TURBULENCE INTENSITY PROFILE IN HIGH REYNOLDS NUMBER PIPE FLOW

Yuki Wada

Nuclear Safety Research Centre
Japan Atomic Energy Agency
2-4, Shirakata, Tokai, Ibaraki, 319-1195, Japan
Email:wada.yuki@jaea.go.jp

Noriyuki Furuichi

National Institute of Advanced Industrial Science and Technology
1-1-1, Umezono, Tsukuba, Ibaraki, 305-8563, Japan

Eisuke Kusano

National Institute of Advanced Industrial Science and Technology
Nagoya University

Yoshiyuki Tsuji

Nagoya University
Furo-cho, Chikusa, Nagoya, Aichi, 464-8603, Japan

ABSTRACT

This paper reports characteristics of turbulence intensity profile obtained in high Reynolds number actual flow facility in Japan. The experiments were performed in a pipe flow with water, and the friction Reynolds number was varied up to $Re_\tau = 2.0\times10^4$. The streamwise velocity was measured by laser Doppler velocimetry (LDV). The new procedure to correct the measurement volume effect of LDV is suggested and we discuss turbulence intensity issues such as the inner peak Reynolds number dependence and the outer logarithmic behavior, it was found that these characteristic behaviors are consistent with previous turbulence studies.

INTRODUCTION

Pipe flow is one of the canonical wall turbulence, which has wide applications in engineering fields. In wall turbulence, due to the existence of wall it is known that characteristic velocity profiles are observed. One of these is the logarithmic law in a mean velocity profile. Similar to mean velocity profiles, recent studies reported a logarithmic law in the outer turbulence intensity profile at high Reynolds numbers. High Reynolds number experiments have been carried out in boundary layer with several facilities. On the other hand in pipe flow, Princeton Superpipe data including Zagalora and Smits (1998) were well known and used to discuss the universal law of wall turbulence for long time. However, Superpipe test was carried out in a compressed air flow facility, therefore it is difficult to understand the compression effect on turbulence measurement. For example, although other wall turbulence experiments reported growth in turbulence intensity inner peak as Reynolds number increases, Superpipe data (Hultmark et al., 2012) shows this peak was independent of Reynolds number. This issue is treated as an open question reported by Marusic et al. (2017). The occasion that high Reynolds number pipe flow data depend only on Superpipe data is not better for the discussion of universality of wall turbulence. Recently in order to overcome this occasion, high Reynolds number pipe flow experiments with non-compressed facility have been conducted in the high Reynolds number actual flow facility (Hi-Reff, Furuichi et al., 2017) with water flow and the Centre for International Cooperation in Long Pipe Experiments (CICLoPE, Örlü et al., 2016) with air flow.

In this paper, based on the high Reynolds number pipe flow LDV measurement data conducted at “Hi-Reff”, characteristics of turbulence intensity profiles such as Reynolds number dependence of inner peak and the outer logarithmic behaviours. For a higher reliability discussion, LDV measurement volume effects on turbulence intensity profiles are also briefly discussed.

EXPERIMENTAL SETUP AND CONDITIONS

Present experiments were conducted using “Hi-Reff” at Advanced Industrial Science and Technology (AIST). Pipe used in this experiment was made from stainless metal and has 100 mm inner diameter, whose inner surface was polished and which has average roughness of 0.8 μm. Inlet length is about 11 m, thus the ratio of the inlet length and the inner diameter is about 110 which is enough to remove an effect of the inlet condition on the flow field at the test section. Maximum flow rate using the present pipe line is 300 m^3/h, then bulk Reynolds number would be approximately 10^6 in the pipe of 100 mm inner diameter. If more details of experimental setup are needed, please refer our previous paper (Furuichi, et al., 2015).

