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# SCALE-BY-SCALE ASSESSMENT OF THE EFFECTS OF MEAN SHEAR ON THE ENERGY BUDGET IN DECAYING TURBULENCE

**Md Kamruzzaman**
Department of Energy and Process Engineering
Norwegian University of Science and Technology
Trondheim, Norway
md.kamruzzaman@ntnu.no

**Lyazid Djenidi**
School of Engineering
University of Newcastle,
University drive, Callaghan,
NSW 2308, Australia
lyazid.djenidi@newcastle.edu.au

**Robert A. Antonia**
School of Engineering
University of Newcastle,
University drive, Callaghan,
NSW 2308, Australia
robert.antonia@newcastle.edu.au

## ABSTRACT

The decay of turbulence in a shear layer generated by two side-by-side grids with different mesh sizes and different solidities is investigated using hot wire anemometry. The single-point energy budget is measured across the grid wake and the two-point energy budget is measured on the wake centreline. It is found that the single point energy budget is dominated by the production dissipation and turbulent diffusion in the central region of the flow. it is also observed that the pressure-velocity correlation term becomes important at the edges of the wake. Measurements of the terms of the scale-by-scale energy budget (see Eq. (3) below) showed that while, as expected, the large-scale motion is the sole contributor of energy at large scales, it also contributes to scales of the order of the Taylor microscale. While the actual physical mechanism of this is yet to be determine, this result demonstrates the impact the mean shear can have on the small-scale motion.

## Introduction

The study of a turbulent shear layer (hereafter denoted TSL) is of fundamental importance in understanding a variety of laboratory and geophysical flows. The importance of the subject is reflected in the very large body of work available in the literature far too vast to be covered here. However, and despite this large body of work, there are still open questions that hinder the development not only of the fundamental understanding of turbulence but also the development of effective control strategies for achieving given outcomes such as drag reduction, mixing enhancement/reduction or, on a geophysical scales, how to improve weather predictions as well as pollutant dispersion either in the atmosphere or oceans. One of these questions relates to the effect of the mean shear on a scale-by-scale (SBS) basis, namely, how does the shear alter the energy distribution at all scales of motion, and can this effect be controlled? Several investigations have been carried out in the past to understand the interaction between two different energy containing regions in mixing layer (Wille, 1936; Liepmann & Laufer, 1947; Sato, 1956; Bradshaw, 1966; Wygnanski & Fiedler, 1970; Brown & Roshko, 1974; Hussain & Husain, 1980; Bell & Mehta, 1990; Guo *et al.*, 2009). Brown & Roshko (1974) investigated the effects of the density ratio on the turbulence mixing in the TSL using flow visualization. They concluded that the large scale coherent structures were the fundamental features of the TSL. Despite the large body of work on the subject of TSL, the main focus of the studies was on single point statistics. There is practically no information on the energy distribution on a scale-by-scale basis in the spatial domain. The knowledge of this SBS energy distribution can be relevant for turbulence control for example. Indeed, to device an effective control strategy, one should first assess how the energy introduced in the flow by the large scale motion is sifted among all scales of motion. Such knowledge should help select and target a range of scales most susceptible to respond to a given control or actuation.
In order to assess how the mean shear affects the flow at all scales of motion, one should try to isolate the effect of the mean shear from other phenomena such as anisotropy, for example. One possibility to achieve this is to investigate turbulence behind by two classical grids of different mesh sizes and solidities put side-by-side. The turbulence generated by flowing a uniform fluid through the two side-by-side grid should be initially approximately homogeneous and isotropic on either side of the grid of the centreline with different turbulent kinetic energy. Since the mean velocities on either side of the centreline are different, a mean velocity gradient develops downstream of the grids.
In the present work, whose main aim is to assess the effect the mean shear can have on the energy transfer among the scales of motion, in particular both the dissi-

pative and scaling ranges, we carry out a SBS analysis of the transport equation for $\overline{(\delta q)^2}$, where $\overline{(\delta q)^2} = (\overline{(\delta u)^2} + \overline{(\delta v)^2} + \overline{(\delta w)^2}); (\delta u_i = u_i(x+r,t) - u_i(x,t)$ is the velocity increment of the $i$-component of the velocity and $r$ is the longitudinal spatial increment; the overbar represents time average). The SBS energy budget equation was tested experimentally in grid turbulence Danaila *et al.* (1999); Hill & Boratav (2001), on the axis of a turbulent round jet (Burattini *et al.*, 2005), and the centreline of a turbulent channel flow (Danaila *et al.*, 2001), where the mean shear is either negligible or zero. Also, analyses based on the SBS energy budget equation were extensively used to investigate self-preservation solutions and their consequences in different turbulent flows (Thiesset *et al.*, 2014; Djenidi *et al.*, 2015*b*; Tang *et al.*, 2015; Djenidi *et al.*, 2015*a*, 2017). The present study is a further exploitation of the SBS analysis to gain further insight into the physics of a TSL.

