11th International Symposium on Turbulence and Shear Flow Phenomena (TSFP11)
Southampton, UK, July 30 to August 2, 2019

A HYBRID IMMERSED BOUNDARY/WALL-MODEL BASED LARGE EDDY SIMULATION METHOD

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ABSTRACT

In the present work, a hybrid immersed boundary/wall-model based large eddy simulation method is developed for high Reynolds number turbulent flows with complex/moving boundary. The large eddy simulation equations are solved on a regular Eulerian mesh. The no-slip condition on the wall is kept by imposing continuous forcing of the immersed boundary (IB) method. Implementation of the wall model can be divided into three steps: calculation of local wall shear, imposing local wall shear by reconstructing near-wall subgrid viscosity dynamically and eliminating the disturbance of immersed boundary forcing. The present method is tested in turbulent plane channel flow, turbulent channel flow with a moving wavy wall and turbulent flow over periodic hills. Results show the capability of the present method in simulating flows with complex/moving boundary and massive flow separation.

INTRODUCTION

Turbulent flow with complex/moving boundary is a fundamental problem in natural phenomena and industrial applications. For high Reynolds number turbulent flows, direct numerical simulation (DNS) will cost enormous computational resource. On the other hand, Reynolds-averaged Navier-Stokes (RANS) simulation, which is widely applied in industrial practice, will lost most of the unsteady information. Large eddy simulation (LES) is a compromise approach that resolves large scale structures with subgrid-scale (SGS) models for small-scale turbulent motions. However, resolving the near-wall eddies in turbulent boundary layer is still expensive (Piomelli et al. 2002), which prevents it from many industrial applications in the current stage. Wall-model based LES is a promising approach to overcome this barrier. The wall model reconstructs local wall shear stress and provides it to the outer layer which is resolved in LES (Kalitzin et al. 2008).

When treating flows with complex/moving boundary, the immersed boundary (IB) method becomes an alternative approach which can capture boundary geometry without body-fitted mesh. Generally speaking, there are two categories of the IB method, i.e. the discrete forcing approach and continuous forcing approach (Mittal and Iaccarino 2005), or similarly the sharp interface approach and the diffused interface approach (Yang and Sotiropolous 2014). Combination of wall-model based LES and the IB method would be promising on engineering demand. Attempts have been made by Tessicini et al. (2002) and Cristallo and Verzicco (2006), in which an embedded mesh around the IB was adopted. Later Roman et al. (2009) imposed the logarithmic law into the reconstruction scheme on the IB points. These studies focused on imposing the wall model into the discrete-forcing IB approach.

The purpose of the present work is to develope a wall-model based LES method in the framework of the continuous forcing IB approach, which can take its advantages of easy implementation and solution smoothness for moving boundary problems. The wall model is based on the thin boundary layer (TBL) equations, and dynamic matching of SGS eddy viscosity is applied. Because the IB forcing is exerted within the diffused layer, the disturbance of IB forcing needs to be eliminated by a modification of the SGS eddy viscosity. The test cases selected in the present study are turbulent flows in a plane channel, passing periodic hills and over a moving wavy wall.

Simulation Method

The governing equations are the filtered incompressible Navier-Stokes equations with an IB forcing term and the continuity equation, i.e.

$$\frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial \tilde{u}_i \tilde{u}_j}{\partial x_j} = -\frac{\partial \tilde{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \frac{1}{\mathrm{Re}} \frac{\partial \tilde{u}_i}{\partial x_j \partial x_j} + f_i$$

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0$$

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Here u_i denotes the filtered velocity components, $\tilde{p}$ is the filtered pressure, and τ_{ij} is the SGS stress tensor.

$$\tau_{ij} = \widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j \qquad (2)$$

The effctive viscosity ν_{SGS} is determined by the dynamic Smagorinsky model (Geormano et al. 1991). The flow solver is based on a second-order central finite-difference scheme. A staggered grid strategy is adopted. Velocity and pressure are decoupled by the block LU decomposition (Kim et al. 2002). In the continuous forcing approach (Peskin 2002), the IB forcing term ***f*** is obtained by spreading the Lagrangian momentum forcing ***F***, which is calculated by a feedback law:

$$f_i(\boldsymbol{x},t) = \int_{\Omega} F_i(\boldsymbol{X},t)\,\delta(\boldsymbol{X}-\boldsymbol{x})\,\mathrm{d}\boldsymbol{x}$$

$$F_i = \alpha \int_0^t (\tilde{u}_{IB,i} - U_i)\,dt' + \beta(\tilde{u}_{IB,i} - U_i) \tag{3}$$

Here $\boldsymbol{X}$ and $\boldsymbol{x}$ are the coordinates of Lagrangian and Eulerian grids, δ is the smoothed delta function, and α and β are large free constants. In Eq. (2), $\tilde{u}_{IB,i}$ is the interpolated velocity at the Lagrangian points and U_i is the velocity of the IB. The above equation physically means that fluid particles and the IB points are linked by a stiff spring with damping.

