FINITE-TIME BLOWUP RELATED TO PARTICLE COLLISION IN DIRECT EULERIAN SIMULATION

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ABSTRACT

This study inspired by Falkovich *et al.* (2002) aims to investigate the motion of particles in turbulence using Eulerian approach under the assumption that particle velocity is a smooth function in space and is uniquely determined by the particle position until collisions between particles occur. When the first collision happens, a discontinuity in the particle velocity gradient appears, resulting in finite-time blowup. Using this approach, the singularities in particle velocity gradient, particle number density, and particle vorticity for various Stokes numbers and gravity factors are investigated numerically in a simple and intuitive Taylor-Green vortex flow, two-dimensional turbulence, and three-dimensional isotropic turbulence.

INSTRUCTION

Particle-laden turbulence has been actively studied for the investigation of natural phenomena and utilization in various industrial fields. Often, collisions between particles play a role in the determination of particle behavior. For example, the precipitation process in clouds is triggered by the collision rate between settling small water droplets affected by background turbulence, thus much research has been carried out to predict the rainfall.

To identify collision between particles, the preferential concentration or clustering of inertial particles has been alternatively investigated using direct numerical simulation (DNS) for decades. The primary mechanism of the preferential concentration is the centrifuge effect of the inertial particles around rotating structures of turbulence (Reade & Collins, 2000). Moreover, a new type of preferential accumulation in columnar structures was observed (Bec *et al.*, 2014; Park & Lee, 2014). Such a clustering naturally leads to more collision between particles. Still, there is a lack of mathematical explanation and understanding of the basic mechanism of turbulent particle interaction. This is because the underlying mechanism of turbulence that causes particle movement is not fully understood due to its nonlinearity and multiscale properties. Moreover, the movement of particles, which has nonlinearity different from fluids, contributes more to the complexity. Choi *et al.* (2016) attempted to investigate directly the collision mechanism tracking colliding finite-sized particle in particle-laden isotropic turbulence. They showed that colliding pattern varies depending on Stokes number. However, an investigation of a collision event in the Lagrangian simulation of particles has limitations since it requires expansive computational cost, and luck to have a collision.

An inspiring characterization of particle collision was proposed by Falkovich *et al.* (2002), who derived a formula for the collision rate of small inertial particle in turbulence using a form that traces random trajectories (Shraiman & Siggia, 2000; Falkovich *et al.*, 2001). They introduced a particle velocity field $\vec{v}(\vec{x},t)$ instead of a particle motion equation established. The governing equation of particle velocity field is determined by relation with particle velocity gradient and particle time scale. When particle collision happens the particle velocity gradient goes to infinity, that is finite-time blowup. This singularity of particle velocity gradient implies that the particle velocity develops blowup and becomes multi-values; in other word, a particle can have two different velocities at the same position. Therefore, the eventual goal of this study is to gain a better understanding of collision process of particles using directly rigorous Eulerian approach.

METHODOLOGY

With specific parameters, it is possible to find a phenomenon in which the behavior of particles moves like a fluid called *flow of particles*. (Park & Lee, 2014; Fouxon *et al.*,

