12th International Symposium on Turbulence and Shear Flow Phenomena (TSFP12)
Osaka, Japan, July 19–22, 2022

LAW OF BOUNDED DISSIPATION AND ITS CONSEQUENCES IN TURBULENT WALL FLOWS

Xi Chen
Key Laboratory of Fluid Mechanics of Ministry of Education
Beihang University (Beijing University of Aeronautics and Astronautics)
Beijing, China
chenxi97@outlook.com

Katepalli R. Sreenivasan
Tandon School of Engineering, Courant Institute of Mathematical Sciences, Department of Physics
New York University
New York, USA
krs3@nyu.edu

ABSTRACT

The dominant paradigm in turbulent wall flows is that the mean velocity near the wall, when scaled on wall variables, is independent of the friction Reynolds number Re_τ. This paradigm faces challenges when applied to fluctuations but has received serious attention only recently. Here, we present a promising perspective, and support it with data, that fluctuations displaying non-zero wall-values, or near-wall peaks, are bounded for large values of Re_τ, owing to the natural constraint that the dissipation rate is bounded. Specifically, $\Phi_\infty - \Phi = C_\Phi Re_\tau^{-1/4}$, where Φ represents the maximum value of any of the following quantities: energy dissipation rate, turbulent diffusion, fluctuations of pressure, streamwise and spanwise velocities, squares of vorticity components, and the wall values of pressure and shear stresses; the subscript ∞ denotes the bounded asymptotic value of Φ and the coefficient C_Φ depends on Φ but not on Re_τ. Moreover, there exists a scaling law for the maximum value in the wall-normal direction of high-order moments, of the form $\langle \varphi^{2q} \rangle_{max}^{1/q} = \alpha_q - \beta_q Re_\tau^{-1/4}$, where φ represents the streamwise or spanwise velocity fluctuation and α_q and β_q are independent on Re_τ. Excellent agreement with available data is observed. A stochastic process for which the random variable has the form just mentioned, referred to here as the 'linear q-norm Gaussian', is proposed to explain the observed linear dependence of α_q on q.

Introduction

Ever since a theory for turbulent shear flows began to develop, the dominant paradigm has been that the flow near the wall scales solely on ν and the wall shear stress, τ_w. This theme has been remarkably successful for the mean velocity, as evidenced by the law of the wall. Similar expectations for turbulent intensities are assumed in engineering models. In practice, this means that turbulence fluctuations, after suitable normalization by the wall stress and viscosity, would be invariant with respect to the friction Reynolds number $Re_\tau = u_\tau \delta / \nu$, where $u_\tau \equiv \tau_w^{1/2}$ is the friction velocity and δ is the flow thickness. Nevertheless, as found in direct numerical simulations (DNS) and in laboratory experiments (EXP), wall-normalized fluctuating quantities increase with Re_τ.

These include (almost) all quantities with wall-values that are non-zero or display near-wall peaks—in particular, in the wall-components of energy dissipation (ε^+_{x-w} and ε^+_{z-w}), diffusion ($\mathscr{D}^+_{x-w}$ and $\mathscr{D}^+_{z-w}$), root-mean-square (rms) vorticity (ω'^+_{x-w} and ω'^+_{z-w}), and rms wall shear stress (τ'^+_{x-w} and τ'^+_{z-w}) and pressure (p'^+_w), absorbing the fluid density in the definition of pressure; the list also includes the near-wall intensity peaks of velocities (u'^+_p occurring at $y^+ \approx 15$ and w'^+_p occurring at $y^+ \approx 45$) and pressure fluctuation (p'^+_p occurring at $y^+ \approx 30$). We adopt the standard convention that the superscript $+$ indicates normalization by u_τ and ν, and the superscript prime represents the rms fluctuation; subscript w represents the wall and p stands for the peak value near the wall, and u, v, w for fluctuation velocities in the streamwise (x), wall-normal (y), and spanwise/azimuthal (z) directions; where two letters are used as subscripts, they indicate wall (w) values and the direction x, y or z.

