\partial}{\partial x_i}\left(U_i U_j\right) = \nu \frac{\partial^2 U_j}{\partial x_i \partial x_i} - \frac{\partial \tau_{ij}^R}{\partial x_i} - \frac{1}{\rho}\frac{\partial P}{\partial x_j} \tag{4}$$

The equations require the closure model for the Reynolds stress tensor τ_{ij}^R. The common approach is the Boussinesq approximation which assumes that the stress tensor is proportional to the strain rate tensor.

$$\tau_{ij}^R = -2\nu_t S_{ij} \tag{5}$$

Here, the eddy viscosity, ν_t, is still unknown so that the current RANS uses the k-ω SST model to determine the eddy viscosity Menter *et al.* (2003).

Flow Conditions and Interfaces

The current computational case to validate the suggested multi-fidelity approach is subharmonic transition on a zero-pressure-gradient flat plate following the Kachanov & Levchenko (1984)'s experiment. It showed the subsonic natural transition induced by subharmonic resonance, i.e. H-type transition, which is triggered by the Tollmien-Schlichting wave and the subharmonic oblique waves. The subsonic natural transition has long laminar boundary-layer, short transition length, and subsequent turbulent boundary-layer. Therefore, it is a good candidate for showing a performance and benefits of the current multi-fidelity method.

Because the instability modes in long laminar flow grow from very small scales as shown in figure 1, the computational cost becomes DNS-like when LES reproduces them. Thus, NPSE which is advanced stability analysis is used to predict both instabilities and nonlinear interactions between them. The obtained instabilities are assigned into the inlet of LES with laminar solution.

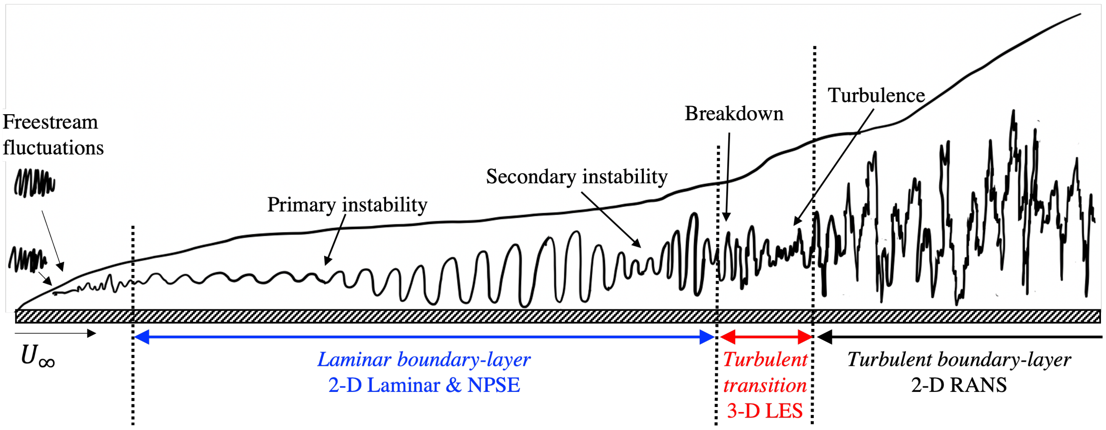

Figure 1. Schematic diagram of the current multi-fidelity method.

Figure 2a shows the amplitude growth of five selected modes. NPSE is conducted until $\sqrt{Re_x} = 681$ where the boundary layer is still laminar although strong nonlinear interactions occur. Thirteen modes determined by the threshold $\max(A_{(m,n)}) > 0.01\% * U_\infty$ following the (Kim *et al.*, 2019), where m and n are temporal and spanwise spatial modes, respectively, are assigned into the inlet of LES with laminar solution. The thick lines which are obtained from LES well follow NPSE results. The amplitude is reduced after $\sqrt{Re_x} = 700$ due to saturating wave-spectrum during turbulent transition. The details of NPSE-coupled LES method are well described in the authors' previous studies (Kim *et al.*, 2019; Lim *et al.*, 2021).

