THEORETICAL AND NUMERICAL ANALYSES OF UNIFORM BLOWING AND SUCTION IN TURBULENT PLANE COUETTE FLOW

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ABSTRACT

We theoretically and numerically investigate the control effect of uniform blowing/suction (UB/US) in a turbulent plane Couette flow under a constant velocity difference (CVD) condition in terms of the net energy saving. The semi-theoretical upper bound of the ratio between UB/US amplitude and the wall velocity, V_w/U_w, for the net energy saving is about 0.45% at very low Reynolds numbers, i.e., $\mathrm{Re}_w \sim 10^3$, while it is less than 0.05% at practically high Reynolds numbers, i.e., $\mathrm{Re}_w \sim 10^9$–10^{10}. According to the present simulations, however, it is found that the amplitude of $V_w/U_w \leq 0.1\%$, (i.e., the amplitude often used for external flows) is too small to obtain substantial control effects in this flow configuration, while the larger amplitude, i.e., $V_w/U_w \geq 0.3\%$, results in a negative net energy saving due to the drastic enhancement of the contribution from the blowing and suction.

INTRODUCTION

Since the skin friction drag in a turbulent flow is much higher than that in the corresponding laminar flow, it is important to establish flow control methods for reduction of the friction drag. Among various control methods, the uniform blowing and suction (UB/US) is a simple and well-known flow control method. Sumitani and Kasagi (1995) investigated the effect of this control in turbulent Poiseuille flow. According to their study, the turbulence is enhanced (suppressed) on the blowing (suction) side, while the drag is decreased (increased) on the blowing (suction) side. Also in a spatially developing turbulent boundary layer flow, the similar effect was confirmed at relatively low (Kametani and Fukagata, 2011) and moderate (Kametani et al., 2015) Reynolds numbers. Based on these studies, UB at the amplitude around 0.1% of the free-stream velocity has also been used in a recent wind-tunnel experiment for airfoil drag reduction (e.g., Eto et al., 2019; Miura et al., 2021). However, this control effect has not sufficiently been investigated for a turbulent plane Couette flow, despite that it is more or less similar to a flat-plate turbulent boundary layer. This is because an enormous computational domain in the streamwise direction is required to capture the large-scale structures in the region far from the walls (Lee and Moser, 2018). Therefore, in the present study, we first analytically investigate whether overall drag reduction and net energy saving is possible with UB/US in turbulent plane Couette flows, then we complement the analysis by numerical simulations.

THEORETICAL ANALYSIS

In this study, all variables are made dimensionless by the fluid density, ρ^*, the wall velocity, U_w^*, and the channel half-width, δ^*, where $*$ denotes dimensional quantities. The subscripts 1, 2, and 3 represent the streamwise, wall-normal, and spanwise directions, respectively. Also, u_i $(i = 1,\, 2,\, 3)$, p, and t denote the velocity components, pressure, and time, respectively. Note that, for convenience, $(x_1,\, x_2,\, x_3)$ and $(u_1,\, u_2,\, u_3)$ are interexchangably denoted by $(x,\, y,\, z)$ and $(u,\, v,\, w)$, respectively. The Reynolds number is defined as $\mathrm{Re}_w = \delta^* U_w^*/\nu^*$, where ν^* is the kinematic viscosity. Similar to Sumitani and Kasagi (1995), the uniform blowing on the lower wall $(y = 0)$ and the uniform suction on the upper wall $(y = 2)$ are imposed. In the present theoretical analysis, study, we investigate the range of the UB/US amplitude required for net energy saving.

The net energy saving rate, S, is defined as (e.g., Kasagi et al., 2009)

$$S = \frac{W_{p0} - \left(W_p + W_a\right)}{W_{p0}}, \tag{1}$$

where W_{p0} is the driving power in the uncontrolled case, i.e.,

$$W_{p0} = U_w \cdot \frac{1}{2\mathrm{Re}_w}\left(\left.\frac{\partial U}{\partial y}\right|_{y=0} + \left.\frac{\partial U}{\partial y}\right|_{y=2}\right) = \left(\frac{1}{U_w^+}\right)^2. \tag{2}$$

Also, W_p and W_a are the driving power and actuation power in the controlled case, respectively. According to the overall energy balance (e.g., Fukagata et al. 2009), the summation of

