12th International Symposium on Turbulence and Shear Flow Phenomena (TSFP12)
Osaka, Japan, July 19–22, 2022

KOLMOGOROV CONSTANT FOR PARTICLE-LADEN TURBULENT CHANNEL FLOW

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ABSTRACT

The Kolmogorov constant is a prefactor used to correlate the spectral distribution of turbulent kinetic energy with the turbulent dissipation rate. The Kolmogorov constant has been used in stochastic turbulence models and turbulence models based on large-scale turbulent structures, like the Smagorinsky model and other eddy viscosity-based models used in large-eddy simulations (LES). Many research works have been performed to estimate the Kolmogorov constant for a wide range of turbulent flows like homogeneous isotropic turbulence, turbulent boundary layers, etc. In earlier work, Sreenivasan (1995) has consolidated and analyzed a large number of experimental data and concluded that at high Reynolds numbers, the prefactor is a universal constant.

In the present work, we have estimated the Kolmogorov constant for particle-laden turbulent channel flow using second-order velocity structure-function based analysis. Our study reveals that, with an increase in particle volume fraction Kolmogorov constant decreases. The present analysis on the variation of Kolmogorov constant with particle volume loading will be helpful to model the two-phase turbulent flows.

INTRODUCTION

The Kolmogorov constant is the proportionality prefactor in Kolmogorov theory which states that the spectral energy in the inertial subrange is $E(k) = C\varepsilon^{2/3}k^{-5/3}$ where ε is the mean viscous dissipation rate of turbulence kinetic energy and k the wavenumber. The Kolmogorov constant is obtained based upon the Kolmogorov hypothesis for different flows such as boundary layers, channel flows, etc. In the seminal work, Sreenivasan (1995) has summarized that the Kolmogorov constant is universal and, independent of the flow configuration and Reynolds number. At sufficiently high Reynolds number where isotropy is satisfied at dissipation scale and inertial range, and will lead to a constant Kolmogorov prefactor. However, at moderate and low Reynolds numbers, the Kolmogorov constant may differ from a universal value. Antonia *et al.* (1997) performed experiments for channel flow and observed a lower value of Kolmogorov constant. They analyzed the second and third-order velocity structure-function and mentioned that small-scale isotropy should be satisfied for the existence of a universal inertial range. Heinz (2002) discussed the variations of Kolmogorov constant for equilibrium turbulent boundary layer and homogeneous isotropic stationary turbulence, and mentioned that the value is near two if anisotropy and acceleration fluctuations dominate in the energy budget. And, the value is near six if these contributions disappear. Yeung & Zhou (1997) mentioned that for the existence of inertial range, isotropy should also be present along with $-5/3$ scaling. If isotropy is not satisfied, the Kolmogorov constant may achieve a different value. The above studies point out the deviation of Kolmogorov constant from universality due to anisotropy at low and moderate Reynolds numbers for unladen cases. In turbulent flows, the local isotropy can be affected by the presence of particles as well. It is evident that particles can affect the turbulence intensity of the gas phase. In the study by Gualtieri *et al.* (2013) for particle-laden shear flow, they concluded that care should be taken while applying Kolmogorov theory as anisotropy is increased for particle-laden cases. Thus, in the present study, we want to explore whether particles can induce anisotropy in the gas phase. And, what is the effect of increased anisotropy on the Kolmogorov theory?

To answer the above question, simulations are performed for turbulent channel flows with direct numerical simulations (DNS) at Reynolds number of 5600 based on average gas velocity and channel width. It is observed that the turbulence intensity of the gas phase decreases with an increase in particle loading, along with the increase in anisotropy across the channel width. The increase in anisotropy affects the Kolmogorov constant, which is discussed in the results section.

SIMULATION PARAMETERS

The fluid phase has been considered to be incompressible and described by Navier-Stokes equation. The discrete smooth point particles are simulated using Newton's second law of the motion. The simulations have been performed in a vertical channel domain with $8\pi\delta * 2\delta * (4/3)\pi\delta$ in streamwise (x), wall-normal (y) and spanwise (z) directions, respectively.

