DIRECT NUMERICAL SIMULATIONS OF TURBULENT PLANAR JETS AND WAKES WITH FENE-P FLUIDS

Mateus C. Guimarães
Departamento de Engenharia Mecânica
IDMEC/LAETA, Instituto Superior Técnico,
Universidade de Lisboa
Avenida Rovisco Pais,
1049-001 Lisbon, Portugal
mateus.carvalho@tecnico.ulisboa.pt

Fernando T. Pinho
Departamento de Engenharia Mecânica
CEFT, ALiCE, Faculdade de Engenharia,
Universidade do Porto
Rua Dr. Roberto Frias,
4200-465 Porto, Portugal
fpinho@fe.up.pt

Carlos B. da Silva
Departamento de Engenharia Mecânica
IDMEC/LAETA, Instituto Superior Técnico,
Universidade de Lisboa
Avenida Rovisco Pais,
1049-001 Lisbon, Portugal
carlos.silva@ist.utl.pt

ABSTRACT

Direct numerical simulations of spatially evolving turbulent planar jets and wakes with viscoelastic fluids are performed in order to study the influence of viscoelasticity on the flow dynamics and to develop a theory for free turbulent shear flows with viscoelastic fluids. The DNS display excellent agreement with the new theoretical results.

INTRODUCTION

It is well-known that the addition of minute amounts of polymers into a Newtonian solvent can lead to drastic changes in the turbulent flow dynamics resulting in *e.g.* massive drag reduction for pipe and channel flows, and the establishment of an elastic range of the energy spectrum for isotropic turbulence. Different theories have been proposed to explain the observed behaviour in wall turbulence or isotropic turbulence. However, there is currently no theory for the description of free turbulent shear flows such as jets and wakes with viscoelastic fluids. The current work presents results from several direct numerical simulations (DNS) of spatially evolving turbulent planar jets and wakes with incompressible viscoelastic polymer solutions in a Newtonian solvent, in order to develop a theory for free turbulent shear flows with viscoelastic fluids.

DIRECT NUMERICAL SIMULATIONS

The rheology of the solutions is characterised by the Finitely Extensible Non-Linear Elastic constitutive model closed with Peterlin's approximation (FENE-P). The momentum equation is given by

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho}\nabla p + \nu^{[s]}\nabla^2 u + \frac{1}{\rho}\nabla \cdot \sigma^{[p]}, \qquad (1)$$

and the continuity equation is

$$\nabla \cdot u = 0. \qquad (2)$$

The polymer stress tensor is given by

$$\sigma^{[p]} = \frac{\rho \nu^{[p]}}{\tau_p}[f(C_{kk})C - I], \qquad (3)$$

and the evolution equation for the conformation tensor C is

$$\frac{\partial C}{\partial t} + u \cdot \nabla C = \nabla u^T \cdot C + C \cdot \nabla u - \frac{1}{\tau_p}[f(C_{kk})C - I]. \qquad (4)$$

In the above, u is the velocity vector and p is the pressure, while polymer and solvent zero-shear kinematic viscosities are $\nu^{[p]}$ and $\nu^{[s]}$, respectively. τ_p is the polymer relaxation time and ρ is the density of the fluid. The Peterlin function used in the present work is given by $f(C_{kk}) \equiv (L^2-3)/(L^2-C_{kk})$, where L is the maximum length of the polymers normalized by their equilibrium radius. The ratio of zero shear rate viscosities is $\beta = \nu^{[s]}/(\nu^{[s]}+\nu^{[p]})$. The polymer concentration in the solution is proportional to $1-\beta$.

The simulations were carried out using a highly accurate code that employs a combination of pseudo-spectral and 6th order 'Compact' finite differences schemes, for the momentum equations, and the Kurganov-Tadmor scheme for the equations governing the conformation of the polymer chains (Kurganov & Tadmor, 2000; Vaithianathan *et al.*, 2006). An explicit 3rd-order Runge-Kutta scheme is used for time-stepping and the pressure-velocity coupling is treated with a fractional step method. Inflow and outflow boundary conditions are used at the boundaries facing the streamwise direction. More details about the numerical methods and code validation can be found in Guimarães *et al.* (2020, 2022) and references therein.