In the present experiment, water was used as a working fluid. LDV system (BSA Flow Software Version 4.10) produced by DANTEC was used for velocity measurement. Measurable velocity component was only streamwise direction. Wave length of the laser as λ_L is 514.5 nm; laser beam diameter at an inducing collecting lens as D_L is 2.2 mm; a spacing of the laser beams at the collecting lens is 38.998 mm; a focal length of the collecting lens as f is 160 mm. Measurement volume of LDV system is well known as an ellipsoidal body, short axis length of the body is calculated as $4\ \lambda_L\ f\ /\ \pi D_L$. In our experimental setup, short and long axis length of the body are calculated as 47.6 μm and 524.9 μm, respectively. To move the LDV unit, a three-dimensional moving system whose moving resolution is 1/160 mm/pulse was utilized.

Experimental flow conditions are the following Reynolds number range: $1000 \leqq Re_\tau \leqq 20000$ ($Re_\tau = u_\tau R / \nu$, u_τ: friction velocity, R: radius of the pipe, ν: dynamic viscosity).

To measure velocity profiles in the test section by different measurement volume especially in the wall normal direction, velocity profile measurements were conducted in different measurement paths of LDV moving system as indicated in Fig.1 for a discussion of the LDV measurement volume effects. In the figure, red and blue lines indicate different measurement path which correspond to using different measurement volume in the wall normal direction, respectively. Table 1 shows example of the relation between the measurement path angle and the spatial resolution at Re_τ=3300. Two different measurement paths of 15 and 22.5 degree against the vertical axis were selected to discuss the measurement volume effects on the velocity measured by LDV.

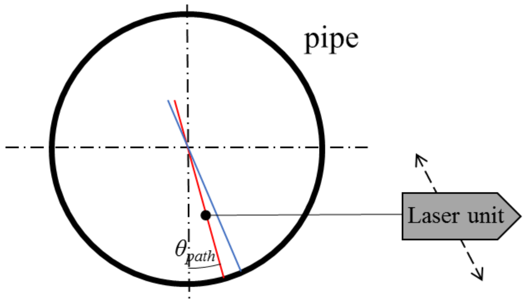

Figure 1. Schematic view of measurement path for different volume size in wall normal direction.

Table 1. Measurement path angle θ_{path} condition against vertical axis at Re_τ=3300.

Spatial resolution	Angle and measurement volume size in vertical direction
High-resolution	~15 degree ($l_y^+ \simeq 5.6$)
Low-resolution	~22.5 degree ($l_y^+ \simeq 10.8$)

MEASUREMENT VOLUME EFFECT OF LDV

First of all, we explain the reason why the turbulent intensity is overestimated due to the measurement volume in LDV measurement under a steady condition in wall turbulence. We consider the situation as measurement volume is enlarged in wall normal direction. Dividing measurement volume into some small volumes in wall normal direction, a probability density function (PDF) calculated from velocity fluctuation measured in each small volume has a different statistic characterized by the wall normal distance. It is considered that PDF obtained in total measurement volume can be obtained as a spatial weighting averaged function of PDF characterized by wall normal distance y_i^+ as shown by equations (1) and (2).

$$p_{OE}(u^+, y^+) = \sum_{i=1}^{n} p_T(u_i^+, y_i^+) \times F(y_i^+) \qquad (1)$$

$$F(y_i^+) = \Delta S(y_i^+) \Big/ \sum_{i=1}^{n} \Delta S(y_i^+) \qquad (2)$$

Here, p (u^+, y^+) indicates probability density function of velocity fluctuation u^+ at y^+. Here, the superscript of + indicates non-dimensionalization by inner variables. Subscript of i indicates a position in which measurement volume is divided for n portions, $y_i^+ = y^+ - l_y^+/2 + (i-1/2)dy^+$ and then, $dy = l_y/n$. $\Delta S(y_i^+)$ is streamwise cross-sectional area at y_i^+; $F(y_i^+)$ is a dominant ratio of $\Delta S(y_i^+)$ to streamwise cross-sectional area at y_i^+; l_y is a wall normal length of total measurement volume. Subscriptions for PDF indicate followings: OE indicates measurement result when wall normal length of measurement volume is l_y at wall distance y^+, T indicates an expected PDF at wall distance of y_i^+ with an infinitely minimal measurement volume. From the relation of equation (1), it is obvious that PDF of velocity measured by LDV is affected by the measurement volume in wall normal direction.