Where δ is half channel width. No-slip boundary conditions are applied on the walls in the y-direction. The bulk Reynolds numbers ($Re_b = \rho_f * \bar{u} * 2\delta/\mu_f$) is fixed at 5600 based on the channel width (2δ) and average fluid velocity ($\bar{u}$) which corresponds to Re_τ = 180 based on the unladen frictional velocity and half-channel width. The detail simulation procedure for calculation of feedback force, near wall corrections in lift and drag, corrections for undisturbed velocity at particle locations are discussed in our earlier work (Muramulla *et al.* (2020) Goswami & Kumaran (2011)). The particles with Stokes numbers ($St = \tau_p/\tau_f$) of 105.47 and 210.93 are simulated where $\tau_p = \rho_p d_p^2/18\mu_f$, $\tau_f = 2\delta/\bar{u}$, ρ_p is the particle density, d_p is the particle diameter and μ_f is the fluid dynamic viscosity.

RESULTS

The anisotropy of small scales across the channel width can be accounted from the ratio of Kolmogorov time scale to mean shear time scale (Antonia & Kim (1992); Saddoughi & Veeravalli (1994))

$$S_c^* = S(\nu/\varepsilon)^{1/2} \tag{1}$$

where S is the mean shear, dU/dy. Antonia & Kim (1992) did the DNS for channel flow and found a value of $S_c^* = 2.5$ at the wall and reduction to low value for $y^+ > 60$ (y^+ is the wall-normal distance normalized with viscous units). The S_c^* is plotted along the wall-normal direction for $St = 210.47$ for different volume fractions (ϕ), Fig. 1. It is found that the S_c^* is 2.42 at the wall for the unladen case. A decrease in the S_c^* is observed away from the wall. The S_c^* increases across the channel as the particle volume loading is increased, suggesting an increase in anisotropy of small scales for particle-laden cases. The S_c^* at the wall for $\phi = 0.0025$ becomes almost 50% higher than the unladen case. The turbulence collapse is observed at $\phi = 0.0028$ (Rohilla *et al.* (2022); Kumaran *et al.* (2020)). This increased anisotropy can affect the Kolmogorov constant for particle-laden cases which is analyzed via a second-order velocity structure-function.

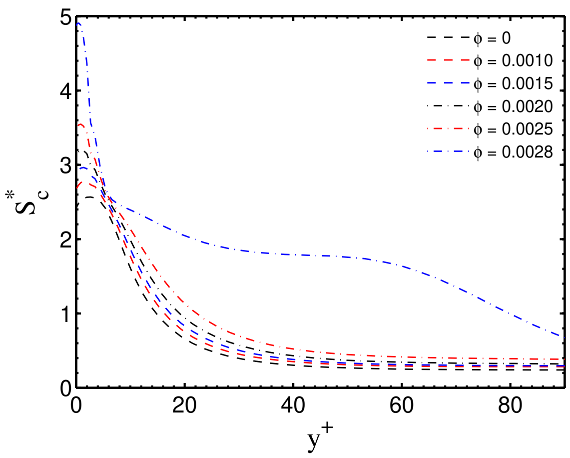

Figure 1: The ratio of Kolmogorov time scale to mean shear time scale for different volume fractions at St = 210.93.

The Kolmogorov constant (C_2) can be calculated using the second-order velocity structure-function (Kolmogorov (1941)) defined as ,

$$\langle(\delta u)^2\rangle = C_2(\varepsilon r)^{2/3}, \tag{2}$$

where r (in the inertial range, $\eta << r << L$) being the distance between the two points, η is the Kolmogorov length scale, L is the integral length scale, ε is the mean viscous dissipation rate and, $\delta u = u(x+r) - u(x)$, with u being the longitudnal fluctuations. The Kolmogorov constant is defined by C_2 and angular brackets denote the time averaging.

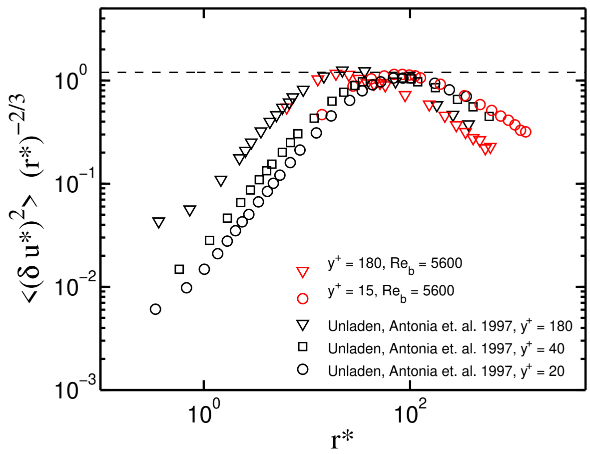

Figure 2: The second-order velocity structure-functions multiplied with $r^{*-2/3}$ for unladen cases. The dashed line is plotted at 1.2.