In the present DNS the Reynolds numbers are $Re_h = U_J h/\nu^{[s]} = 3500$ for jets and $Re_d = U_\infty d/\nu^{[s]} = 14000$ for wakes, where h is the inlet slot-width of the jet and d is the transverse length scale of the wake generator object, U_J is the inlet jet velocity and U_∞ is the wake free-stream velocity. The Weissenberg number ($Wi = \tau_p \Delta U_0/d$ for wakes and $Wi = \tau_p U_J/h$ for jets) and polymer concentration parameter β are varied in order to observe the influence of the fluid rheology in the dynamics of the resulting viscoelastic turbulent flow. The number of grid points of the (uniform) computational mesh in each direction is given by n_x, n_y and n_z, and domain sizes are L_x, L_y and L_z. For the turbulent planar wake simulations we have $n_x = 4032$, $n_y = 1152$ and $n_z = 288$ with $L_x/d = 84$, $L_y/d = 24$ and $L_z/d = 6$. For the turbulent planar jet DNS we have $n_x = 1152$, $n_y = 1152$ and $n_z = 144$ with $L_x/d = 40$, $L_y/d = 40$ and $L_z/d = 5$.

The DNS show that the polymers in the solution lead to drastic changes in the evolution of several flow statistics such as the spreading rates and the rates of decay of both the mean velocity and passive scalar concentration, Reynolds stresses and viscous dissipation rate (Guimarães *et al.*, 2020, 2022). Two-dimensional contours of the instantaneous vorticity modulus and the passive scalar concentration are shown in figures 1 and 2, respectively, to illustrate the simulations of jets and wakes. When the inlet Weissenberg number is large, the mean flow structure at the fully turbulent far field can be split onto two different regions; a first region where a large reduction of the spreading and decay rate and a suppression of the normalized turbulent velocity fluctuations is observed, and a subsequent region where the local Deborah number is low and viscoelastic effects are milder (Guimarães *et al.*, 2022).

THEORY

A new theory describing the far-field fully-developed region of turbulent viscoelastic jets and wakes is presented, based on a scaling analysis of the equations governing the mean flow and polymer conformation fields, and results in several scaling laws for the shear layer width, centreline velocity/velocity-deficit, polymer stresses and polymer chain conformation. Some results have already been published at Guimarães *et al.* (2020), where it is shown that Newtonian and viscoelastic shear layers follow the same scaling laws for the centreline velocity decay and shear layer width, but with different values of the scaling law coefficients. The new theory is also able to predict the scales that yield self-similarity of the profiles of polymer stress and polymer chain elongation and the condition for similarity of the profiles of Reynolds shear stress. Additionally, the scaling laws for the streamwise evolution of the maximum polymer shear stress $\sigma_c(x)$ at the initial highly elastic region of the far field are derived. For jets, the result is $\sigma_c(x) \sim x^{-5/2}$ while for wakes we have $\sigma_c(x) \sim x^{-2}$. Comparisons between the analytical predictions resulting from the new theory and the DNS results display excellent agreement (Guimarães *et al.*, 2020, 2022). The new theory is thus an extension of the classical theory of free turbulent shear flows of Newtonian fluids (Tennekes & Lumley, 1972; Pope, 2000) to the more complex case involving non-Newtonian viscoelastic fluids.

Conclusions

Several direct numerical simulations (DNS) of spatially evolving turbulent planar jets and wakes with viscoelastic FENE-P fluid have been performed. Large viscoelastic effects on the flow dynamics and evolution have been observed when the Weissenberg number is high. An analytical theory for the far field fully turbulent region of the flow has been developed using similarity analysis and scaling arguments; excellent agreement is obtained between the results of the DNS and the analytical predictions. We expect that the ideas used in our theory of viscoelastic turbulent planar jets and wakes will also be useful in the description of other spatially evolving canonical turbulent flows such as free shear layers in the axisymmetric (round) configuration, isotropic turbulence decaying behind a grid of bars and the outer region of a flat plate boundary layer.

ACKNOWLEDGEMENTS

M.C.G acknowledges Fundação para a Ciência e Tecnologia (FCT) through contract UI/BD/151060/2021. C.B.S. acknowledges Fundação para a Ciência e Tecnologia (FCT) through IDMEC, under LAETA, project UIDB/50022/2020. F.T.P. is thankful for funding by FCT and ALiCE through project LA/P/0045/2020 and FCT and CEFT through projects UIDP/00532/2020.