Law of Bounded dissipation and the $Re_\tau^{-1/4}$ defect power law

We present a broad explanation for the growth of quantities just mentioned, on the basis of the law for bounded wall-dissipation advanced by Chen & Sreenivasan (2021), hereafter as CS. We recall that the latter is expressed as

$$\varepsilon^+_{x-\infty} - \varepsilon^+_{x-w} = C_\varepsilon Re_\tau^{-1/4} \tag{1}$$

where $\varepsilon_x = \nu \langle |\nabla u|^2 \rangle$ is the streamwise wall-dissipation ($\langle \cdot \rangle$ denotes average), C_ε is a constant independent of the Reynolds number and $\varepsilon_{x-\infty}$ is the asymptote as $Re_\tau \to \infty$ of ε_{x-w}, which is the wall value of ε_x. After normalization using u_τ^4/ν, $\varepsilon^+_{x-\infty}$ is thought to be bounded by 1/4, which is the constraint imposed by the exact maximum production. CS verified this scaling by comparisons with available data, and also provided the following physical rationale for (1). What controls the turbulence peak values at any Reynolds number is the peak energy dissipation, which equals the maximum production only at infinitely large Reynolds number; and at any finite Reynolds number, it is the departure of the dissipation rate from its limiting value that determines the finite Reynolds number dependence. Specifically, the peak dissipation falls short of the peak energy production of 1/4 at finite Reynolds number by transmitting outwards in the amount $\varepsilon_d = u_\tau^3/\eta_0$ where η_0 the outer flow Kolmogorov length scale, and hence $\varepsilon_d^+ = \varepsilon_d/(u_\tau^4/\nu) \sim Re_\tau^{-1/4}$ leading to equation (1). For more details of the argument, one may consult CS.

A natural generalization of the above result is

$$\Phi_\infty - \Phi = C_\Phi Re_\tau^{-1/4}, \tag{2}$$

where Φ is any of the quantities ε_{x-w}^+, ε_{z-w}^+, $\mathscr{D}_{x-w}^+$, $\mathscr{D}_{z-w}^+$, $\omega_{z-w}^{\prime+2}$, $\omega_{x-w}^{\prime+2}$, $\tau_{x-w}^{\prime+2}$, $\tau_{z-w}^{\prime+2}$, $p_w^{\prime+}$, $p_p^{\prime+}$, $u_p^{\prime+2}$ and $w_p^{\prime+2}$, the subscript ∞ denotes their bounded asymptotic values and the coefficient C_Φ depends on the quantity Φ in question but not on the Reynolds number. We also note that $\varepsilon_{x-w}^+ = \tau_{x-w}^{\prime+2} = \omega_{z-w}^{\prime+2} = \langle(\partial u^+/\partial y^+)^2\rangle_w$ and $\varepsilon_{z-w}^+ = \tau_{z-w}^{\prime+2} = \omega_{x-w}^{\prime+2} = \langle(\partial w^+/\partial y^+)^2\rangle_w$ due to the no-slip wall condition (i.e. $\nabla u = \partial_y u$ and $\nabla w = \partial_y w$ at the wall), while $\mathscr{D}_{x-w}^+ = \varepsilon_{x-w}^+$ and $\mathscr{D}_{z-w}^+ = \varepsilon_{z-w}^+$ because of the Reynolds stress balances at the wall. Moreover, we argue the maximum values of moments would also be bounded as $Re_\tau \to \infty$, and that the finite-Re_τ dependence is the same 1/4-power. Accordingly, we write

$$\langle \varphi^{2q} \rangle_{max}^{1/q} = \alpha_q - \beta_q Re_\tau^{-1/4} \tag{3}$$

where φ represents either u or w fluctuations, and α_q (for $q = 1-5$) represents different asymptotes for different q' when $Re_\tau \to \infty$; β_q is independent of Re_τ. Note that for $q = 1$, $\alpha_1 = \Phi_\infty$ and $\beta_1 = C_\Phi$ in (2).

Figure 1 shows Re_τ-variations for the wall values of dissipations ε_{x-w}^+ and ε_{z-w}^+ (top panels) and near-wall peaks of $\langle u^{2q}\rangle^{+1/q}$ and $\langle w^{2q}\rangle^{+1/q}$ for $q = 1-5$ (bottom panels). Solid lines denote the $Re_\tau^{-1/4}$ defect power law fittings by (2) for dissipations and (3) for moments, where values of α_q, shown in the figure 2, are of particular interest because they represents the asymptotic values of the moments.