## Experimental set up

The experiment is carried out in an open circuit wind tunnel. The air flow is driven by a centrifugal blower, which is controlled by a variable-cycle (0-1500 rpm) power supply. To minimise vibration, the blower is supported by dampers and is connected to the tunnel by a flexible joint. The wind tunnel has a working section with dimensions 2.4 m × 0.35 m × 0.35 m. Measurements are carried out using hot-wire anemometry. Single and X-hot wires are used to measure the $x$ (streamwise), $y$ (vertical) and z (spanwise) components of the velocity, $u$, $v$ and $w$, respectively; the mean shear is in the $y$ direction. The diameter of the sensitive part of the hot wires is $d = 2.5\mu m$ and their length is $l = 200d$. The hot wires are made of coil of Wollaston (platinum) and operated using an in-house constant temperature anemometer (CTA) at an overheat ratio of 1.8.

The turbulent shear layer is generated by a "composite" grid, which is a perforated plate with square holes of different sizes on either side of the midsection of the grid (Figure 1). The mesh size is $M_L$ = 19.9 mm on the upper half and $M_S$ = 10.2 mm on the lower half (the subscripts $S$ and $L$ hereafter refer to the small and large sections of the grid). The solidity $\sigma$ is 30% and 43%, respectively, leading to $U_1 = 9.2 m/s$ and $U_2 = 6.2 m/s$, for the lower and upper freestream velocities, respectively. Measurements were taken across the shear layer and over a distance $x$ ranging from $20M_L$ to $85M_L$ downstream of the grid. Figure 2 shows examples of normalized mean velocity profiles at various position downstream of the grid. Notice the perfect collapse of the profiles, indicating that the TSL is evolving in a self-preservation state.

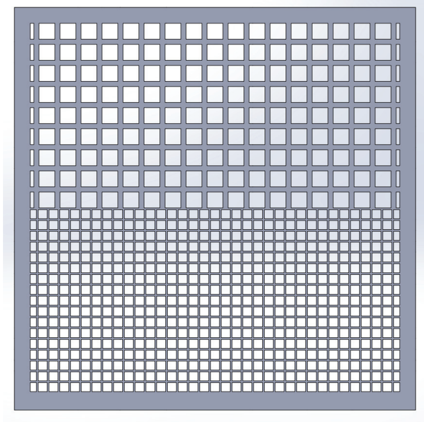

Figure 1. Grid used to initialize the shear layer. The upper and lower parts of the grid are of different solidity.

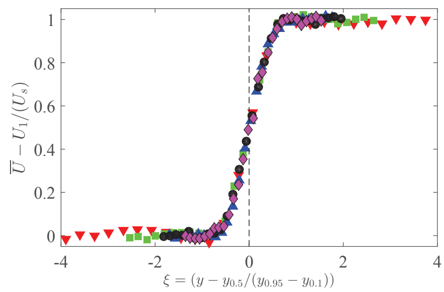


Figure 2. Normalised mean velocity profiles at different downstream locations, $x/M_L = 20$, 40, 60, 65 and 85.

## Results

### Turbulent kinetic energy budget for $q^2$

We first present the steady state budget of the turbulent kinetic energy (TBK), $q^2$, written as:

$$\frac{1}{2}\overline{U_j}\frac{\partial \overline{q^2}}{\partial x_j} = -\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - \frac{1}{2}\frac{\partial \overline{q^2 u_j}}{\partial x_j} - \frac{1}{\rho}\frac{\partial \overline{p u_j}}{\partial x_j} + \frac{1}{2}\nu\frac{\partial^2 \overline{q^2}}{\partial x_j \partial x_j} - \nu\overline{\left(\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}\right)} \quad (1)$$

where $\frac{1}{2}\overline{q^2} = \frac{1}{2}\overline{u_i u_i}$ is the total kinetic energy. The term on the left side of the equation represents the advection, and the terms on the right side represent, respectively, the production, turbulent diffusion, pressure velocity correlation, the molecular or viscous diffusion and turbulent mean kinetic energy dissipation ($\varepsilon$); the overbar represents time averaging. For the present flow, (1) reduces to

$$\frac{1}{2}\overline{U}\frac{\partial \overline{q^2}}{\partial x} + \overline{uv}\frac{\partial \overline{U}}{\partial y} + \frac{1}{2}\left[\frac{\partial \overline{q^2 u}}{\partial x} + \frac{\partial \overline{v q^2}}{\partial y}\right] + \frac{1}{2}\left[\frac{\partial \overline{pu}}{\partial x} + \frac{\overline{pv}}{\partial y}\right] + \varepsilon = 0 \quad (2)$$