Expected PDF of p_T in equation (1) can be calculated by equation (3) using universal PDFs: p_U which are characterized by only wall normal distance of y^+, mean velocities: U^+ and root mean squares (RMSs) of the fluctuating velocity: u^+_{rms}.

$$p_T(u^+, y^+) = p_U((u^+ - U^+)/u^+_{rms}, y^+)/u^+_{rms} \qquad (3)$$

Here, a division in the right-hand side of equation (3) means a normalization to satisfy that the integration of equation (3) should be 1. From this relation, substituting expected mean velocity and RMS value of fluctuating velocity component to the equation (3), we can obtain an expected PDF profile at each wall normal location.

Here, we check the statistical characteristics of velocity measured by LDV. Nakao, et al. (1987) reported that the time averaged statistics (several moments obtained from PDF) of velocities measured by LDV can be correlated by the penetration frequencies of tracer particles into the measurement volume. In the present paper, we developed this correlation to the PDF including all time averaged statistics of velocities using weighting method of the inverse of velocity. Thus, actual PDF at the measurement location can be calculated by equation (4) which indicates weighting method by a probability variable of u.

$$p(u_i) = p_f(u_i) \times \frac{1}{|u_i|} \Big/ \sum_{i=1}^{n} \frac{1}{|u_i|} \qquad (4)$$

Here, subscription of f in the right-hand side indicates a probability density calculated by using original measured velocities. It is noted that an average and a standard deviation (STD) weighted by the particle penetration frequency should be used at the separation of *bin size* to calculate $p_f(u_i)$.

CORRECTION METHOD

In order to utilize the above measurement volume effect of LDV for correction of turbulence intensity profiles, by applying the equation (1) to all measurement locations, equation (5) can be obtained as a matrix expression. Here, i and j which are subscripts of the penetration frequency F correspond to wall normal distances of y_i and y_j, respectively. $F_{i,j}$ can be calculated by the equation (2). Thus, the relation between profiles of the expected PDF: p_T and the PDF affected by the LDV measurement volume: p_{OE} in all measurement locations are obtained.

$$\begin{pmatrix} F_{1,1} & \cdots & F_{1,n} \\ \vdots & \ddots & \vdots \\ F_{n,1} & \cdots & F_{n,n} \end{pmatrix} \begin{pmatrix} p_T(u^+, y_1^+) \\ \vdots \\ p_T(u^+, y_n^+) \end{pmatrix} = \begin{pmatrix} p_{OE}(u^+, y_1^+) \\ \vdots \\ p_{OE}(u^+, y_n^+) \end{pmatrix} \qquad (5)$$

Although the equation (5) can be solved analytically as an inverse problem, during a solving process the equation (5) experimental uncertainties would affect on an obtained profile which would be a quite different profile from an expected profile. Thus, it is considered that the calculation of expected PDFs by the analytical method to solve equation (5) is difficult for practical usages. From this point of view, a more practical correction method to estimate expected PDFs should be proposed.

Here, the equation (5) can be transformed to the equation (6) when there are universal PDFs; only RMS velocity should be required to reconstruct expected PDFs by using the equation (3). Equation (6) indicates that overestimated RMS velocity profile can be calculated by only inputs of an expected RMS velocity profile: $\boldsymbol{u}_{rms}^+$ and the penetration frequency: $\boldsymbol{F}$ when there are universal PDFs. A new correction procedure is proposed by using the equation (6).

$$\boldsymbol{u}_{rms}^{+OE} = f_{OE}(\boldsymbol{u}_{rms}^{+}, \boldsymbol{F}) \quad (6)$$

If RMS profile obtained from Eq.(6) agrees with the original RMS profile measured by LDV, input RMS profile can be considered to be expected RMS profile measured with a sufficient small measurement volume by LDV. In this paper, we treat these results as correction results.