The second-order velocity structure-function for unladen case is plotted in Fig. 2 using the Eqn. 2. The (*) denotes the normalization with Kolmogorov scales. The profiles are plotted for two locations, one in the near-wall region ($y^+ \sim 15$) and the other at the channel center ($y^+ \sim 180$), and validated against the experimental data of Antonia *et al.* (1997). There is a good agreement between the experimental data of Antonia *et al.* (1997) and the present DNS results for both the channel locations. The majority of the reported values of C_2 in the literature are around 2 or more (Heinz (2002); Choi *et al.* (2004); Sawford & Yeung (2011); Saddoughi & Veeravalli (1994); Sreenivasan (1995)). However, a value of C_2 can be lower than two due to lower Reynolds number or if there is a deviation of isotropy in dissipation and inertial-range (Sreenivasan (1995); Antonia *et al.* (1997); Yeung & Zhou (1997)).

In Fig. 3, the second-order velocity structure-function is shown for different particle volume loadings for Reynolds number of 5600 in the channel center location. There is a decrease observed in the value of second-order velocity structure-function at all the r^* locations with an increase in volume fraction. The peak value decreases nearly to 0.4 for $\phi = 0.0025$ from 1.2 for unladen flow. The decrease observed in the value of second-order velocity structure-function, Fig. 3, is due to an increase in anisotropy across the channel, as shown in Fig. 1.

In Fig. 4, the peak value of the second-order velocity structure-function is plotted for two channel locations, one in the near-wall and the other at the channel center. It is seen that the Kolmogorov constant (C_2) decreases significantly with

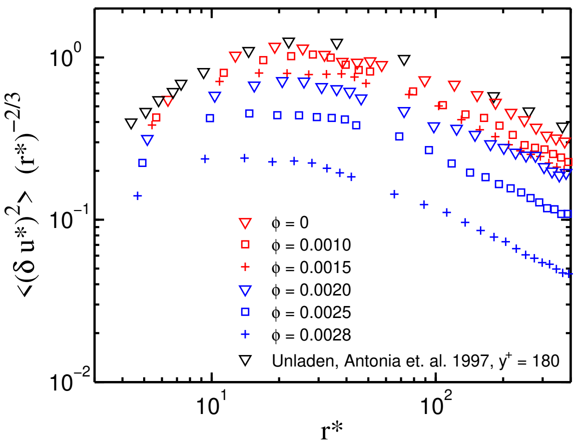

Figure 3: The second-order velocity structure-functions multiplied with $r^{*-2/3}$ for different volume fractions at channel center location ($y^+ = 180$) with St = 210.93.

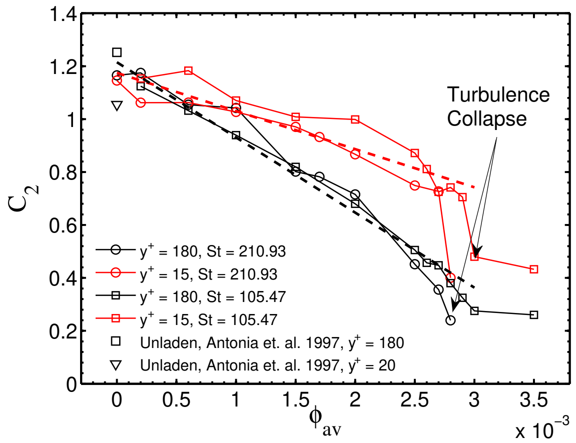

Figure 4: The peak of second-order velocity structure-function plotted for different volume fractions. The dashed lines are the fitting curves.

increase in volume loading. The C_2 value is not relevant at volume loading of $\phi = 0.0028$ and 0.003 for St = 210.93 and 105.47 respectively as turbulence has collapsed at these volume loadings (Rohilla *et al.* (2022); Kumaran *et al.* (2020)). Also, it is observed that the C_2 increases from near-wall region to the channel center for unldaden case (Antonia *et al.* (1997); Choi *et al.* (2004)). However, the decrease in the peak value of C_2 is more for the channel center location than the near-wall region with an increase in volume loading. It is interesting to note that a crossover occurs as the volume loading is increased and the peak value of C_2 becomes less in the channel center than the near-wall region for both the Stokes numbers. It is also observed that this crossover occurs at low volume loading for low Stokes number, Fig. 4.