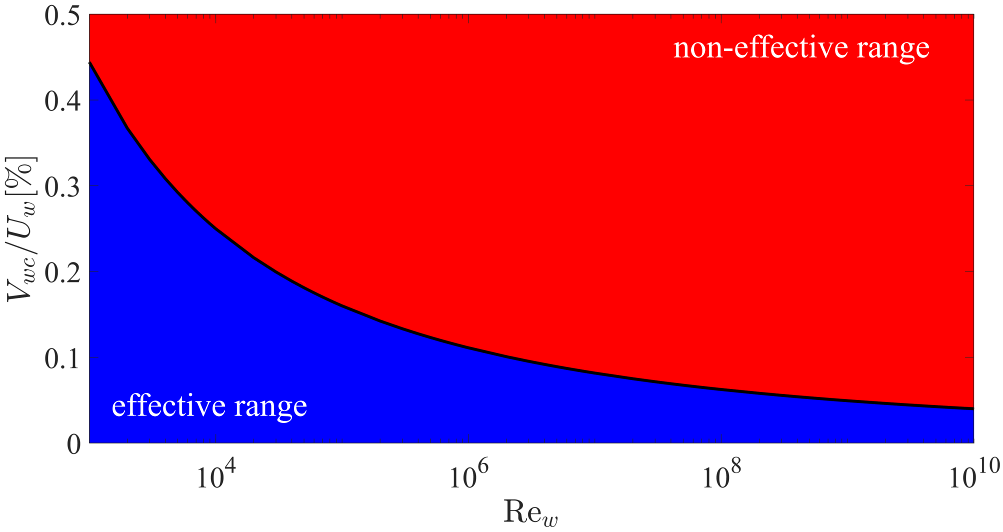

Figure 1. Relationship between the upper bound of the UB/US amplitude, V_{wc}/U_w, and the Reynolds number, Re_w, according to the obtained theoretical formula (Eq. (10)). Blue and red represent the effective and non-effective ranges of the UB/US amplitude, respectively.

these values, i.e., $W_p + W_a$, can be computed using the dissipation rate, i.e.,

$$W_p + W_a = \frac{1}{V}\int_V \frac{2}{\mathrm{Re}_w}\left(S_{ij}S_{ji}\right) dV, \tag{3}$$

where S_{ij} and V are the strain-rate tensor and the volume of the computational box, respectively.

In the present study, the effective range of the UB/US amplitude is derived under a hypothesis that the lower bound of $W_p + W_a$ is the value corresponding to the laminar flow with UB/US, i.e., $\left(W_p + W_a\right)_{\mathrm{lam}}$:

$$\left(W_p + W_a\right)_{\mathrm{lam}} \leq W_p + W_a, \tag{4}$$

where the subscript "lam" denotes quantities in the corresponding laminar flow with UB/US. In the situation where the positive net energy saving rate is achieved, $W_p + W_a$ should also be less than the pumping power in the uncontrolled case, i.e.,

$$\left(W_p + W_a\right)_{\mathrm{lam}} \leq W_p + W_a \leq W_{p0}, \tag{5}$$

which suggests that the net energy saving is possible only when this range exists. Using Eq. (3), $\left(W_p + W_a\right)_{\mathrm{lam}}$ is computed as

$$\begin{aligned}\left(W_p + W_a\right)_{\mathrm{lam}} &= \frac{1}{V}\int_V \frac{2}{\mathrm{Re}_w}\cdot\frac{\partial U_{\mathrm{lam}}}{\partial y} dV \\ &= V_w \frac{\exp\left(2V_w \mathrm{Re}_w\right)+1}{\exp\left(2V_w \mathrm{Re}_w\right)-1},\end{aligned} \tag{6}$$

where V_w denotes the blowing or suction velocity amplitude ($V_w > 0$). Note that the laminar solution with UB/US can analytically be derived as

$$U_{\mathrm{lam}} = 2\frac{\exp\left(V_w \mathrm{Re}_w y\right)-1}{\exp\left(2V_w \mathrm{Re}_w\right)-1} - 1. \tag{7}$$