REFERENCES

Guimarães, M. C., Pimentel, N., Pinho, F. T. & da Silva, C. B. 2020 Direct numerical simulations of turbulent viscoelastic jets. *Journal of Fluid Mechanics* **899**.

Guimarães, M. C., Pinho, F. T. & da Silva, C. B. 2022 Turbulent planar wakes of viscoelastic fluids analysed by direct numerical simulations. *Journal of Fluid Mechanics, submitted* .

Kurganov, A. & Tadmor, E. 2000 New high-resolution central schemes for nonlinear conservation laws and convection-diffusion equations. *J. Comp. Phys.* **160** (1), 241–282.

Pope, S. B. 2000 *Turbulent Flows*. Cambridge University Press.

Tennekes, H. & Lumley, J. L. 1972 *A first course in turbulence*. The MIT Press.

Vaithianathan, T., Robert, A., Brasseur, J. G. & Collins, L. R. 2006 An improved algorithm for simulating three-dimensional, viscoelastic turbulence. *J. Non-Newt. Fluid Mech.* **140** (1), 3–22.

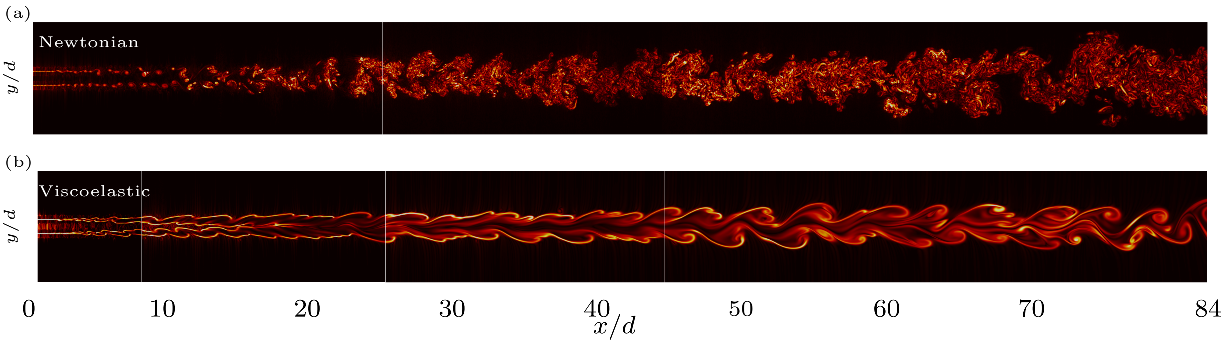

Figure 1. Vorticity magnitude contours for (a) Newtonian wake and (b) viscoelastic wake with $Wi = 4.2$, $L = 100$ and $\beta = 0.8$.

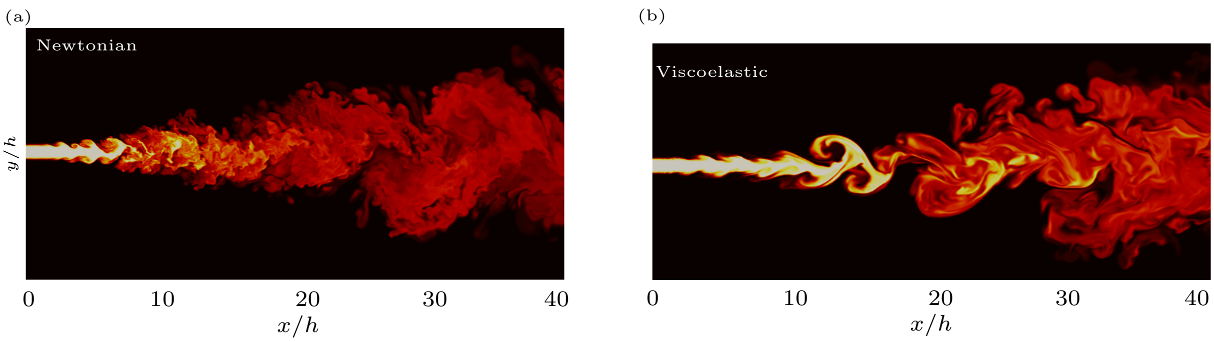

Figure 2. Passive scalar contours for (a) Newtonian jet and (b) viscoelastic jet with $Wi = 4$, $L = 100$ and $\beta = 0.8$.