RANS simulation is applied in the fully turbulent flow after LES computation for turbulent transition. LES is able to provide Reynolds-averaged variables. From the given variables from LES and the Boussinesq relation, the variables for RANS simulation can be reconstructed. Figure 2b shows the inlet profiles of the RANS simulation. In the inlet of RANS, three variables are used: velocity vector, turbulent kinetic energy, and turbulent dissipation rate. The Reynolds-averaged velocity vector and turbulent kinetic energy can be straightforwardly obtained from the results of LES. However, the turbulent dissipation rate ω cannot be calculated directly from the LES results. Thus, the turbulent dissipation rate is obtained using turbulent kinetic energy and eddy viscosity. The following equations show the procedures for the calculation for ω.

$$\omega = \frac{\bar{k}}{\nu_{t.RANS}} \tag{6}$$

While eddy viscosity of LES exists, it differs from eddy viscosity of RANS because of the different physics between them. Eddy viscosity for the RANS, which will be used for the calculation of ω assigned into the inlet of RANS, should be also reconstructed. Here, relying on the Boussinesq approximation, eddy viscosity for RANS simulation is calculated as shown in equation 7 (Monier *et al.*, 2018).

$$\nu_{t.RANS} = \frac{||\bar{\tau}^r_{ij}||}{||\langle \bar{S}_{ij} \rangle - 1/3\delta_{ij} \langle \bar{S}_{kk} \rangle ||} \tag{7}$$

where $||.||$ denotes norm, and $\bar{\tau}^r_{ij}$ means traceless stress tensor of LES.

Computational Results

In order to show the efficiency of the current method, the number of grid counts are compared with previous studies in table 1. As a result, a number of grid count is reduced by two orders-of-magnitude compared to the simulation where the turbulent boundary-layer is also computed by LES. While DNS study (Lozano-Durán *et al.*, 2018) requires 250 million grid points and LES (Kim *et al.*, 2019) requires 30 million grid points for the same transitional boundary-layer, the current multi fidelity needs only 6 million grid points.

Table 1. A number of grid points comparison.

Computational methods	grid points
DNS (Lozano-Durán *et al.*, 2018)	250×10^6
PSE-LES (Kim *et al.*, 2019)	30×10^6
Current multi-fidelity method	6×10^6

The computational results are shown in figure 3. Judiciously combined three methods successfully predict transitional boundary layer (see figure 3b). Because of the high-fidelity and unsteady nature of LES, the instantaneous fluctuating boundary-layer and vortical structures are captured as shown in figure 3a and figure 3a. The staggered $\Lambda-$structures which are generally observed in subharmonic transition are well captured in LES computation (see, figure 3b). The subsequent hairpin vortices and small eddies are also reproduced.

Skin-friction coefficients of laminar, LES, and RANS simulation are smoothly connected in figure 3c. LES predicts that skin friction deviates from laminar value due to turbulent transition and rapidly grows to turbulent value. RANS simulation well expects skin friction of turbulent boundary layer. Skin friction of RANS, however, slightly overshoot near the inlet. It is conjectured that ω which is constructed from LES variables rather than directly obtained from the transport equation does not suit to RANS equations.

For a remedy of the discrepancy at the interface of LES and RANS, the inlet of the RANS is slightly modified. The profiles are shown in Figure 4a The reconstructed turbulent kinetic energy is different from the desired value for RANS computations, red line. Therefore, turbulent kinetic energy