Note that since the molecular diffusion was found to be negligible across the TSL it was dropped from the equation. All terms, except for the pressure-velocity correlation term, were measured; this latter term is obtained by balancing the TNK equation. A comment is warranted for the measurement of $\varepsilon$, which strictly requires the measurement of twelve terms, which is practically impossible when using hot-wire anemometry. One is required to use a surrogate for $\varepsilon$. One first choice is the local isotropic form of $\varepsilon$, $\varepsilon_{iso} = 15\nu\overline{(\partial u/\partial x)^2}$. In order to assess if this surrogate is adequate here, we tested the local isotropy (LI) of the flow. We thus ascertained the LI hypothesis by calculating the ratio $\alpha = \overline{(\partial u/\partial x)^2}/\overline{(\partial v/\partial x)^2}$ which should be equal to 0.5 if LI is satisfied. It is found that for $x/M_L = 60$, $\alpha =$ 0.502, 0.511, 0.528, 0.506 and 0.522 at $(y - y_c) = -83$, -37, 0, 37 and 83, respectively; $y_c$ corresponds to the grid

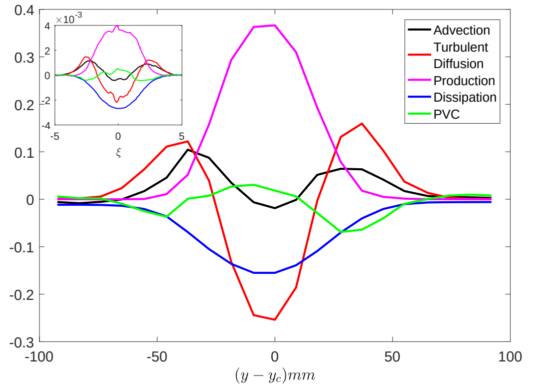


Figure 3. Terms of the transport equation for $q^2$ at $x/M_L = 60$, normalised by $U_S^3/x^{-1}$. Inset: DNS data of Rogers & Moser (1994), normalised by $U_S^3$ and $\delta_m$, the momentum thickness of the mixing layer.

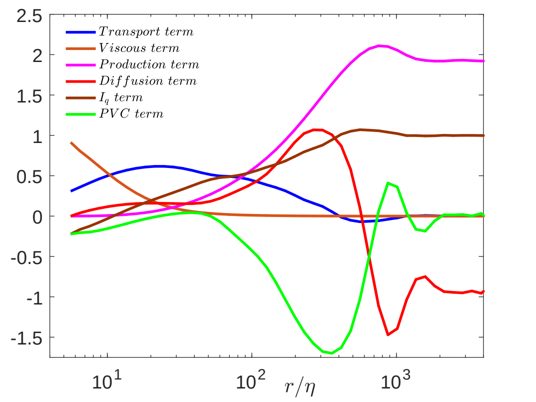


Figure 4. Different terms of the SBS energy budget (Equ. 3) on the TSL centreline. The data are normalized by $(4/3)\overline{\varepsilon}r$.

centreline. This yields an average of approximately 0.514 across the TSL, which is only 2% higher than the isotropic value, indicating that LI is relatively well approximated in the present TSL and that $\varepsilon_{iso}$ is an adequate surrogate $\varepsilon$, thus providing confidence in the measurement of the TKE budget. Of interest, we estimated the difference between $\overline{\varepsilon}_{iso}$ and $\overline{\varepsilon}$ obtained using the spectral chart method (see, Djenidi & Antonia (2012)) on the grid centreline and found to be only 7%, lending support to the assumption of local isotropy.

The measurement of the TKE budget across the TSL is reported in Figure 3 for the downstream position $x/M_L = 60$. The molecular diffusion is practically zero across the TSL, which, as mentioned earlier, was the reason it was dropped from the TKE equation. We also show in the figure (see inset) the DNS data of a temporally evolving turbulent mixing layer (Rogers & Moser (1994)) for comparison. There is a remarkable similarity between the present budget and that of the temporally evolving turbulent mixing layer. The production is important across the TSL. The turbulent diffusion, advection and pressure terms appear to act as mechanisms for redistributing the energy produced around the TSL centreline. In particular, the turbulent diffusion plays a significant role in transferring the energy from the TSL centreline towards the edges. We also observe this mechanism in the DNS data.

Figure 3 illustrates clearly the effect of the non-homogeneity on the TBK budget. However, it only shows how the redistribution of energy contained by the large scales of motion across the TSL is impacted by the mean shear. As mentioned in the introduction, one would like to assess how the energy at all scales of motion is affected by the non-homogeneity in order to device appropriate turbulence control strategies. Thus, we now turn our attention to the scale-by-scale energy budget on the centreline of the TSL, which should provide us with information on the effect of the mean shear on the energy redistribution on a scale-by-scale basis.