DATA ANALYSIS METHOD

In this section, data analysis method for the discussion of turbulence intensity profiles are presented.

Inner peak of turbulence intensity profile

In wall turbulence, it is well-known that turbulence intensity profiles have an inner peak at about y^{+}=15. As described above, LDV measurement for wall turbulence especially in the near wall region, the measurement volume effect on the turbulence intensity becomes significant. By using the present correction method for LDV measurement, the issue of inner peak of turbulence intensity profile is discussed.

Logarithmic behaviour of outer turbulence intensity

We also discuss about characteristic of outer turbulence intensity profiles such as a logarithmic behaviour. In this discussion, we use original turbulence intensity profiles because the effect of LDV measurement volume is almost completely negligible for the outer profiles as presented later. For the logarithmic behaviour of outer turbulence intensity profile, the following relation between the turbulence intensity and the wall normal distance normalized by pipe radius as indicated by equation (7) has been well-known.

$$(u_{rms}^{+})^2 = B_1 - A_1\,log(y/R) \quad (7)$$

Here, according to the research by Marusic, et al. (2017), $A_1 = 1.26$ and $B_1 = 2.10$ were suggested for turbulent boundary layer.

After comparison with equation (7) with two constants previously proposed, to determine the best constants for the present experimental turbulence intensity profiles, below two indicator functions are utilized. The equation (8) indicates an indicator function for a slope of equation (7) for turbulence intensity profile in the logarithmic scale of *y/R*. The equation (9) indicates an indicator function for an additive constant of equation (7) with an arbitrary value of A_1. It is expected that turbulence intensity profile has a logarithmic relation when these profiles calculated by equations of (8) and (9) has a flat region with a constant value. The beginning and the ending positions of the logarithmic relation for the wall normal distance are also discussed by using a plot of the indicator functions of additive constant normalized by an inner and an outer variables for various Reynolds number data.

$$\Xi_1 = \frac{d(u_{rms}^{+})^2}{d[\log\left(\frac{y}{R}\right)]} \quad (8)$$

$$\Psi_1 = (u_{rms}^{+})^2 + A_1 log(y/R) \quad (9)$$

RESULTS AND DISCUSSION

In this section, turbulence intensity profiles measured in the present experiments without correction, corrected results and

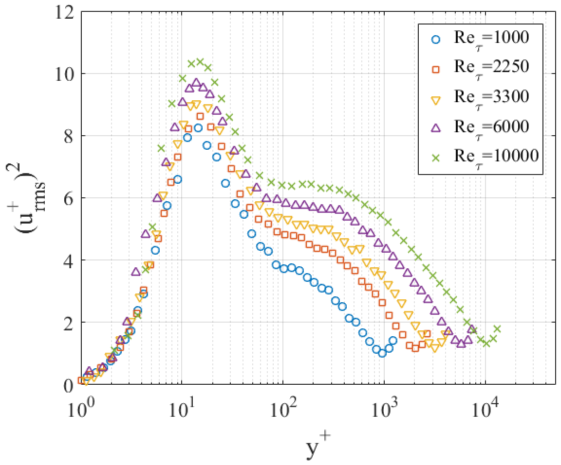

Figure 2. Turbulence intensity profile without correction.

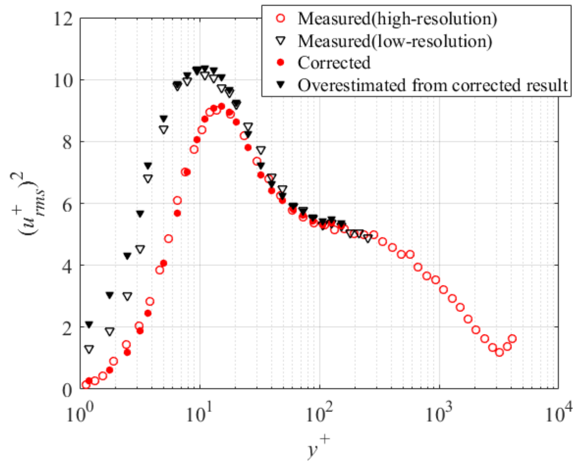

Figure 3. Turbulence intensity profile applied present correction method for LDV measurement.

