UNIVERSALITY OF PROBABILITY DENSITY FUNCTION IN HIGH REYNOLDS-NUMBER TURBULENT BOUNDARY LAYER

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ABSTRACT
The probability density function (pdf) of a streamwise velocity component is studied in zero-pressure gradient boundary layers. From analyzing the data up to$R_\theta \approx 80000$, it is found that pdfs have self-similar profiles in the log-law region of mean velocity. In the outer part where the turbulent intensity shows another log-law, the stream-wise velocity pdf shows the self-similar property.

Probability density function profiles asymptote to the universal shape close to the Gaussian, but are positively skewed at the core region, indicating smaller values in the tail parts. In the log-law region of turbulence intensity, however the pdf is negatively skewed. These characteristics are summarized depending on the Reynolds number.

INTRODUCTION
It is clear from the Reynolds averaged equation that the mean velocity is strongly related with turbulence intensity. Thus, the mean velocity profile can be potentially discussed from the view point of turbulence intensity distribution. This idea was originally confirmed by Tsuji et al. (1999, 2005) in the case of low Reynolds numbers subject to the invariance of pdfs in the overlap region. A logarithmic velocity profile was derived from the pdf equation with adopting two empirical relations. In this paper the idea is reexamined in the case of higher Reynolds numbers. It has been experimentally observed that the mean velocity in the lower part of the log-region tends to over chute, that is, the velocities here are slightly higher than if they would follow the log-law.

In the outer region, there is another log-law of turbulent intensity profile. It is predicted by the attached eddy model by Townsend (1956). This relation is only confirmed in higher Re number shear flows. We found that the pdf of streamwise velocity indicates the self-similar profile in the outer log-region. Following these results, the universality of pdfs in mean velocity and turbulent intensity is discussed in relation with log-law region.

EXPERIMENTAL CONDITION
The zero-pressure gradient turbulent boundary layers in higher Reynolds numbers are studied through an experiment at the Railway Technical Research Institute (RTRI) low-noise wind tunnel in Maibara Japan. The wind tunnel was constructed to promote research and development on aerodynamic noise and other subjects related to Shinkansen and other high-speed railways (see Fig.1). The closed test section, 5m width × 3m height × 20m length, is composed of a windward wind tunnel (length 6.5m) and a leeward wind tunnel (length 13.5m), both separately movable, and used to test and improve the characteristics of the aerodynamic force and drag work on train models and study the flow around them.

The flat plate is placed inside the test section and the fluctuating stream-wise velocity component is measured by a constant temperature anemometry with I-type proce at the position 6.0, 14.4 and 18.7m from the leading edge (Fig.2). The measurements are performed in high Reynolds number, $15000 \leq R_\theta \leq 80000$. The mean skin friction is determined using oil-film interferometry. As the detailed experimental conditions are mentioned in Tsuji et al. (2021).

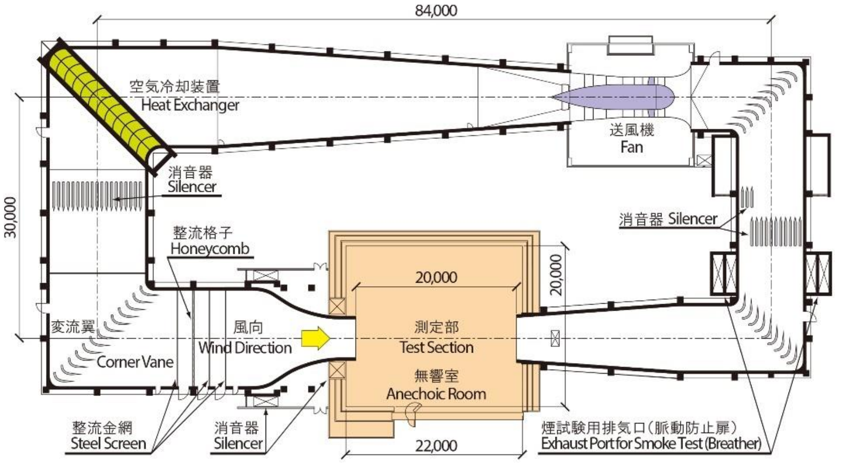

Fig.1 Plain view of wind tunnel facility at RTRI wind tunnel technical center (http://www.rtri.or.jp/rd/maibara-wt/INDEX.HTML).

Table 1 Statistical properties of boundary layers at (a) $x = 6.0$ m, (b) $x = 14.4$ m, (c) $x = 18.7$ m.