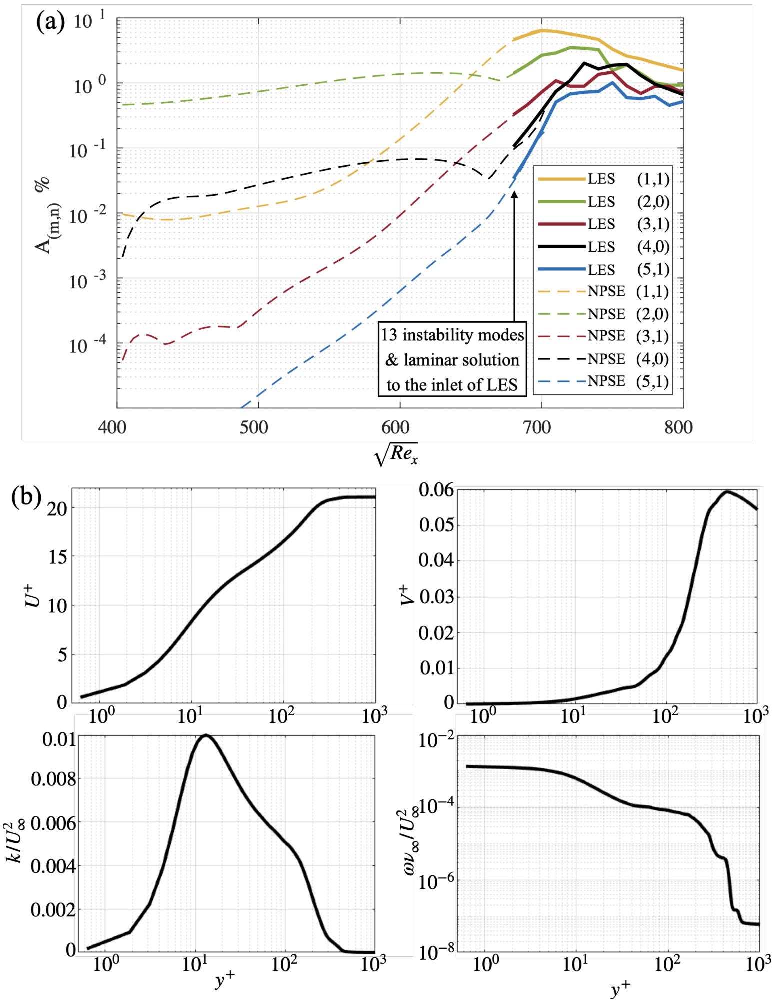

Figure 2. Computational data for (a) NPSE- LES and (b) LES-RANS interfaces: (a) amplitude growth of 5 major modes from NPSE analysis among 13 instabilities assigned at the LES inlet (b) mean velocity (U^+ and V^+) and turbulence quantities (k and ω) assigned at RANS inlet from the LES computation.

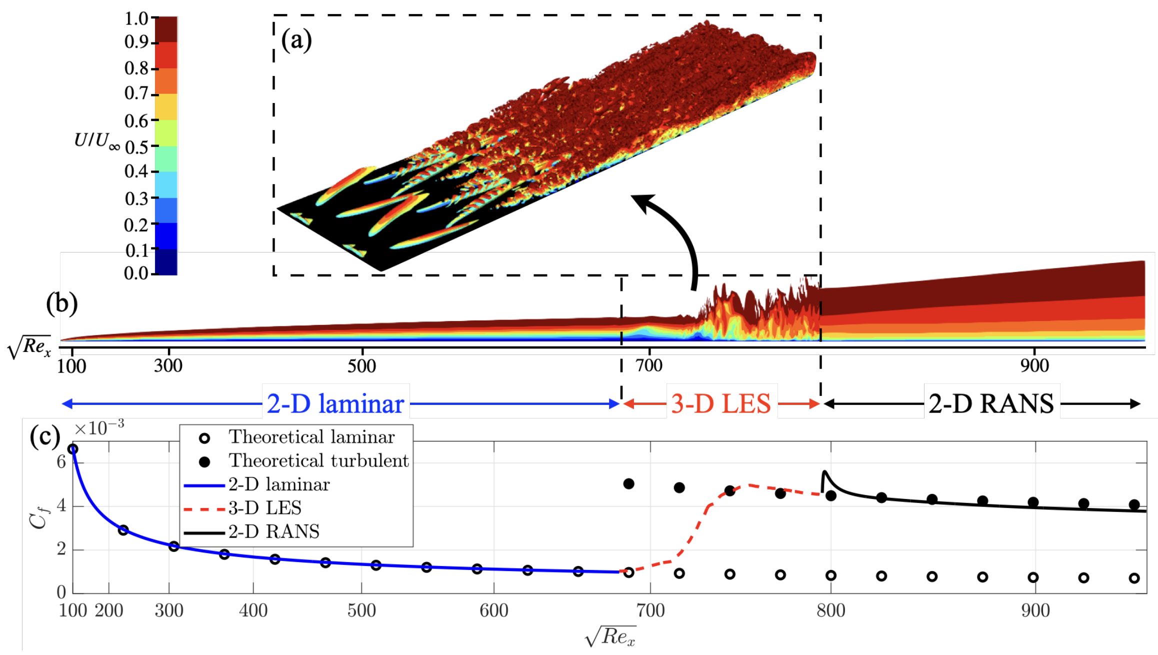

Figure 3. Selected computational results: (a) Vortical structures visualization using iso-surface of $Q = 20$ coloured by normalized streamwise velocity in the LES domain. (b) Normalized instantaneous streamwise velocity fields on a spanwise-normal plane; the scale x:y=1:6 is used for the visualization purpose. (c) Averaged skin friction coefficient C_f.