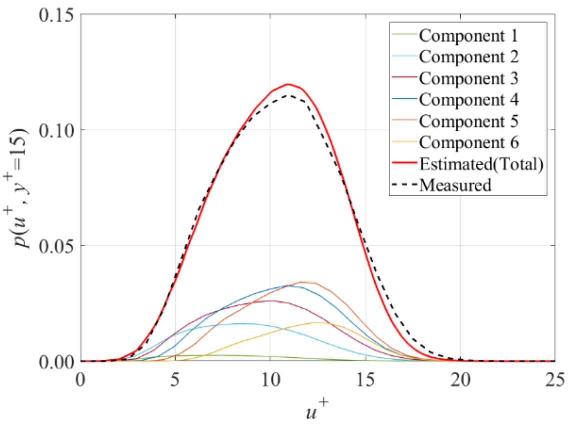

Figure 4. PDF profiles compared between measured and estimated result at Re$_\tau$ =3300.

the issues of the characteristics of turbulence intensity profiles are presented.

Turbulence intensity profiles without correction

Figure 2 shows turbulence intensity profiles measured by LDV under relatively high resolution conditions without any correction at Re$_\tau$ range from 1000 to 10000. It is observed that

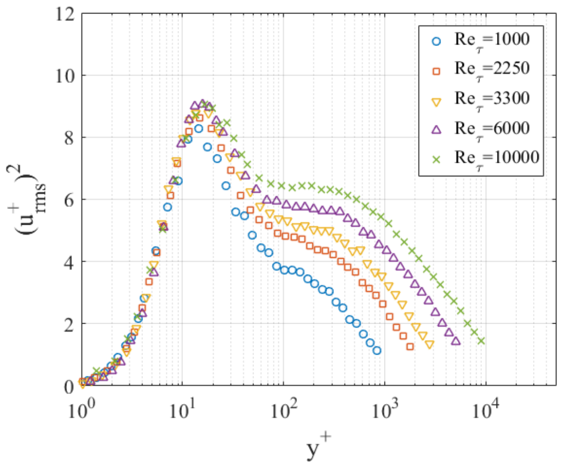

Figure 5. Turbulence intensity profile after correction.

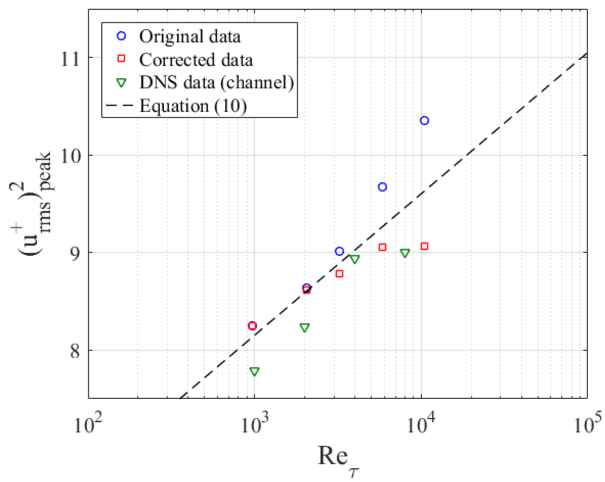

Figure 6. First peak of the turbulence intensity profile plotted against Re_τ.

each profile has a clearly inner peak at y^+=15. However the Reynolds number dependence of the inner peak value is significant. It is considered that the significant Reynolds number dependence should be affected by LDV measurement volume. On the other hand, outer profiles are found to have no significant effect of measurement volume. In the below subsection, the present corrected results are discussed in detail.