IMPLEMENTATION OF WALL MODEL IN IB METHOD

In the present method, implementation of wall model within the framework of the continuous forcing approach is divided into three steps: calculation of local shear force, dynamic matching of SGS eddy viscosity and modification based on the IB forcing. The local shear is obtained by solving the TBL equations (Balaras et al. 1996; Cabot and Mion 2000):

$$\frac{\partial}{\partial x_n}\left[(\nu+\nu_t)\frac{\partial \tilde{u}_i}{\partial x_n}\right] = F_i$$

$$\frac{\partial \tilde{u}_i}{\partial x_i} = 0$$

$$F_i = \frac{\partial \tilde{u}_i}{\partial t} + \frac{\partial}{\partial x_i}(\tilde{u}_n \tilde{u}_i) + \frac{\partial \tilde{p}}{\partial x_i} \tag{4}$$

where x_n denotes the wall-normal distance. Two simplified versions of the wall model are applied in present work, with $F_i = 0$ and $F_i = \partial \tilde{p} / \partial x_i$. The former one is called the equilibrium-stress model. The model viscosity ν_t is obtained by a RANS-like mixing-length eddy viscosity model with near-wall damping (Cabot and Moin 2000), i.e.

$$\frac{\nu_t}{\nu} = \kappa \mathrm{x}_n^+ (1 - e^{-x_n^+/A})^2 \tag{5}$$

Here the superscript “+” denotes the normalization based on the local wall viscous unit, κ denotes Karman’s constant, and *A*=19. This equation is solved in a body-fitted embedded mesh, which is refined in the wall-normal direction. No-slip boundary condition is imposed at the wall. Upper-boundary conditions of velocity and pressure are interpolated from the nearby Eulerain grid points. By solving the above TBL equation at every time step, the local wall shear τ_w is obtained.

Then the calculated wall shear is inserted into LES solution. Because of the limitation of IB method, the stress boundary condition can not be applied directly.Here a equivalent approach is adopted. Based on the local shear and averaged SGS eddy viscosity on a reference plane above the wall, the near-wall SGS eddy viscosity can be dynamically calculated:

$$\frac{\nu_{SGS}}{\nu} = (\langle \nu_{SGS,r} \rangle / (y_r^{+}(1 - e^{-y_r^{+}/A})^2))\, y^{+}(1 - e^{-y^{+}/A})^2 \tag{6}$$

Here y_r is the normal distance between wall and the reference plane.

However, the IB forcing in the continuous forcing approach is spread over several layers of Eulerian grids, which blurs the boundary and is blended into the SGS eddy viscosity near the IB. Thus, a further modification to the IB forcing is needed. Considering the TBL equation with the IB forcing, a modified SGS eddy viscosity can be calculated by the TBL equation with IB forcing, i.e.

$$\frac{\partial}{\partial x_n}\left[(\nu + \nu_{SGS} + \nu_{IB})\frac{\partial \tilde{u}_i}{\partial x_n}\right] = F_i + f_i \tag{7}$$

In summary, the wall model in the continuous forcing IB method is implemented as follows:

(1) LES velocities are interpolated at the top of the embedded mesh;

(2) The local wall shear stress is calculated at the IB points by solving the TBL equation;

(3) The SGS viscosity is reconstructed at the near-wall Eulerian grid points by dynamic matching;

(4) The near-wall SGS eddy viscosity is modified to eliminate the disturbance of IB forcing.