The drag coefficient in the uncontrolled flow, C_{f0}, is computed as

$$C_{f0} = \frac{2\tau^*_{w0}}{\rho^* U_w^{*2}} = 2\left(\frac{1}{U_w^+}\right)^2, \tag{8}$$

where τ_{w0} denotes the wall shear stress in the uncontrolled flow. Also, using the empirical formula proposed by Robertson (1959), C_{f0} is expressed as

$$C_{f0} = \frac{G}{\left(\ln \mathrm{Re}_w\right)^2}, \tag{9}$$

where G is the empirical coefficient, and we use the value proposed by Pirozzoli et al. (2014), i.e., $G = 0.424$. Substituting Eqs. (2), (6), (8), and (9) into (5), we can obtain the following inequality,

$$V_w \frac{\exp\left(2V_w \mathrm{Re}_w\right)+1}{\exp\left(2V_w \mathrm{Re}_w\right)-1} \leq \frac{0.212}{\left(\ln \mathrm{Re}_w\right)^2}. \tag{10}$$

Solving Eq. (10) in the range of $V_w > 0$, we can obtain the effective range of V_w, i.e., $0 < V_w < V_{wc}$, where V_{wc} denotes the upper bound of the UB/US amplitude having the possibility of net energy saving.

Figure 1 shows the relationship between V_{wc}/U_w and Re_w. According to this figure, V_{wc}/U_w decreases as the Reynolds number increases, and it is less than 0.05% at significantly high Reynolds numbers, i.e., $\mathrm{Re}_w \sim 10^9$–10^{10}. Even at very low Reynolds numbers, i.e., $\mathrm{Re}_w \approx 1000$, V_{wc}/U_w is about 0.45%. Although the UB/US can relaminarize the turbulent Couette flow, this semi-theoretical analysis suggests that the net energy saving is possible only when the UB/US amplitude is very small.

DIRECT NUMERICAL SIMULATION

As a complementary investigation, we perform a direct numerical simulation (DNS) of a fully developed turbulent

Table 1. Computational conditions for the validation and verification.

Case	Re_τ	$L_x \times L_y \times L_z$	$N_x \times N_y \times N_z$	Δx^+	Δy^+	Δz^+
Case 1	180.6	$8\pi \times 2 \times 4\pi$	$256 \times 96 \times 256$	17.7	0.9–6.0	8.8
Case 2	181.2	$8\pi \times 2 \times 4\pi$	$512 \times 96 \times 512$	8.8	0.9–6.0	4.4
Case 3	178.9	$16\pi \times 2 \times 4\pi$	$512 \times 96 \times 256$	17.7	0.9–6.0	8.8
Case 4	180.2	$32\pi \times 2 \times 4\pi$	$1024 \times 96 \times 256$	17.7	0.9–6.0	8.8
Case 5	179.8	$64\pi \times 2 \times 4\pi$	$2048 \times 96 \times 256$	17.7	0.9–6.0	8.8
Avsarkisov et al. (2014)	180	$20\pi \times 2 \times 6\pi$	$1296 \times 251 \times 768$	13.1	0.92–5.9	6.6