Linear q-norm Gaussian (LQNG) process

In this section, we show that the linear q-dependence in figure 2 can result from a Gaussian random variable via an exponential transformation. Let us first define the q-norm for a (random) variable ϕ as $\phi_q = \langle \phi^q \rangle^{1/q}$, where $\langle \cdot \rangle$ represents the expectation value. If ϕ_q depends linearly on q, it then satisfies

$$\phi_q = \ln(\chi_q), \tag{4}$$

where χ_q is the q-norm of a log-normal variable χ, i.e. $\chi = e^\kappa$ with κ Gaussian distributed. This is demonstrated as follows.

For a Gaussian variable κ with its mean μ and variance 2σ, one has

$$\ln(\langle e^{q\kappa} \rangle) = \mu q + \sigma q^2. \tag{5}$$

Accordingly,

$$\chi_q = \langle \chi^q \rangle^{1/q} = \langle e^{q\kappa} \rangle^{1/q} = [e^{\mu q + \sigma q^2}]^{1/q} = e^{\mu + \sigma q}; \tag{6}$$

$$\phi_q = \ln(\chi_q) = \mu + \sigma q. \tag{7}$$

We may refer the random variable ϕ which has a linear q-norm (7) as the 'linear q-norm Gaussian' (LQNG) process generated by the Gaussian seed κ. The above procedure is summarized as follows:

$$\kappa \xrightarrow{\mathbf{E}} \chi \xrightarrow{\mathbf{Q}} \chi_q \xrightarrow{\mathbf{E}^{-1}} \phi_q \xrightarrow{\mathbf{Q}^{-1}} \phi; \qquad \phi = \mathbf{Q}^{-1}\mathbf{E}^{-1}\mathbf{Q}\mathbf{E}(\kappa). \tag{8}$$

Here, $\mathbf{E}$ and $\mathbf{Q}$ indicate operations of exponential transform and q-norm, respectively, which are non-commutable for random variables; and the superscript -1 indicates the inverse operation (supposing ϕ is determined by its moments). In other words, the LQNG process satisfies the following operator-reflection symmetry

$$\mathbf{E} \circ \mathbf{Q}(\phi) = \mathbf{Q} \circ \mathbf{E}(\kappa). \tag{9}$$

If ϕ and κ are non-random, it is trivial that $\phi = \kappa$; instead, ϕ and κ here are random variables, and by assigning κ as a Gaussian variable, we obtain a linear dependence of ϕ_q on q,

For wall turbulence, the asymptotes for the near-wall peaks of $\langle u^{2q}\rangle^{1/q}$ and $\langle w^{2q}\rangle^{1/q}$ when $Re_\tau \to \infty$ are LQNG processes. That is, substituting $\phi = u^2$ in (7) we have

$$\alpha_{u,q} = \mu_u + \sigma_u q \tag{10}$$

where $\mu_u \approx 5.5$ and $\sigma_u \approx 5.9$ according to figure 2(left). Similarly, substituting $\phi = w^2$ in (7) we have

$$\alpha_{w,q} = \mu_w + \sigma_w q \tag{11}$$

where $\mu_w \approx 0$ and $\sigma_w \approx 3.9$ according to figure 2(right). The fact that $\mu_w \approx 0$ may reflect the absence of inactive motion in the w-component of the velocity.

Conclusion

The paper shows that the averages of turbulent fluctuations which possess non-zero wall values or near-wall peaks are bounded and follow a universal $Re_\tau^{-1/4}$ defect law. The paper also extends the same argument to wall-normal peaks in high-order (even) moments of velocity fluctuations.

REFERENCES

Chen, X. & Sreenivasan, K.R. 2021 Reynolds number scaling of the peak turbulence intensity in wall flows. *J. Fluid Mech.* **908**, R3.

Hutchins, N., Nickels, T. B., Marusic, I. & Chong, M. S. 2009 Hot-wire spatial resolution issues in wall-bounded turbulence. *J. Fluid Mech.* **635**, 103–136.

Lee, M. & Moser, R.D. 2015 Direct numerical simulation of turbulent channel flow up to Re_τ = 5200. *J. Fluid Mech.* **774**, 395–415.

Meneveau, C. & Marusic, Ivan 2013 Generalized logarithmic law for high-order moments in turbulent boundary layers. *J. Fluid Mech.* **719**, R1.

