TURBULENT TRANSITIONS IN AXISYMMETRIC FLOW

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ABSTRACT

We investigate a transition between two turbulent states in an axisymmetric flow. These are two-dimensional states with two and three velocity components and will be indicated by 2D2C and 2D3C, respectively. This transition is studied using direct numerical simulations in toroidal geometry. We show that in our set-up the 2D2C flow-state is caracterized by reduced fluctuations and reduced radial transport. We suggest that the 2D3C-2D2C transition is relevant to explain a similar transition in tokamak fusion plasmas.

BACKGROUND

Statistically axisymmetric turbulent flow is observed when a strong body force, such as the Coriolis force, renders a system invariant along an axis (Batchelor (1946); Chandrasekhar (1950)). This is different from strictly axisymmetric flow, considered here, where not only the averages but also the instantaneous fields are invariant in the azimuthal direction. This case has been the subject of recent investigations (Leprovost *et al.* (2006); Naso *et al.* (2010); Thalabard *et al.* (2014)) in order to extend statistical mechanics, succesfully applied to 2D flows, to a case between 3D and 2D. Numerical simulations were performed (Qu *et al.* (2017, 2018)) in order to corroborate these theorical investigations. Axisymmetric flows seem indeed to be an intermediate state between two dimensional and three dimensional flows. For instance 2D turbulence is caracterised by a self-organization of large structures caused by an energy transfert from smaller turbulent structures to large vortices (Kraichnan (1967); Kraichnan & Montgomery (1980); Paret & Tabeling (1997)), which is also observed in axisymmetric turbulence.

In the case of a cylindrical domain (Qin *et al.* (2020, 2021), it has been demonstrated that it is possible to induce with a linear forcing a transition between two dimensional states, one with a three-component velocity (2D3C), and an other with two velocity components (2D2C). The second state is caracterised by the large structures of a two dimensional flow (Qu *et al.* (2017, 2018) The first one is an intermediate state between two and three dimensional flow, with large structures breaking up in smaller vortices. The transition is piloted by the ratio of the toroidal and poloidal components of the forcing.

Even though few flows in nature are close to strictly axisymmetric, there exists a direct and very important, application of the concept of axisymmetric turbulence. This application is magnetically confined fusion. A tokamak is a torus-shaped metallic chamber in which a plasma of hydrogen is confined by a strong magnetic field in order to produce energy by nuclear fusion. Indeed a strong magnetic field renders a velocity field invariant in the direction in which it is applied (Moffatt (1967); Favier *et al.* (2010); Agoua *et al.* (2021)), here in the azimuthal direction. To maintain a sufficiently intense temperature, turbulence which induces a loss of confinement of the plasma must be reduced as far as possible. We propose to demonstrate that the 2D3C-2D2C transition can be linked to the LH transition between a low confined turbulent state (L-mode) and a higher quality confined state (H-mode) observed in tokamaks (Wagner *et al.* (1982)). For that we show that this phenomenon observed in a cylindric system (Qin *et al.* (2020)) can be observed in a torus, and that it can be linked to a loss of confinement.

MODEL AND NUMERICAL METHOD

The domain of the simulations is a disk, corresponding to a poloidal cross-section of the torus. We consider a neutral fluid described by the axisymmetric Navier-Stokes equation (1) in the coordinate system of the disk (e_r, e_θ, e_ϕ) with a forcing of the velocity $\mathbf{F}$.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{1}{\rho}\nabla P + \nu\Delta\mathbf{u} + \mathbf{F} \quad (1)$$

$$\frac{\partial T}{\partial t} + (\mathbf{u}\cdot\nabla)T = \kappa\Delta T + S(r) \quad (2)$$

$$\mathbf{F_{pol}} = c_p T(r)\mathbf{e_r} - \mathbf{u_{pol}} \times \beta \mathbf{e}_\phi \quad (3)$$

$$\mathbf{F_{tor}} = c_t \left(u_{tor} - \frac{\overline{u_{tor} y}}{y} \right) \mathbf{e}_\phi \quad (4)$$