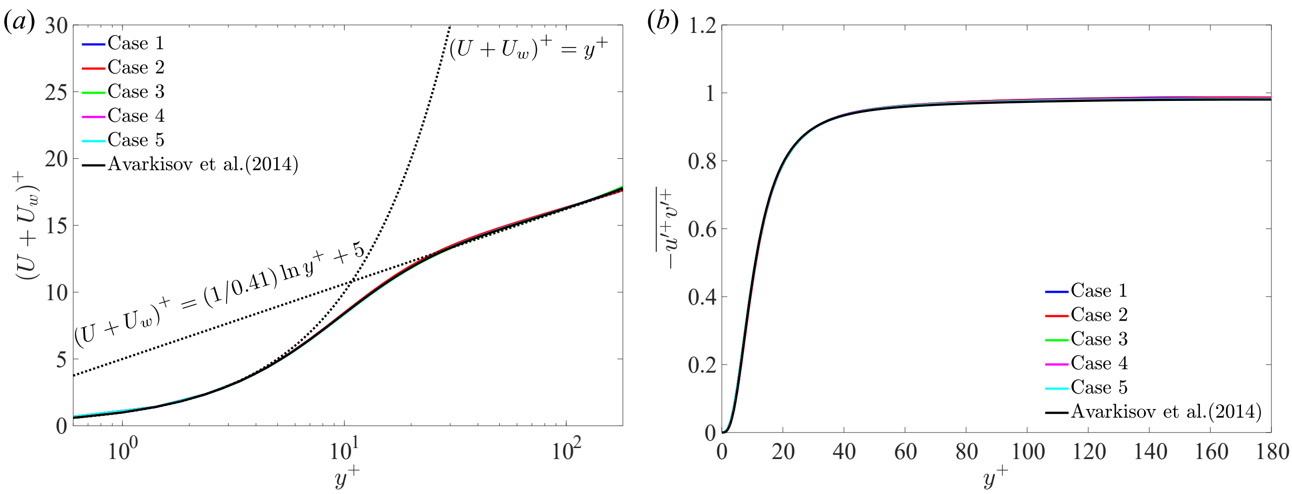

Figure 2. Mean velocity and the Reynolds shear stress (RSS) of the uncontrolled flow in the different computational conditions: (*a*) mean velocity profile; (*b*) RSS profile.

plane Couette flow controlled using the UB/US. The governing equations are the incompressible continuity and Navier-Stokes equations, i.e.,

$$\frac{\partial u_i}{\partial x_i} = 0, \tag{11}$$

$$\frac{\partial u_i}{\partial t} = -\frac{\partial (u_i u_j)}{\partial x_j} - \frac{\partial p}{\partial x_i} + \frac{1}{\mathrm{Re}_w}\frac{\partial^2 u_i}{\partial x_j \partial x_j}. \tag{12}$$

The Reynolds number is set to $\mathrm{Re}_w = 3200$, which corresponds to the friction Reynolds number of $\mathrm{Re}_\tau \simeq 180$ in the uncontrolled flow.

The periodic boundary condition is applied in the streamwise (x) and spanwise (z) directions. The boundary conditions of the streamwise and spanwise velocities on the walls are applied as

$$\begin{cases} u = U_w,\ w = 0 \quad (\text{upper wall}), \\ u = -U_w,\ w = 0\ (\text{lower wall}). \end{cases} \tag{13}$$

Also, the boundary condition of the wall-normal velocity on the wall in the uncontrolled and controlled cases as follows:

$$\begin{cases} v(x,0,z) = v(x,2,z) = 0 \quad (\text{uncontrolled}), \\ v(x,0,z) = v(x,2,z) = V_w\ (\text{UB/US}). \end{cases} \tag{14}$$

The velocity amplitude of the UB/US is varied as 0.05%, 0.1%, 0.3%, 0.5%, and 1.0% of the wall velocity, U_w.

The present DNS code is developed based on the DNS code developed by Fukagata et al. (2006). The governing equations are spatially discretized by means of the energy-conserving fourth-order finit difference scheme (Morinishi et al., 1998) in the streamwise and spanwise directions and second-order finite difference scheme in the wall-normal direction. The time integration is performed by means of the low-storage, third-order Runge-Kutta/Crank-Nicolson scheme (Spalart et al., 1991) with the high-order SMAC-like velocity-pressure coupling scheme (Dukowicz and Dvinsky, 1992).

First, we validate and verify the present DNS code. Table 1 shows the computational conditions for the validation and verification of the present simulation. Here, L_i, N_i, and Δx_i denote the length of the computational box, the number of the grid points, and the grid spacing, respectively, in the i-th direction. Figure 2 shows the mean velocity profile and Reynolds shear stress (RSS) of the uncontrolled flow in all cases. As shown in Fig. 2, these statistics in all cases are in good agreement with the reference results by Avsarkisov et al. (2014). Figure 3 shows the comparison of the mean velocity profile in the relaminarization case, i.e., $V_w/U_w = 1.0\%$ (explained later). As can be observed in Fig. 3, the dependence of the computational domain can hardly be observed. Therefore, it is found that the computational condition of Case 1 is sufficient for the present simulation.