In Fig. 5, the ratio of streamwise root mean square (rms) fluctuations to the Kolmogorov velocity scale (u_k) is plotted for $St = 210.93$. It is observed that the ratio decreases faster in the channel center than in the near-wall region with increased particle volume loading. This shows that the separation of velocity scales (large to small-scales) decreases more in the channel center than in the near-wall region. This explains the larger

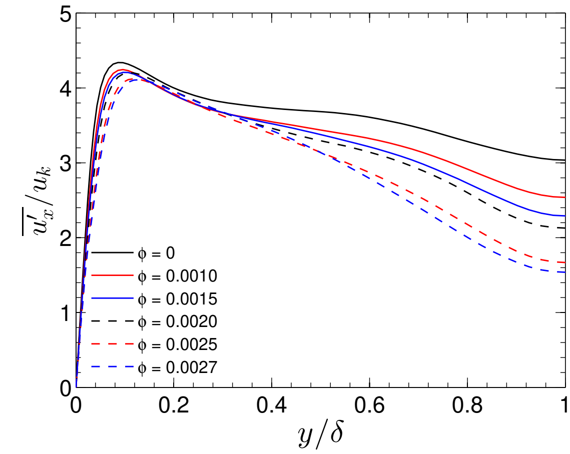

Figure 5: The ratio of streamwise root mean square fluctuations (u'_x) and Kolmogorov velocity scale (η) plotted in the wall-normal direction for a range of volume fractions for St = 210.93.

decrease of the Kolmogorov constant in the channel center region.

SUMMARY

We report that the Kolmogorov constant decreases with an increase in volume loading at moderate Reynolds number due to an increase in anisotropy. In the case of unladen wall-bounded flows, the Kolmogorov constant increases away from the wall. However, it is observed in the present study that the Kolmogorov constant in channel center location is less than the near-wall region for higher volume loadings. The demonstrated variation in the Kolmogorov constant will be helpful to develop better turbulence models in stochastic modeling and LES for particle-laden flows.

REFERENCES

Antonia, RA & Kim, J 1992 Isotropy of small-scale turbulence. In *Proc. Summer Program of the Center for Turbulence Research, Stanford*.

Antonia, RA, Zhou, Tongming & Romano, GP 1997 Second- and third-order longitudinal velocity structure functions in a fully developed turbulent channel flow. *Physics of Fluids* **9** (11), 3465–3471.

Choi, Jung-Il, Yeo, Kyongmin & Lee, Changhoon 2004 Lagrangian statistics in turbulent channel flow. *Physics of fluids* **16** (3), 779–793.

Goswami, Partha S. & Kumaran, V. 2011 Particle dynamics in the channel flow of a turbulent particle gas suspension at high Stokes number. Part 1. DNS and fluctuating force model. *J. Fluid Mech.* **687**, 1–40.

Gualtieri, Paolo, Picano, Francesco, Sardina, Gaetano & Casciola, Carlo Massimo 2013 Clustering and turbulence modulation in particle-laden shear flows. *Journal of Fluid Mechanics* **715**, 134–162.

Heinz, Stefan 2002 On the kolmogorov constant in stochastic turbulence models. *Physics of Fluids* **14** (11), 4095–4098.

Kolmogorov, AN 1941 Energy dissipation in locally isotropic turbulence. In *Dokl. Akad. Nauk. SSSR*, , vol. 32, pp. 19–21.

Kumaran, Viswanathan, Muramulla, NSP, Tyagi, Ankit & Goswami, PS 2020 Turbulence collapses at a threshold particle loading in a dilute particle-gas suspension. *EPL (Europhysics Letters)* **128** (6), 64001.

Muramulla, P, Tyagi, A, Goswami, PS & Kumaran, V 2020 Disruption of turbulence due to particle loading in a dilute gas–particle suspension. *Journal of Fluid Mechanics* **889**.

Rohilla, Naveen, Muramulla, Pradeep & Goswami, Partha S 2022 Applicability of large eddy simulations to capture turbulence attenuation in particle-laden channel flows. *Physical Review Fluids* **7** (2), 024302.

Saddoughi, Seyed G & Veeravalli, Srinivas V 1994 Local isotropy in turbulent boundary layers at high reynolds number. *Journal of Fluid Mechanics* **268**, 333–372.

Sawford, Brian L & Yeung, Pui-Kuen 2011 Kolmogorov similarity scaling for one-particle lagrangian statistics. *Physics of Fluids* **23** (9), 091704.

Sreenivasan, Katepalli R 1995 On the universality of the kolmogorov constant. *Physics of Fluids* **7** (11), 2778–2784.

Yeung, PK & Zhou, Ye 1997 Universality of the kolmogorov constant in numerical simulations of turbulence. *Physical Review E* **56** (2), 1746.