e_ϕ corresponds to the toroidal direction and velocity can be decomposed in $\mathbf{u} = \mathbf{u_{pol}} + \mathbf{u_{tor}}$. The toroidal magnetic field is not included in the equations, but is at the origin of the axisymmetric nature of the system. In order to introduce a realistic forcing, we propose that poloidal fluctuations $\mathbf{u_{pol}}$ are induced (3) by a scalar field T modelled by the heat equation (2). An anisotropic friction $\mathbf{u_{pol}} \times \beta \mathbf{e}_\phi$ is added to allow the development of zonal flows observed in the plasma (Lin *et al.* (1999); Diamond *et al.* (2005)). Toroidal perturbations are generated by a simple linear forcing (4). A correction is added to avoid the generation of angular momentum $\overline{u_y y}$ with y the distance to the vertical axis of the tokamak. The source of the scalar field and the toroidal forcing is chosen such that the generation of velocity perturbations increases at the edge of the plasma, as observed in experiments (Holland *et al.* (2011)).

Numerical simulations are performed using the open-source code Nek5000. It uses the spectral element method (Patera (1984)) to solve the Navier-Stokes and scalar equations. Velocity fluctuations are initialy chosen randomly around a null average. A Dirichlet boundary condition is imposed at the edge of the domain.

RESULTS

We study the evolution of the behaviour of the flow with the parameter $\alpha = c_T/c_P$ which determines the relative intensity of toroidal and poloidal forcing strengths. The evolution of the behaviour of the flow is illustrated by the stream function ϕ defined by $\Delta\phi = \omega_\phi$ with $\omega_\phi = (\nabla \times u) \cdot e_\phi$ the azimuthal component of the vorticity. The role of the friction is observed in 1. Without friction in the figure at left two counter-rotating vortices are obtained as it is usually observed in 2D flow. The presence of the friction occurs the formation of two large and concentric structures in the 2D2C regime as observed in the figure at right. In contrary when α increases the behaviour of the flow changes as shown in figure 2. In this case the structures are smaller and more chaotic as observed in 3D turbulence, which should weaken the confinement.

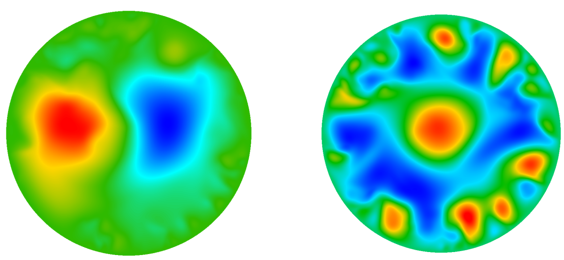

Figure 1. Stream function field in H mode without and with friction

The parameter $\gamma = E_T/E_P$, with E_T and E_P energy fluctuations in the toroidal direction and poloidal plane respectively, is used to determinate quantitatively the behaviour of the flow between 2D3C and 2D2C states. The relation between α and γ is illustrated on figure 3. When γ is closed to zero, the flow is purely poloidal without toroidal perturbations, which corresponds to a 2D2C state. Its nature is similar to the H-mode in tokamas. In figure 3 this state coincides with a low parameter γ. When γ increases, toroidal fluctuations are more and more predominant and the flow is in a 2D3C state. Figure 3 shows a critical transition for α above a critical value $\alpha_{crit} \approx 1$.

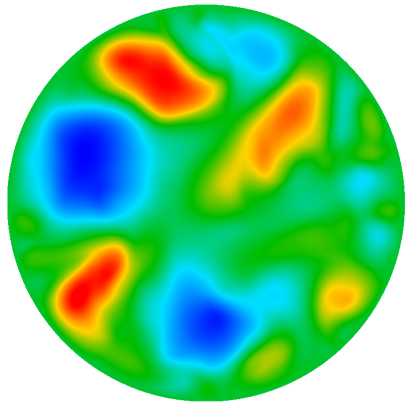

Figure 2. Stream function field in L mode

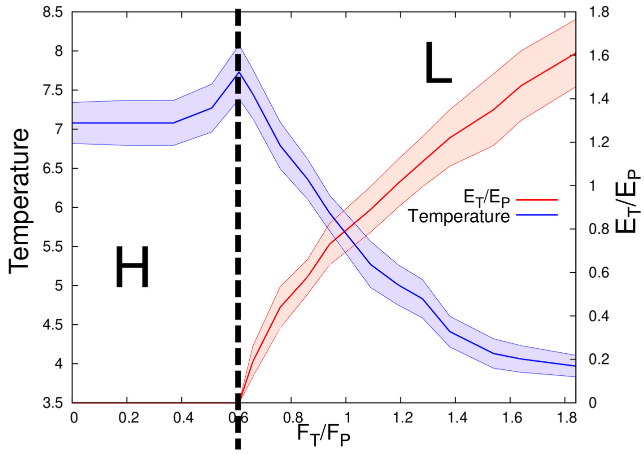

Figure 3. Relation of γ and the temperature of the tracer at the center of the disk with the control parameter α.