Figure 4 shows the comparison of the mean velocity pro-

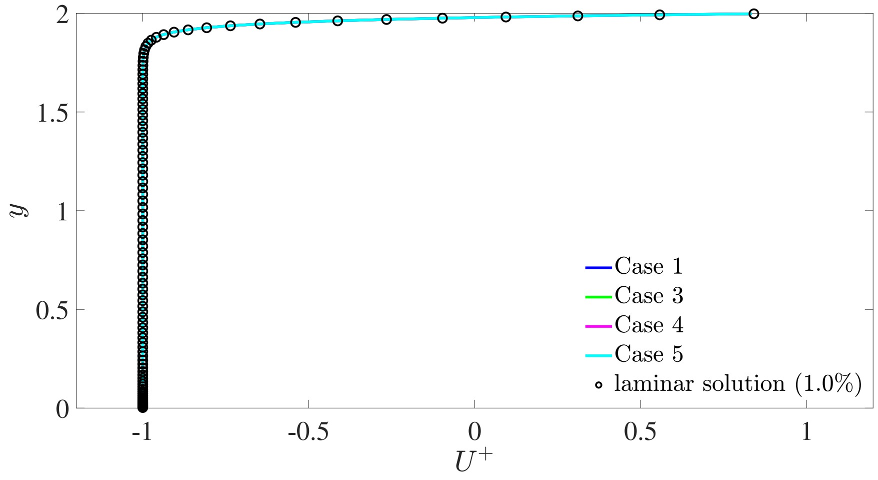

Figure 3. Mean velocity profiles of the controlled flow ($V_w/U_w = 1.0\%$) in the different computational conditions.

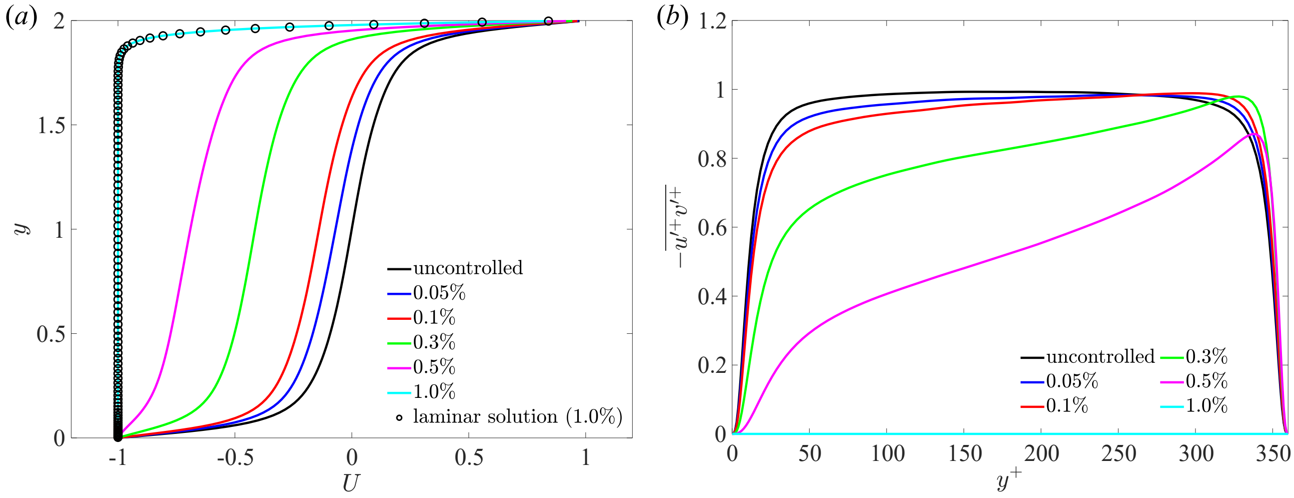

Figure 4. Mean velocity profiles and RSS for different UB/US amplitudes, V_w/U_w, obtained by DNS: (*a*) mean velocity profile; (*b*) RSS.

Table 2. Net energy saving rate, S, for different UB/US amplitudes, V_w/U_w, computed from DNS results.