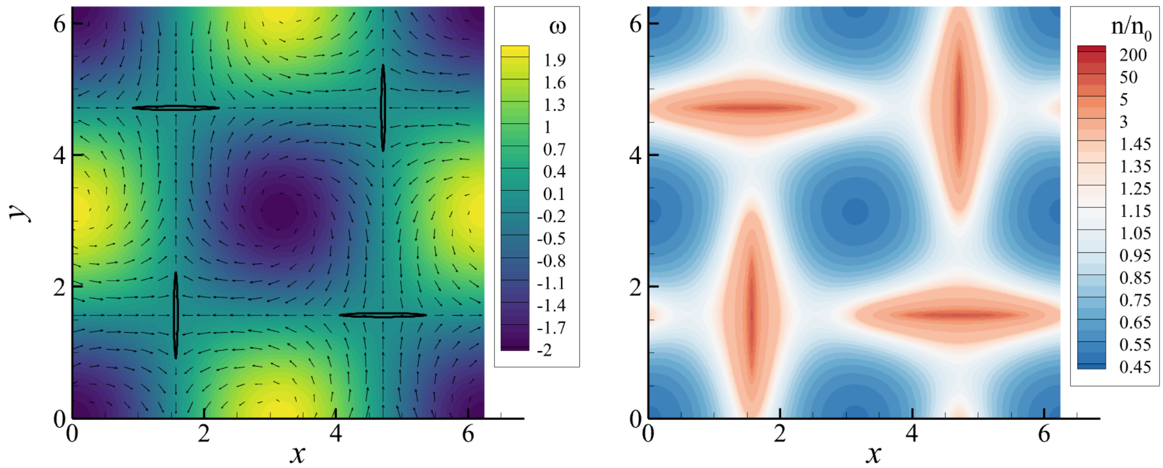

Figure 1. Plot of representing collision of particle ($St = 1, W = 0$): Colored contours represent the vorticity level of the background flow (left) and particle number density (right). The solid lines for the negative divergence level of particles are denoted.

2015). Assuming that the behavior of these particles is smooth in space, the particle velocity equation and particle number density ($n(\vec{x},t)$) can be derived in the Eulerian frame as follows:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = \frac{\vec{u}-\vec{v}}{\tau_p} + \vec{g} \tag{1}$$

$$\frac{\partial n}{\partial t} + (\vec{v}\cdot\nabla)n = -\nabla\cdot(n\vec{v}) \tag{2}$$

where $\vec{v}$ is the particle velocity, $\vec{u}$ is the fluid velocity, $\vec{g}$ is the gravity acceleration, and τ_p is the time scale of particle.

Direct numerical simulation was performed to understand the mechanism of particles suspended in turbulence. For Taylor-Green vortex, the exact solutions were used and for the other background fluids as two-dimensional decaying turbulence and three-dimensional isotropic turbulence, the spatial discretization was calculated using the Fourier spectral method, and the time advancement was carried out using the 3rd-order Runge-Kutta method. Both fluid and particle governing equations were calculated in the Eulerian framework. For the two-dimension and three-dimension computational regions, periodic boundary conditions were set for a square domain $[2\pi \times 2\pi]$ and a regular hexahedron domain $[2\pi \times 2\pi \times 2\pi]$, respectively.

The dimensionless numbers that determine the physical properties of suspended particles through dimensional analysis are the Stokes number (St) and the gravity factor (W), which are defined as follows:

$$St = \frac{\tau_p}{\tau_f}, \quad W = \frac{v_T}{v_f}$$

where τ_f and v_f depend on each background fluid. In the case of 2D flow, $\tau_f = 1/\sqrt{\omega_i \omega_i}$ and $v_f = \sqrt{v_i v_i}$ were used and in the case of 3D flow, the Kolmogorov scale was used.

In this study, we investigated the case where the Stokes number is less than or greater than 1 and the gravitational constant is 0-20 using Eulerian direct simulation. That is, the analysis was performed in the range of the case where the collision did not occur and the case where the collision occurred depending on the presence or absence of gravity.

RESULTS AND DISCUSSION

Taylor-Green vortex has an exact solution in two-dimensional space, and it is easy to intuitively understand the behavior of particles. It was confirmed in Figure 1 that when the particle number density has the maximum value, the negative value of the particle velocity gradient increases at the same time where the collision happens in the x- and y-axis direction. Moreover, since there is a high probability that the collision direction is not in the x-axis and y-axis directions in two-dimensional and three-dimensional turbulence, the minimum value of eigenvalue on symmetric part of particle velocity gradient (λ_D: D is dimension of simulation) was investigated, and it was confirmed the results for the Taylor-Green vortex. Finally, such statistics were examined for three-dimensional turbulence.

To understand the mutual mechanism of particles and fluids, the particle equation of motion at the Eulerian lattice points obtained by the new approach can be used to investigate the finite-time blowup (Beale *et al.*, 1984; Majda, 1991; Bustamante & Kerr, 2008). Therefore, the solution of Eq. (1) can be differentiated to obtain a solution that diverges within a finite time, and is as follows (Falkovich *et al.*, 2002):

$$\sigma(t) \sim \frac{1}{t - t_c} \tag{3}$$

where the blowup of $\sigma = \triangledown\vec{v}$ means the collision of two particles, and t_c is the collision time.