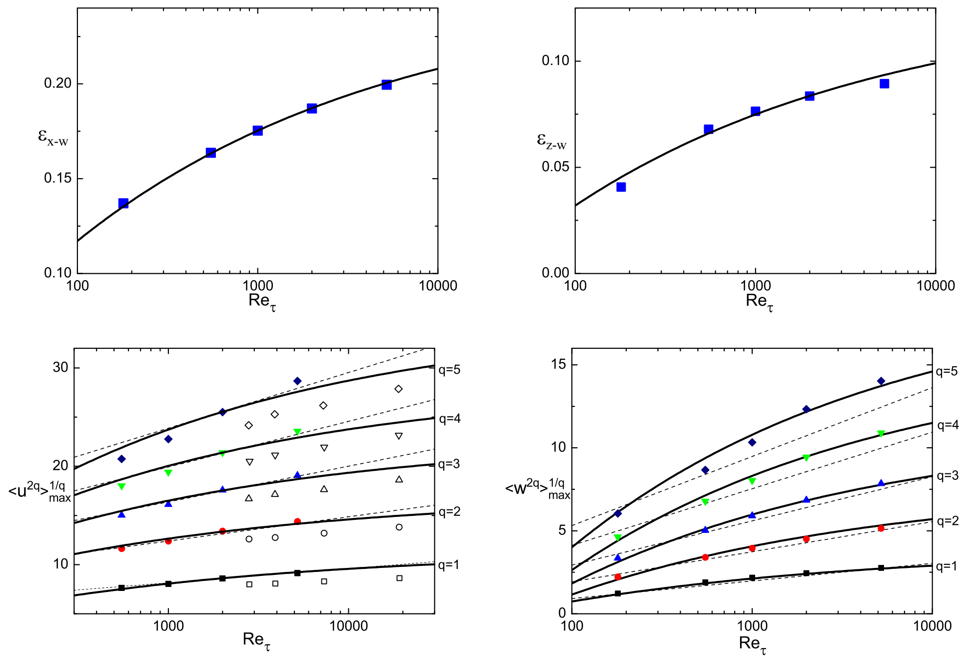

Figure 1. (top) Re_τ-variations of wall dissipation rates after normalization in viscous units: streamwise velocity component ε_{x-w} - left, and spanwsie velocity component ε_{z-w} - right. (bottom) Re_τ variations of the maximum u (left) and w (right) moments near the wall. Solid symbols represent the DNS channel data of Lee & Moser (2015); open symbols are EXP TBL data by Hutchins *et al.* (2009), extracted from Meneveau & Marusic (2013). Dashed lines from the Gaussian logarithmic model by Meneveau & Marusic (2013), while solid lines denote the $Re_\tau^{-1/4}$ defect power law fittings by (2) for dissipations, i.e. $\varepsilon_{x-w} = 1/4 - 0.42 Re_\tau^{-1/4}$ and $\varepsilon_{z-w} = 0.13 - 0.31 Re_\tau^{-1/4}$, and (3) for moments, with α_q shown in the figure 2.

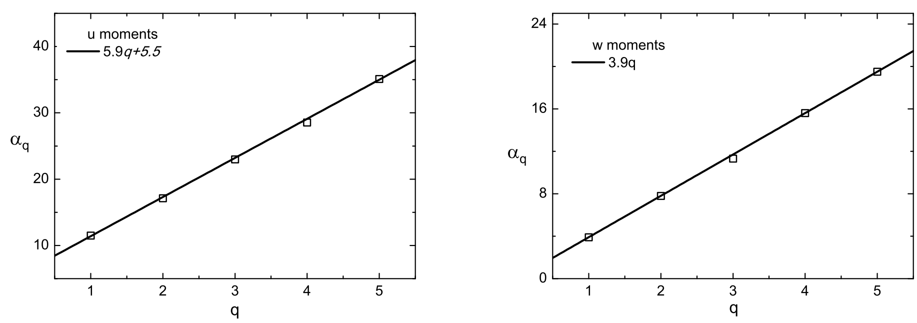

Figure 2. Variations with q of α_q for the moments of u (left) and w (right). Solid lines indicate linear dependence, explained in (10) and (11), given by the LQNG process defined in (8).