Correction results

Figure 3 shows turbulence intensity profiles measured by LDV (high- and low-resolution), corrected profile by the present method and the overestimated result based on equation (6) from the present corrected result at Re_τ = 3300. These profiles are indicated by opened circles and triangles, closed circles and triangles, respectively. Here, we obtained different spatial resolution results due to changing the inclination angle of LDV measurement volume against pipe inner wall. It was confirmed that spatial resolution effect on high-resolution measured results is almost negligible. As can be seen from Figure 3, the difference between low- and high-resolution measured results are found to be significant because of the effect of measurement volume. Applying the present correction method to the measurement data with low-resolution, we obtained corrected result which agrees with high-resolution measured result. It is observed that the overestimated result from the present corrected result based on equation (6) indicated by closed triangles corresponds to the measurement result with low-resolution. From these results, it is concluded that the present correction method works well to improve the measurement volume effect of LDV.

Figure 4 shows PDF profiles compared between the low-resolution measured result and the estimated result by equation (1) based on universal PDF profiles of near wall region. Black dashed and red bold solid lines indicate measured PDF profile and PDF profile calculated by equation (1) at y^+ = 15, respectively. Other fine lines indicate universal PDF profiles of near wall region. As can be seen from figure, the measured profile and the estimated profile which consists of some universal PDF components are found to be overlapped with each other. This result confirms the validity of present correction method based on PDF and measurement volume.

Figure 5 shows turbulence intensity profiles corrected by the present correction method. As can be seen from the comparison between Figure 2 and 5, it is found that Reynolds number dependence of the turbulence intensity profile especially in near wall region is weakened by the correction. It is also clearly observed that outer turbulence intensity profiles are not affected by the present correction method. This means that LDV measurement volume has no effect on the turbulence statistics in the outer region. This Reynolds number dependence and a logarithmic behaviour of outer turbulence intensity profile are discussed in below subsections.

Inner peak of turbulence intensity profile

Applying the present correction method to the measurement results under high resolution condition, expected turbulence intensity profiles are obtained as indicated in Figure 5. Based on these corrected profiles, the inner peak values of turbulence intensity profiles are plotted against Reynolds number in Figure 6. Circles and squares indicate original data and corrected data, respectively. Dashed line indicates Reynolds number dependence as described by equation (10) reported by Marusic et al. (2017). Difference between the present original data measured under relatively high resolution condition and the equation becomes gradually larger as Reynolds number increases. While the corrected inner peak values of turbulence intensity profiles correspond to equation (10) at Reynolds number less than 8000. On the other hand, at Reynolds number larger than 8000, the inner peak values are found to keep an almost constant value which means the inner peak value may be independent of Reynolds number. In pipe flow, Hultmark et al. (2012) have reported that turbulence intensity inner peak values are independent of Reynolds number, and the value is about 9.0. The present results agree with the report by Hultmark, et al.. Figure 6 also shows the peak values of turbulence intensity profiles of channel DNS conducted by Yamamoto and Tsuji (2018). The channel DNS result shows a stronger Reynolds number dependency than that of the present pipe experimental result at relatively low Reynolds number. For relatively high Reynolds number, it is observed that the inner peak values of turbulence intensity profiles are closer to constant value around 9.0 which is corresponds to pipe experimental results. From this fact, it is considered that the internal flow such as pipe and channel is different from the external flow such as boundary layers from the aspect of the near wall turbulence intensity. However, it is noted that the present correction method would give an expected turbulence intensity profile from the turbulence

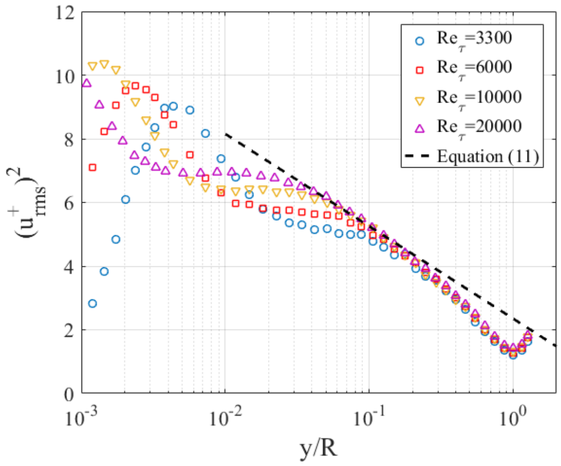

Figure 7. Turbulence intensity profile in outer region.