For particle number density, the solution of Eq. (2) for the initial uniform particle number density can be expressed as:

$$n(t) = n_0 \exp\left[-\int_0^t \triangledown\cdot\vec{v}(\vec{x}(t'),t')dt'\right] \tag{4}$$

If $\triangledown\cdot\vec{v} \sim 1/(t-t_c)$ near blowup, the solution of Eq. (4) can be derived as:

$$\frac{n(t)}{n_0} \sim \frac{t_c}{|t-t_c|} \tag{5}$$

where n_0 is the initial uniform particle number density.

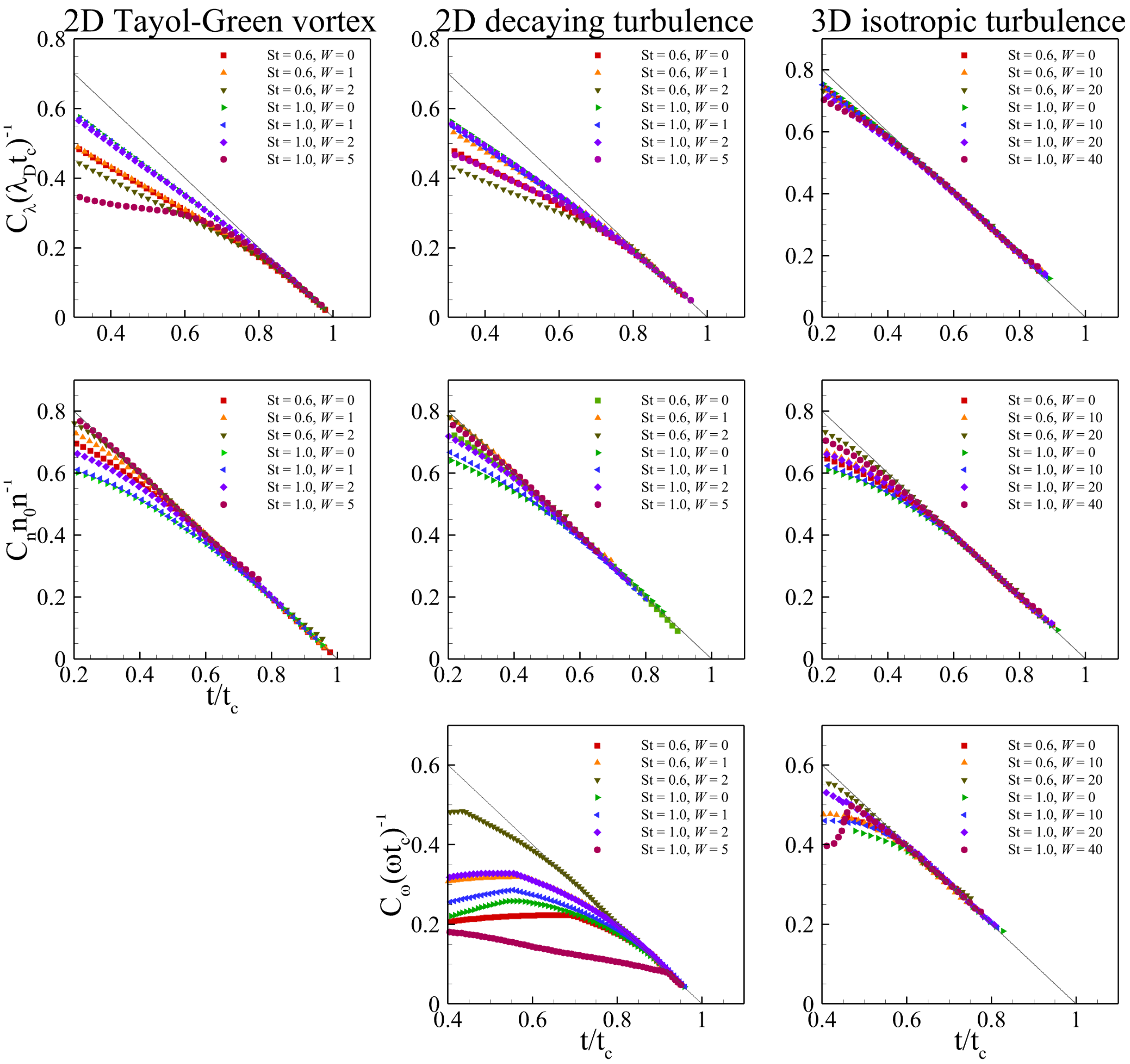

Figure 2. Time evolution of particle velocity gradient, particle number density, and particle vorticity for various Stokes numbers with gravity factors in three background fluids until the collision between particles: C_λ, C_n, C_ω are the compensation factors.