(a)

δ *[mm]	θ [mm]	u_τ [m/s]	U_0[m/s]	δ_{99}[mm]	Δ [mm]	R_θ	procedure
1.61E+01	1.22E+01	6.85E-01	1.99E+01	1.11E+02	4.68E+02	1.578E+04	⑯
1.54E+01	1.19E+01	9.81E-01	2.93E+01	1.09E+02	4.61E+02	2.258E+04	⑯
1.44E+01	1.12E+01	1.29E+00	3.87E+01	1.09E+02	4.34E+02	2.787E+04	⑯
1.39E+01	1.09E+01	1.56E+00	4.79E+01	1.04E+02	4.27E+02	3.346E+04	⑯

(b)

δ *[mm]	θ [mm]	u_τ [m/s]	U_0[m/s]	δ_{99}[mm]	Δ [mm]	R_θ	procedure
2.87E+01	2.22E+01	6.60E-01	2.01E+01	2.06E+02	8.72E+02	2.850E+04	⑭
2.76E+01	2.16E+01	9.63E-01	3.02E+01	2.07E+02	8.66E+02	4.192E+04	⑭
2.66E+01	2.09E+01	1.25E+00	3.98E+01	1.98E+02	8.50E+02	5.253E+04	⑭
2.64E+01	2.08E+01	1.51E+00	4.91E+01	1.98E+02	8.61E+02	6.372E+04	⑭

(c)

δ _*[mm]	θ [mm]	u_τ [m/s]	U_0[m/s]	δ_{99}[mm]	Δ [mm]	R_θ	procedure
3.06E+01	2.39E+01	6.61E-01	2.02E+01	2.29E+02	9.36E+02	3.192E+04	③
3.05E+01	2.40E+01	9.56E-01	3.02E+01	2.31E+02	9.65E+02	4.697E+04	⑭
2.94E+01	2.32E+01	1.23E+00	4.00E+01	2.24E+02	9.58E+02	6.127E+04	③
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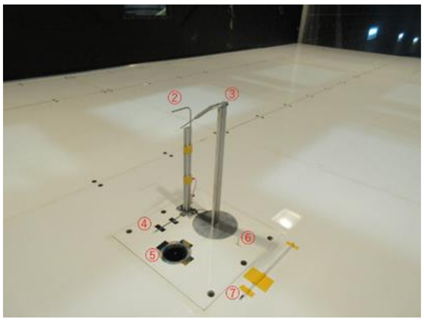

Fig.2 Test section for probe setting at calibration. ②Pitot tube, ③ hot-wire, ④ ⑤ thermistor, ⑥ thermocouple, ⑦ thermometer.

RESULTS AND DISCUSSIONS

The mean veclcity profiles are plotted in Fig.3. We can observe the clear log-law profile

$$U^+ = \frac{1}{\kappa}\ln(y^+) + B \tag{1}$$

raiging from $y^+ \cong 180$ to $0.15\delta^+$. Upper subscript + denotes the normalization by inner variables. Kármán constant κ and additive constant B are evaluated as $\kappa = 0.385 \pm 0.003$, $B = 4.26 \pm 0.15$, respectively. The skewness (third order moments) of stream-wise velocity fluctuation is plotted in Fig.4. The skewness decreases toward the outer edge of boundary layer. But it shows the constant value as marked (1) and (2) in Fig.4. Therefore the pdf becomes invariant in these regions. They are called inner and outer pdf invariant regions, for convenience. Probability distributions are plotted in Fig.5. They are positively skewed in (1) but negatively skewed in (2). It is confirmed that pdf profiles are self-similar in each region.

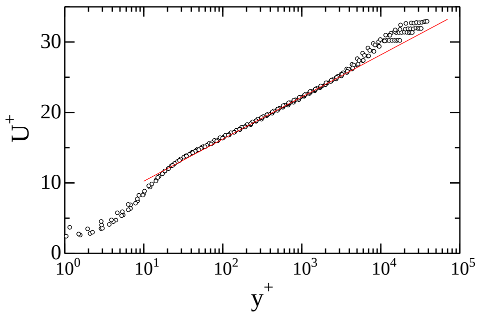

Fig.3 Mean velocity profiles measured at x=14.4m with $U_0 = 20.1, 30.2, 39.8, 49.1$ m/sec.

Compared with the mean velocity profile, the log-law region consists of two invariant pdf regions. But the second invariant pdf region was not observed in low Reynolds numbers.

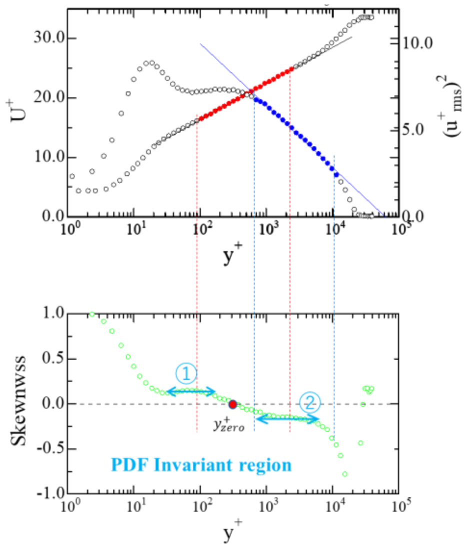

Fig.4 Mean velocity and turbulent intensity distribution (upper) and the skewness distribution at $R_\theta = 814200$. Invariant pdf regions are indicated by (1) and (2) in lower graph.