V_w/U_w	0.05%	0.1%	0.3%	0.5%	1.0%
S	3.9%	1.9%	−20.3%	−60.9%	−216.0%

file and RSS in the uncontrolled and controlled cases. Note that circle denotes the laminar solution of $V_w/U_w = 1.0\%$ in Eq. (7). As can be observed in Fig. 4(*a*), the mean velocity profile shifts upward as the UB/US amplitude increases. The profile in the case of $V_w/U_w = 1.0\%$ is in perfect agreement with the laminar solution of Eq. (7). In other words, in the case of $V_w/U_w = 1.0\%$, the flow is relaminalized by UB/US. According to Fig. 4(*b*), the RSS in the case of $V_w/U_w = 1.0\%$, i.e., the relaminarization case, is perfectly suppressed. In other cases (not relaminarization), although the RSS are enhanced in the region near the wall of the suction side, those in the other region are suppressed compared with the uncontrolled case. The net energy saving rates in all cases are shown in Table 2. Although the small net energy saving effect can be observed for $V_w/U_w \leq 0.1\%$, the net energy saving effect cannot be obtained for $V_w/U_w > 0.1\%$ despite the relaminarization. This is because the UB/US amplitude of $V_w/U_w = 1.0\%$ is greater than the upper bound at $\mathrm{Re}_w = 3200$, i.e., $V_{wc}/U_w \approx 0.33\%$ shown in Fig. 1.

In order to investigate the detailed effect of UB/US control, we investigate different contributions to the wall shear stress using the Fukagata-Iwamoto-Kasagi (FIK) identity (Fukagata et al., 2002). The FIK identity in the turbulent plane Couette flow with UB/US control under CVD condition is expressed as

$$\tau_w = \underbrace{\frac{1}{\mathrm{Re}_w}}_{\tau_w^l} + \underbrace{\frac{1}{2}\int_0^2 \left(-\overline{u'v'}\right) dy}_{\tau_w^r} \underbrace{-V_w U_b}_{\tau_w^{bs}}, \tag{15}$$

where $()^l$, $()^r$, and $()^{bs}$ denote the contribution from the laminar flow, the RSS, and the blowing and suction, respectively.

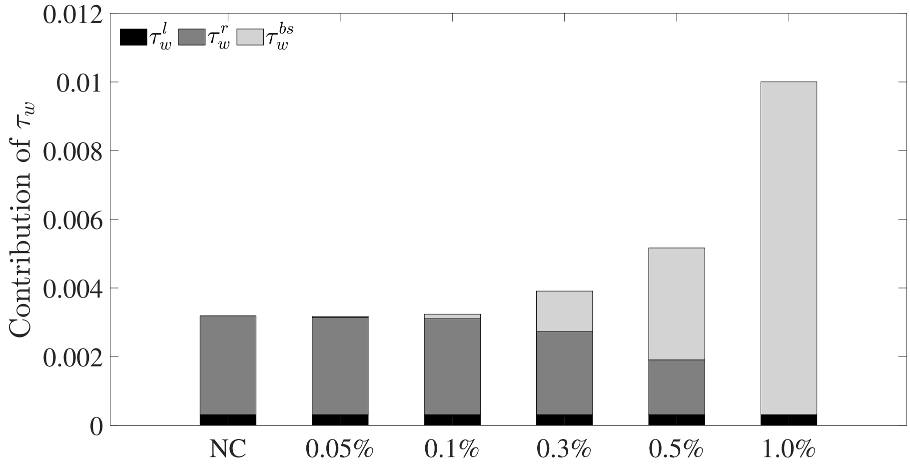

Figure 5. Contribution to the wall shear stress, τ_w, from each term of Eq. (15) in the uncontrolled and controlled cases. Black, dark gray, and light gray represent the contribution from the laminar flow, the RSS, and the blowing and suction, respectively.

Also, U_b denotes the bulk-mean velocity. Figure 5 shows the contribution from each term in right-hand-side (RHS) of Eq. (15). As shown in Fig. 5, the contribution from the blowing and suction increases and the contribution from the RSS decreases as the UB/US amplitude increases. In the relaminarization case, i.e., $V_w/U_w = 1.0\%$, there is no contribution from the RSS, and that from the blowing and suction is drastically enhanced. In the cases of the smaller UB/US amplitude, i.e., $V_w/U_w \leq 0.1\%$, the contribution from the RSS is nearly unchanged, and there is little contribution from the blowing and suction, so that the net energy saving effect is hardly obtained. On the other hand, in the cases of larger UB/US amplitude, i.e., $V_w/U_w \geq 0.3\%$, the amount of decrease of the contribution from the RSS is smaller than that of increase of the contribution from the blowing and suction, so that the total drag increases. Large increase of the total drag leads to large negative net energy saving rate.