The transition between 2D3C and 2D2C states, which we assimilate with the transition between L-mode and H-mode is illustrated in figure 4. From a first simulation with $c_T/c_P = 3.52$ which induces a 2D3C state with important toroidal perturbations, α is set to $c_T/c_P = 0.24$ at $t = 90s$. This modification induces a dramatic and fast drop of the toroidal fluctuating energy to zero and then a transition to a 2D2C state for the flow. A remarkable feature is that the poloidal forcing is kept constant, but nevertheless the poloidal fluctuations drop dramatically. This shows that the nature of the poloidal flow is changed and not only its amplitude.

In order to investigate the link between this transition and a loss of confinement in the tokamak, a passive scalar field modelled by equation (2) is introduced in the disk. Its source is located at the center. Figure 5 shows the behaviour of this tracer in the flow for the two states. The radial density profile represents the diffusion of the tracer in the flow from the center

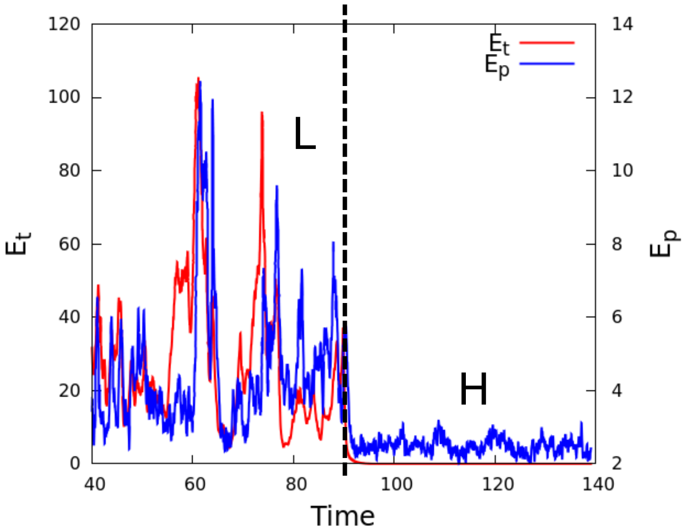

Figure 4. Transition between the L-mode and the H-mode.

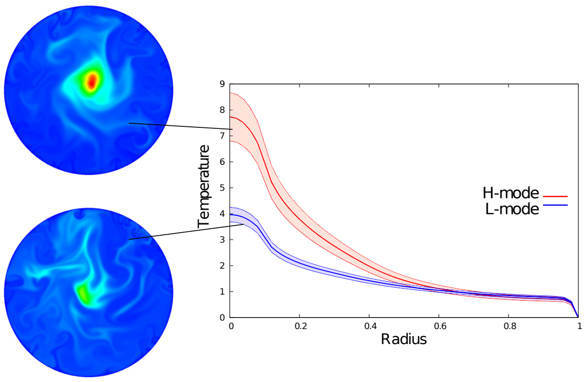

Figure 5. Temperature of the tracer in H-mode and L-mode

to the edge caused by the turbulent transport. For the H-mode, the turbulent transport is weaker and the tracer remains more concentrated around the center than for the L-mode. The relation between temperature at the center and α is represented on figure 3, which shows the coïncidence between H-mode/2D2C state and high temperature.

CONCLUSION

The link between the LH transition observed in tokamaks and a 2D3C-2D2C transition is illustrated by simulations of an axisymmetric flow in a torus. This transition is associated with a better confinement of the temperature in the 2D2C state similar to the H-mode behaviour. In the full paper we will further investigate the turbulent dynamics underlying the transition by assessing the energy cascade of both toroidal and poloidal flow components.

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