For particle vorticity ω_p in finite-time singularity, the derived equation follows as:

$$\omega_p(t) \sim \frac{1}{t - t_c} \tag{6}$$

that is equivalent to the particle velocity gradient.

The derived Eqs. (3), (5), and (6) look simple, but their solutions are determined by very complex processes. In particular, when the motion of particles is driven by turbulence, the particle velocity gradient can easily go to infinity in a finite time due to quadratic nonlinearity. A discontinuity in the particle velocity gradient means a collision between two particles physically. The major results in all background fluids depicted in Figure 2 show the process of finite-time blowup for particle velocity, particle number density, and particle vorticity. The solutions of particle velocity gradient, particle number density, and particle vorticity follow well Eqs. (3), (5), and (6), respectively. Therefore the results are collected altogether in a linear line. With increasing gravity, the particles do not collide even though the collision happens before increasing gravity. This means that *flow of particles* exists. This study has started with an ideal assumption. From this, particle vorticity ω_p is ideally derived. Thus, the finite-time blowup of particle vorticity can not be observed for 2D Taylor-Green vortex.

CONCLUSION

In order to provide a complete mathematical description of a particle collision in particle-laden turbulence, we studied Eulerian approach to the motion of particles, which was derived under the assumption that particle velocity is a smooth function of space. Finite-time blowups of particle velocity gradient, particle number density, and particle vorticity were observed as a result of the mutual collision between particles. The simulation results showed that it is consistent with the solution obtained by Eulerian approach. Therefore, the characteristics of the background fluid can be grasped in the vicinity of the collision of the three-dimensional turbulence, and this leads to an understanding of the mechanism of collision.

REFERENCES

Beale, J. T., Kato, T. & Majda, A. 1984 Remarks on the breakdown of smooth solutions for the 3-d euler equations. *Communications in Mathematical Physics* **94** (1), 61–66.

Bec, Jérémie, Homann, Holger & Ray, Samriddhi Sankar 2014 Gravity-driven enhancement of heavy particle clustering in turbulent flow. *Physical Review Letters* **112** (18).

Bustamante, Miguel D. & Kerr, Robert M. 2008 3d euler about a 2d symmetry plane. *Physica D: Nonlinear Phenomena* **237** (14-17), 1912–1920.

Choi, Jung-Il, Park, Yongnam, Kwon, Ohjoon & Lee, Changhoon 2016 Interparticle collision mechanism in turbulence. *Physical Review E* **93** (1).

Falkovich, G., Fouxon, A. & Stepanov, M. G. 2002 Acceleration of rain initiation by cloud turbulence. *Nature* **419** (6903), 151–154.

Falkovich, G., Gawdzki, K. & Vergassola, M. 2001 Particles and fields in fluid turbulence. *Reviews of Modern Physics* **73** (4), 913–975.

Fouxon, Itzhak, Park, Yongnam, Harduf, Roei & Lee, Changhoon 2015 Inhomogeneous distribution of water droplets in cloud turbulence. *Physical Review E* **92** (3).

Majda, Andrew J. 1991 Vorticity, turbulence, and acoustics in fluid flow. *SIAM Review* **33** (3), 349–388.

Park, Yongnam & Lee, Changhoon 2014 Gravity-driven clustering of inertial particles in turbulence. *Physical Review E* **89** (6).

Reade, Walter C. & Collins, Lance R. 2000 Effect of preferential concentration on turbulent collision rates. *Physics of Fluids* **12** (10), 2530.

Shraiman, Boris I. & Siggia, Eric D. 2000 Scalar turbulence. *Nature* **405** (6787), 639–646.