CONCLUSIONS

We theoretically and numerically investigate the effect of the uniform blowing and suction control in turbulent plane Couette flow under the constant velocity difference condition. According to the present semi-analytical solution, the upper bound of the UB/US amplitude decreases as the Reynolds number decreases, and it is less than 0.05% of the wall velocity at significantly high Reynolds numbers, i.e., $\mathrm{Re}_w \sim 10^9$–10^{10}. The results of a direct numerical simulation (DNS) shows that the net energy saving can be achieved only when the blowing amplitude is very small, $V_w/U_w \leq 0.1\%$, and we cannot obtain the net energy saving for larger amplitudes even if the flow is relaminarized. According to the analysis of the FIK identity, in the cases of larger UB/US amplitude, the drag is drastically increased due to the large contribution from the blowing and suction beyond the amount of decrease of the contribution from the Reynolds shear stress.

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REFERENCES

Avsarkisov, V., Hoyas, S., Oberlack, M., and Garcia-Galache, J. P., 2014, "Turbulent plane Couette flow at moderately high Reynolds number," J. Fluid Mech., Vol. 751 (2014).

Dukowicz, J. K. and Dvinsky, A. S., 1992, "Approximate factorization as a high order splitting for the implicit incompressible flow equations," J. Comput. Phys., Vol. 102, pp. 336–347.

Eto, K., Kondo, Y., Fukagata, K., and Tokugawa, N., 2019, "Assessment of friction drag reduction on a Clark-Y airfoil by uniform blowing," AIAA J., Vol. 57, pp. 2774–2782.

Fukagata, K., Iwamoto, K., and Kasagi, N., 2002, "Contribution of Reynolds stress distribution to the skin friction in wall-bounded flows," Phys. Fluids, Vol. 14, L73–L76.

Fukagata, K., Kasagi, N., and Koumoutsakos, P., 2006, "A theoretical prediction of friction drag reduction in turbulent channel flow by superhydrophobic surfaces," Phys. Fluids, Vol. 18, 051703.

Fukagata, K., Sugiyama, K., and Kasagi, N., 2009, "On the lower bound of net driving power in controlled duct flows," Physical D, Vol. 238, pp. 1082–1086.

Kametani, Y. and Fukagata, K., 2011, "Direct numerical simulation of spatially developing turbulent boundary layers with uniform blowing or suction," J. Fluid Mech., Vol. 681, pp. 154–172.

Kametani, Y., Fukagata, K., Örlü, R., and Schlatter, P., 2015, "Effect of uniform/suction in a turbulent boundary layer at moderate Reynolds number," Int. J. Heat Fluid Flow, Vol. 55, pp. 132–142.

Kasagi, N., Hasegawa, Y., and Fukagata, K., 2009, "Toward cost-effective Control of Wall Turbulence for Skin Friction Drag Reduction," In: Eckhardt, B. (eds) Advances in Turbulence XII. Springer Proceedings in Physics, vol 132. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-642-03085-7_48

Miura, S., Ohashi, M., Fukagata, K., and Tokugawa, N., "Drag reduction by uniform blowing on the pressure surface of an airfoil," AIAA J. 60, 2241-2250 (2022).

Morinishi, Y., Lund, S. T., Vasilyev, V. O., and Moin, P., 1998, "Fully conservative higher order finite difference schemes for incompressible flow," J. Comput. Phys., Vol. 143, pp. 90–124.

Pirozzoli, S., Bernardini, H., and Orlandi, P., 2014, "Turbulence statistics in Couette flow at high Reynolds number," J. Fluid Mech., Vol. 758, pp. 327–343.

Robertson, J. M., 1959, "On turbulent plane Couette flow," Proc. 6th Midwestern Conf. on Fluid Mech., University of Texas, Austin, pp. 169–182.

Spalart, P. R., Moser, R. D., and Rogers, M. M., 1991, "Spectral methods for the Navier-Stokes equations with one infinite and two periodic directions," J. Comput. Phys., Vol. 96, pp. 297—324.

Sumitani, Y. and Kasagi, N., 1995, "Direct numerical simulation of turbulent transport with uniform wall injection and suction," AIAA J., Vol. 33, pp. 1220–1228.