Turbulent intensity profiles are plotted in Fig.4. Townsend's original idea (Townsend, 1956) was extended by Perry et al. (1986) , who reported that the distribution of eddies with a population density was inversely proportional to the distance from the wall, and consequently the intensity of the stream-wise velocity component $(u^+_{rms})^2$ is in proportion to the logarithmic relation of the distance from the wall. This relation is expressed as

$$(u^+_{rms})^2 = A_u \ln\,(y/\delta_{99}) + B_u \tag{2}$$

Where δ_{99} is the boundary layer thickness where the local mean velocity is equal to 99% of U_0. The coefficients A_u and B_u are evaluated as $A_u = -1.38 \pm 0.05$, $B_u = 1.8 \pm 0.1$, respectively. The second invariant pdf region is closely related to the log-law region of Eq.(2). The pdf profiles are also seen in Fig.5, and they are negatively skewed around core region and in tail parts. The skewness is positive near the wall but decreases toward the outer region. There is a zero-crossing at y_{zero} or the skewness is zero at this position. We carefully study the pdf shape around y_{zero}. Here, one of the measures characterizing pdf profiles is introduced. It is the Kullback-Leibler divergence (KLD or KL-divergence) or relative entropy. This measure, which was introduced by Kullback (1959) and adopted as a basic quantity in the information theory, is defined as follows:

$$D(P||Q) \equiv \sum_{\{s\}} P(s_i)\ln\{P(s_i)/Q(s_i)\}$$

where $P(\boldsymbol{s})$ and $Q(\boldsymbol{s})$, $\{\boldsymbol{s}\} = \{s_1, s_2, s_3 \cdots\}$ are discrete probability distributions. KLD has a non-negative value for any $P(\boldsymbol{s})$ and $Q(\boldsymbol{s})$, and it is zero only when $P(\boldsymbol{s})$ is exactly equal to $Q(\boldsymbol{s})$. The more the $P(\boldsymbol{s})$ and $Q(\boldsymbol{s})$ come to resemble each other, the smaller is the KLD. Thus KLD indicates quantitatively the resemblance between $P(\boldsymbol{s})$ and $Q(\boldsymbol{s})$. A simple numerical example was presented earlier in Tsuji and Nakamura (1999). In this analysis, $Q(\boldsymbol{s})$ is taken to be the probability at zero-crossing and $Q(\boldsymbol{s})$ is set to the pdf that is measured at distance y from the wall.

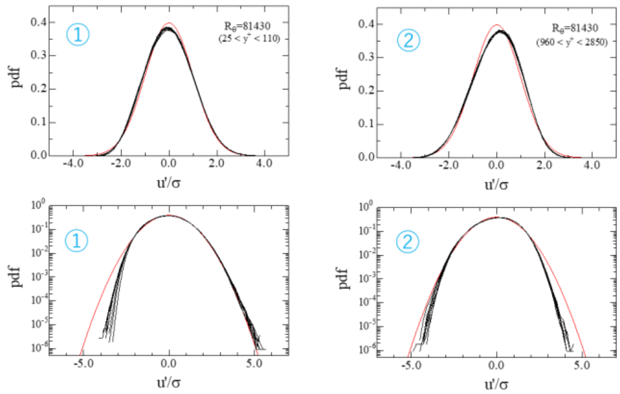

Fig.5 Probability density function in two invariant pdf regions.
① Inner invariant region, ② Outer invariant region.

Figure 6 shows the KL-divergence distribution around zero crossing point at y_{zero} for different Reynolds numbers. The KL-divergence profiles are overlap sufficiently, therefore the pdf is universal independent of Reynolds numbers. In low Reynolds numbers, there is no zero-crossing. Skewness has positive values in the overlap region.

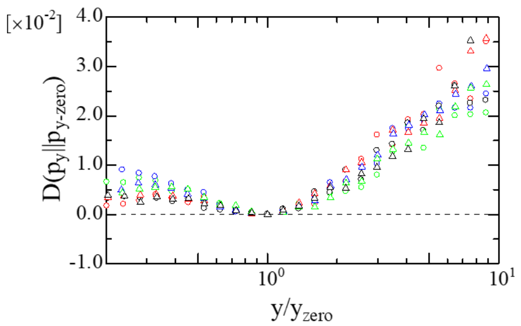

Fig.5 Probability density function in two invariant pdf regions. Different colours indicate different Reynolds numbers.

In the present high Re number data, the probe resolution effect on the PDF can not be avoided in the inner invariant PDF region. However the invariance of PDF in this region was discussed in Tsuji et al.(2005) keeping the sufficient resolution of hot wire.

SUMMARY

High Reynolds number data measured at RIRI wind tunnel are analyzed. The probability density functions of stream-wise velocity fluctuation become invariant in two different parts across the turbulent boundary layers. Mean velocity log-law as described by Eq. (1) consists of these two invariant pdf regions. The turbulent intensity log-law following Eq.(2) is consistent with the outer invariant pdf region. This region is observed only in very high Reynolds numbers. At the zero-crossing point y_{zero}, where the skewness is zero, the pdfs are universal independent of Reynolds number. Zero-crossing point locates around the center of mean velocity log-law region.

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