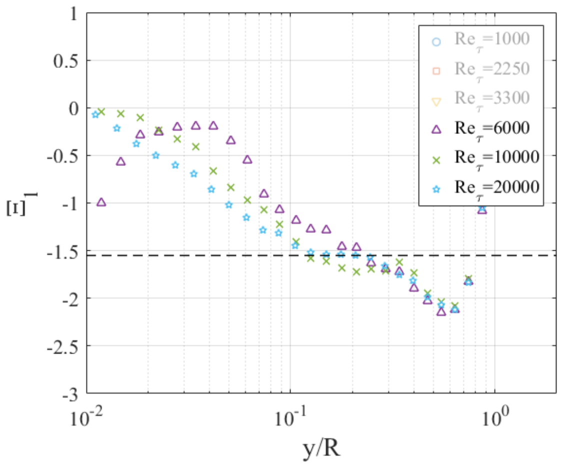

Figure 8. Ξ_1 plotted against wall normal distance normalized by outer variable.

intensity profile affected by measurement volume under the condition that the affected profile was measured with a certain level of accuracy. It is considered that more accurately measurements for high Reynolds number flow are required to conclude that the inner peak of turbulence intensity profile would be independent of Reynolds number at least very high Reynolds number.

$$(u_{rms}^{+})^{2}{}_{peak} = 3.80 + 0.63\log(Re_{\tau}) \qquad (10)$$

Logarithmic behaviour of outer turbulence intensity

We also discuss about the characteristics of the outer turbulence intensity profile such as a logarithmic behaviour. In this discussion, we use original turbulence intensity profile because the effect of LDV measurement volume on the outer turbulence intensity profile is almost negligible as described in the above. In Figure 7, turbulence intensity profiles are plotted against wall normal distance normalized by outer variable of a pipe radius. A difference of symbols and colour variation indicates a difference of Reynolds number. Measured profiles are found to be well collapsed with each other in the outer region. Dashed line indicates the logarithmic behaviour as described by equation (11). Here, C indicates a modified constant which would be dependent on flow field such as pipe,

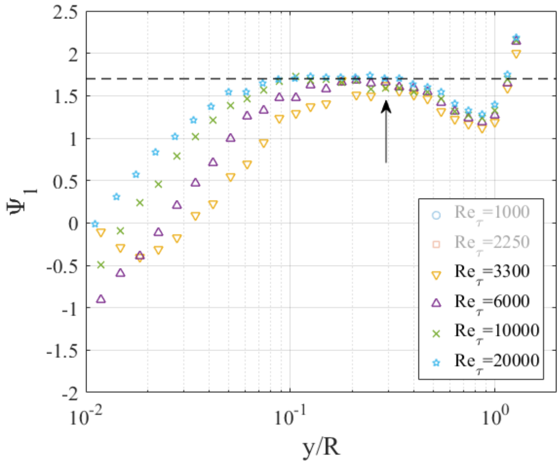

Figure 9. Ψ_1 plotted against wall normal distance normalized by outer variable with $A_1 = 1.55$.

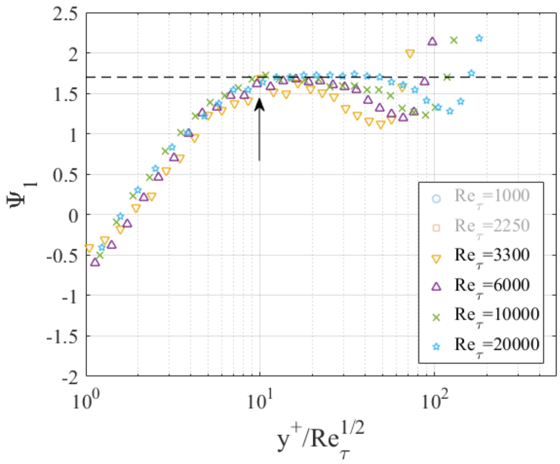

Figure 10. Ψ_1 plotted against wall normal distance normalized by intermediate variable with $A_1 = 1.55$.