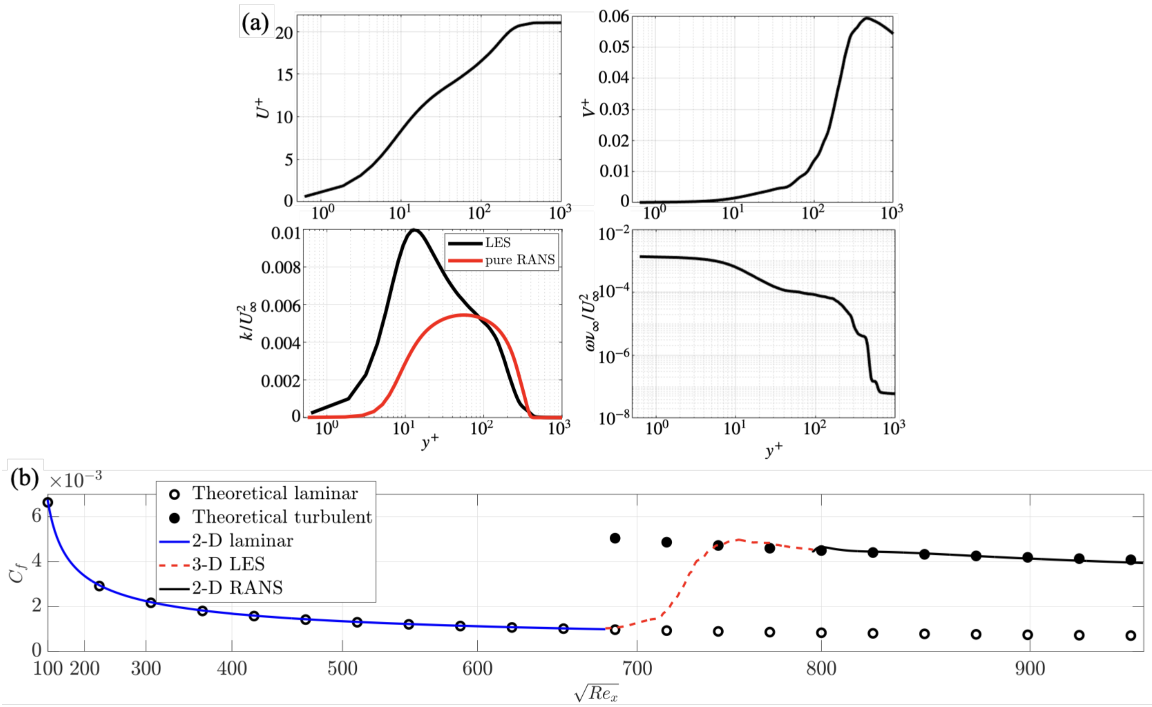

Figure 4. Selected computational results from the modified RANS inlet: (a) The mean velocity (U^+ and V^+) and turbulence quantities (k and ω) assigned at RANS inlet. (b) Averaged skin friction coefficient C_f.

taken from the pure RANS, which has the same boundary-layer thickness with the current LES results at the LES-RANS interface, is assigned into the inlet of RANS. The steady 2-D RANS simulation for a flow over a flat plate is additionally conducted in order to obtain the pure RANS results.

As a result, the current modification works as intended. There is no weird pick near the inlet in the RANS computation. The skin friction curve becomes much smoother compared to figure 3c. The limitation of the current method is that it demands pure RANS data. Also, the current modification mitigates the transient behavior near the RNAS inlet while small modulation still exists. Thus, the further investigation will be needed and conducted.

Conclusions

A multi-fidelity approach is suggested in the current study. Three methods, NPSE, LES, and RANS, are incorporated carefully to describe a whole transitional boundary layer. The suggested approach provides high fidelity and high efficiency in numerical computations. NPSE is used in the laminar boundary layer to predict interactions and growth of instabilities with low computational cost. LES is carried out for turbulent transition and the early turbulent flow. LES well reproduces vortical structures and boundary-layer growth. RANS is applied to the subsequent turbulent boundary layer. The computational cost is drastically decreased compared to previous studies. While DNS and LES require 250 million and 30 million grid points (Lozano-Durán *et al.*, 2018; Kim *et al.*, 2019), respectively, the current approach only needs 6 million grid points. Also, streamwise velocity fields and the skin friction show that the current multi-fidelity approach well predicts the whole transitional boundary layer. The initial pick near the RANS inlet is shown due to the different solution required for the RANS even though the solution is obtained from the high-fidelity computation. The additional modification for the LES-RANS interface successfully removes this pick.

In the current study, the suggested multi-fidelity method is demonstrated in the canonical case. Furthermore, the authors plan to explore the suggested multi-fidelity approach to more complex transitional flows. The current NPSE-LES-RANS method can be extended to other transitional flows covering a wide range of transition paths, because NPSE can handle both natural and non-natural transition paths, and both LES and RANS would resolve the subsequent turbulent flow adequately.

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