Result and discussion

Figure 1 shows the mean velocity profile and the velocity fluctuations obtained by the present hybrid method for turbulent channel flow at different Reynolds numbers. Grid numbers in the three directions are (128, 93, 64), with the first grid point above the wall located at $y^+ = 18 \sim 174$. It is seen that the present results show good agreements with the DNS data

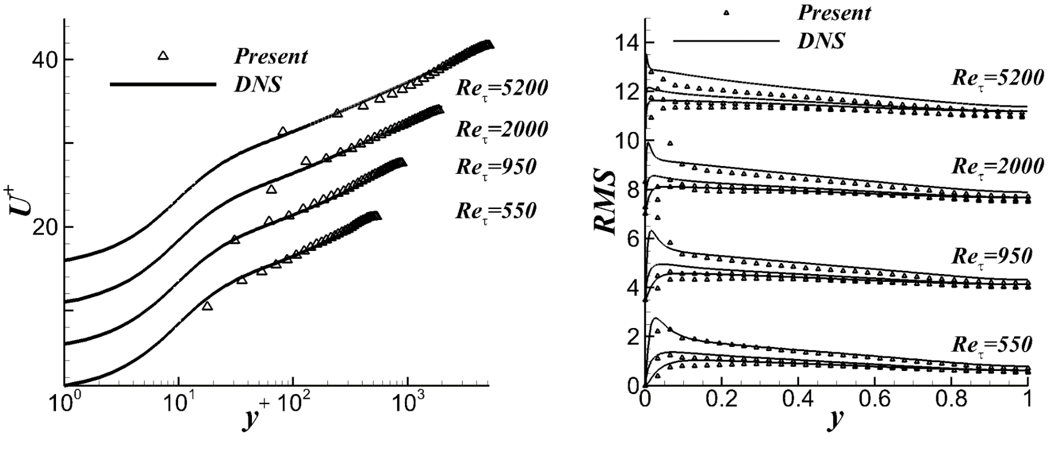

Figure 1. Mean velocity profiles (a) and resolved velocity fluctuations (b) of turbulent channel flow at $\mathrm{Re}_\tau = 550 \sim 5200$. Solid lines and diamond circles denote the DNS data and the present results, respectively.

(Moser et al. 1999, Del Álamo et al. 2004, Hoyas & Jiménez 2006, Lee & Moser 2015). As the Reynolds number increases, the height of first off-wall grid point in wall unit increases, which can no-longer captures the first near-wall peaks of the RMS velocity fluctuations. The streamwise velocity fluctuation is overestimated in the near-wall region and underestimated far from the wall. The normal and spanwise velocity fluctuations are underestimated in the whole flow region. Moreover, it is found that the constant mixing length model without dynamic matching leads to considerably poor results, which is caused by over-prediction of drag. The wall model including the IB forcing modification predicts drag coefficient precisely. When the Reynolds number reaches $\mathrm{Re}_\tau = 5200$, the 'oscillation' of IB forcing will be no longer smoothed by the fluid viscosity (Figure 2), which degenerates the accuracy of wall model. A high-viscosity buffer is then applied beneath the immersed boundary to smooth this 'oscillation'. Velocity predicted after smoothing shows good agreement with DNS data.

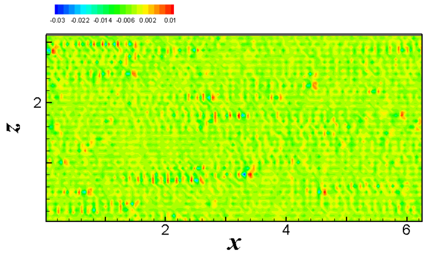

Figure 2. Fluctuation of IB force at $\mathrm{Re}_\tau = 5200$.

Figure 3 shows the mean velocity of the turbulent channel flow with a moving wavy wall, in order to demonstrate the capability of the present method in dealing with the moving boundary problems. Reynolds number is $\mathrm{Re}_b = 10000$, based on the half height of channel δ and bulk velocity U_b . The upper and lower walls are placed near the bottom and top boundaries of the computational domain and mimicked by the IBs. The lower wall is oscillating vertically. The vertical displacement of the lower IB points is prescribed as $\eta(x,t) = 0.5\delta + a \sin k(x - ct)$, where $a = 0.2\delta$ denotes the wave amplitude, $k = 2$ denotes the wave number and $c = 0.4U_b$ denotes the wave speed. As seen in Figure 3, the overall agreement with the resolved LES results (Zhang et al. 2019) is satisfactory. No separation occurs for the mean flow.

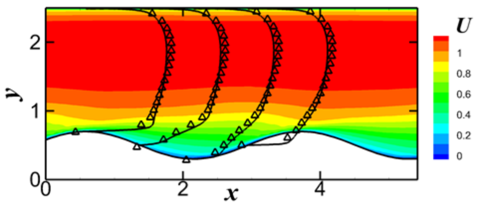

Figure 3. Contours of the mean streamwise velocity of turbulent channel flow with a moving wavy wall at $\mathrm{Re}_b = 10000$ and the mean velocity profiles at four different phases. The wave amplitude is 0.2 of the half channel with, and the wave speed is 0.4 of the bulk velocity. Solid lines and triangle circles denote the fully resolved LES data using the body-fitted mesh and the present results, respectively.