channel and boundary layer. For the present results, C becomes approximately 0.25. As can be seen from the comparison between equation (11) and plotted profiles in Figure 7, it is obviously observed that the outer turbulence intensity profile in pipe flow also indicates logarithmic behaviours like other wall turbulence.

$$(u_{rms}^{+})^{2} = 2.10 - 1.26\,log(y/R) + C \qquad (11)$$

From Figure 7, outer profile of turbulence intensity measured in the present experiments are found to almost agree with equation (11) with $A_1 = 1.26$ suggested by Marusic, et al. (2017). However, there is some difference between the present measured profiles and the equation (11). It is considered that the discussion of coefficients of the logarithmic relation is important from the aspect of the dependence of the facility or working fluid. To distinguish the coefficient of the logarithmic relation for the present measured turbulence intensity profiles without any previous reported results, equation (8) is utilized. When the calculated profile has a flat region, this region would be considered as a region in which turbulence intensity can be described by equation (7). Figure 8 shows calculated profiles by using equation (8) for data at high Reynolds number larger than 6000. Dashed line indicates the constant value of -1.55. Relatively low Reynolds number profiles of 6000 and 10000 are not found to have a clearly flat region. On the other hand, the profile at the highest Reynolds number of 20000 is obviously found to have a flat region from approximately 0.12 to 0.24 in *y/R*. The constant value is about -1.55 which is different from 1.26 suggested by the previous study. Based on the present measured data, it is concluded that outer turbulence intensity profiles have a logarithmic relation, while the constant value A_1 is 1.55 in equation (7).

To discuss the beginning and the ending point of the logarithmic behaviour in the outer turbulence intensity profile, the additive constant in equation (7) is calculated by using equation (9) with $A_1 = 1.55$. Figure 9 and 10 shows additive constant profiles plotted against wall normal distance normalized by the outer variable of R and the intermediate variable which is a combination of the inner variable of ν/u_τ and a root of the outer variable of Re_τ, respectively. Dashed line indicates the constant value of 1.70. In each figure, only relatively high Reynolds number data are plotted. In Figure 9, outer side profiles are found to be collapsed, and it is observed that there is a flat region whose constant value is about 1.70. The ending point of the logarithmic relation is considered as equation (12) as indicated by the black arrow in Figure 9. For the beginning point of the logarithmic relation, it is observed that inner profiles in Figure 10 are well collapsed. The beginning point is considered as equation (13) as indicated by the black arrow in Figure 10.

$$y_{ed} = 0.30R \qquad (12)$$

$$y_{st}^{+} = 10\sqrt{Re_\tau} \qquad (13)$$

CONCLUSIONS

Characteristics of turbulence intensity profile in high Reynolds number pipe flow based on LDV measurement data with a correction were presented. The results are summarized as follows. Present correction method gives reasonable results for LDV turbulent pipe flow measurement. Based on the present correction results, turbulence intensity inner peak was found to be dependent on Reynolds number less than 8000 and its trend is close to equation (10) suggested by Marusic et al. (2017). While at higher Reynolds number larger than 8000 the inner peak value is found to keep constant about 9.0 which means the inner peak value may be independent of Reynolds number at least sufficient high Reynolds numbers. The present outer turbulence intensity profiles were found to be collapsed with each other, and they were close to equation (11) suggested by Marusic et al. (2017) with modified constant of 0.25. We also discussed best fit coefficients of the logarithmic relation for the present experimental data by using the indicator functions for a slope and an additive constant of equation (7). These coefficients were obtained as $A_1 = 1.55$ and $B_1 = 1.70$. The beginning and the ending points of the logarithmic relation were assessed by using the outer variable scaling and the intermediate variable scaling, then equations (13) and (12) were obtained for the beginning and the ending, respectively.

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