The streamwise component of the Reynolds stresses are plotted in Figure 4. Results of the present method show good agreements with those obtained by the fully resolved LES. There are two near-wall peaks in the vertical direction, which are accurately captured by the present method.

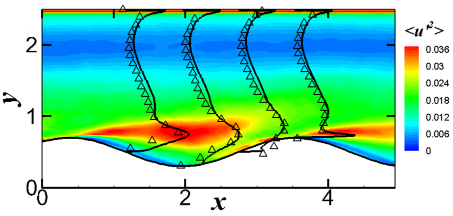

Figure 4. Contours of the streamwise component of the Reynolds stresses of turbulent flow over a traveling wavy wall and the corresponding profiles at four phase locations. Solid lines and triangle circles are the same as Figure 3.

By simulating flow over periodic hills, we test the reliability of the present method in case of large flow separation. Here the local pressure gradient needs to be included in the wall model. The Reynolds number is $\mathrm{Re}_b = 10570$ based on the hill height *h* and the bulk velocity U_b . The computational domain size and grid numbers are (9*h*, 4.035*h*, 4.5*h*) and (256,123,64), respectively, in the three directions. The two walls are mimicked by the IBs in

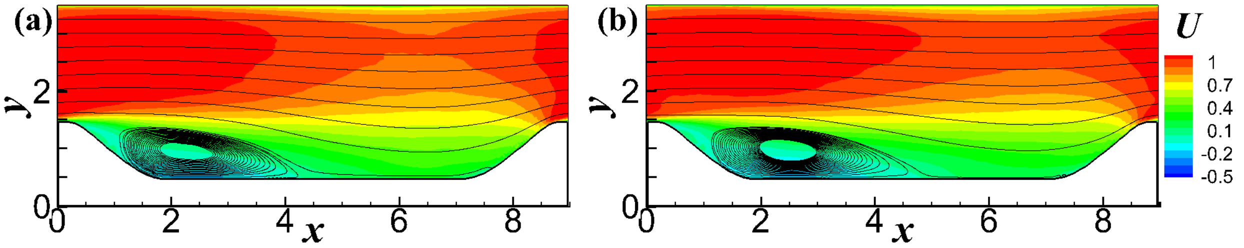

Figure 5. Contours of the mean streamwise velocity and the streamlines of the mean flow field for flow over periodic hills at $\mathrm{Re}_b = 10570$: (a) no wall model; (b) the present method.

conjunction with the wall model. Flow rate is kept constant in simulation. In Figure 5, the separation bubble behind the periodic hill can be seen from the streamlines of the mean velocity field. Result of the present method is compared with that without wall model. It is clearly seen that the size of the separation zone is under-predicted without wall model. The locations of flow separation and reattachment points in different cases are listed in Table 1. By using the present wall model, the reattachment position agrees well with the resolved LES data, which is about 4.7 according to Fröhlich et al. (2005). This result is better than the one predicted by the DES method (Xia et al. 2013). However, the flow separation point is still delayed as compared with the resolved LES result of 0.22.

Table 1. Locations of the separation (x_{sep} / n) and reattachment (x_{reatt} / h) points of the turbulent flow over periodic hills.

Case	x_{sep} / h	x_{reatt} / h
Coarse-mesh LES without wall model	0.5	4.25
Present method	0.35	4.78
Xia et al. (2013)	0.186	4.23
Fröhlich et al. (2005)	0.22	4.72

Conclusions

A hybrid immersed boundary/wall-model method based on large eddy simulation was developed and tested. The IB method is based on the continuous forcing approach. The present method showed good capability of simulating fully developed turbulent channel flow at high Reynolds numbers. Modification of the SGS eddy viscosity has eliminated the disturbance of IB forcing and improved the prediction of wall shear stress. Both the mean velocity and turbulent fluctuations were accurately predicted in the case of turbulent flow over a moving wavy wall. Then the capability of the present method for capturing massive separation was tested by simulating turbulent flow over periodic hills. In conclusion, the proposed method is capable for handling high Reynolds number turbulent flows with complex/moving boundary.

Currently, the work is in progress to test the performance of the present method in cases with more complex boundaries, toward engineering applications. The use of local pressure gradient needs to be checked more carefully.

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