## Scale-by-scale energy budget on the centerline of TSL

Following Danaila *et al.* (2001), this budget equation, which represents the transport of $\overline{(\delta q)^2}$, can be expressed as follows

$$
\begin{aligned}
&\underbrace{-\overline{(\delta q)^2(\delta u)}}_{\text{transport}} + \underbrace{2\nu\frac{d}{dr}\overline{(\delta q)^2}}_{\text{viscous}} \\
&\underbrace{-\frac{U}{r^2}\int_0^r s^2\frac{\partial}{\partial x}\overline{(\delta q)^2}ds}_{\text{advection}} \underbrace{-\frac{2}{r^2}\frac{dU}{dy}\int_0^r s^2\overline{(\delta u\delta v)}ds}_{\text{production}} \\
&\underbrace{-\frac{1}{r^2}\int_0^r s^2\frac{\partial\overline{(v(\delta q)^2)}}{\partial y}ds}_{\text{turbulent diffusion}} + PVC = \underbrace{\frac{4}{3}\overline{\varepsilon}r}_{\text{energy dissipation}} \quad , \quad (3)
\end{aligned}
$$

where the assumption of LI was used. As for the single point budget equations, all terms of (3) but the pressure term (PVC) were measured; the PVC term is obtained by balancing the equation. Figure 4 shows all the terms of (3) on the grid centreline. We also report a term denoted $I_q$ which represents the total contribution of the non-homogeneous terms (advection, production, turbulent diffusion and PVC) of (3) and evaluated by subtracting the right side to the summation of first and second terms of the left side.

As expected, the viscous term dominates the dissipative range ($r/\eta \to 0$), the transport term is important in the range $10 \le r/\eta \le 100$ and the overall contribution from the integral terms, represented by the $I_q$ term, dominates the budget in the large scale range. While the viscous and transport terms present "classical" behaviour (the viscous term is maximum in the dissipative range and decreases with increasing $r$, the transport term is maximum in the scaling range and drops to zero at small and very large $r$), the $I_q$ term shows a rather uncharacteristic behaviour in the range $10 \le r/\eta \le 100$; expect for grid turbulence (not shown here), it exhibits a noticeable hump-like behaviour. Considering that no such such hump exists in the $I_q$ term measured in a classical grid turbulence, this behaviour is likely to stem from the balance between the production, PVC and diffusion terms; these terms are not present in the grid turbulence. Noteworthy is the strong "oscillation" of both the PVC and diffusion terms; both terms reach their maximum magnitude for $r/\eta \sim 300-400$ and present an antiphase-like behaviour, suggesting a possible (anti)correlation between the two mechanisms, as if they act against each other, arguably, leading to the development of the hump in the $I_q$

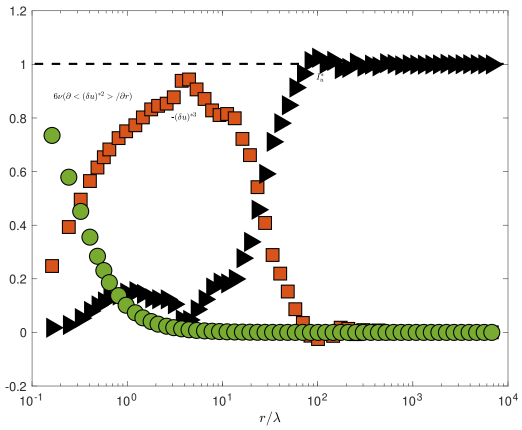


Figure 5. Different terms of the SBS energy budget (Equ. 4) in a rough wall turbulent boundary layer at $y/\delta = 0.23$ (Kamruzzaman *et al.* (2015). The asterisk represents normalization by $(4/3)\overline{\varepsilon}r$.

term. The "uncharacteristic" behaviour of the $I_q$ term is reminiscent of that observed (as shown here in Figure 5) in its counterpart $I_u$ term on the SBS budget for $\overline{(\delta u)^2}$ (Eq. (4) below) in a turbulent boundary layer

$$-\overline{(\delta u)^3} + 6\nu \frac{d}{dr}\overline{(\delta u)^2} + I_u = \frac{4}{5}\overline{\varepsilon}r. \qquad (4)$$

Not shown here, the hump exhibited by $I_u$ is observed for different y-positions ($y/\delta = 0.029, 0.23, 0.3$ and $0.5$) in the boundary layer. The hump in the boundary layer is more clearly defined than in the present TSL, perhaps reflecting a stronger effect of the mean shear on the small scales.

## 1 Conclusions

Hot-wire velocity measurements have been made in a turbulent shear layer (TSL) generated by a composite grid made of two side-by-side grids with different mesh sizes and different solidities. The emphasis of the study, which is still in progress, is to assess the effect the mean shear can have on the energy transfer among the scales of motion, in particular both the dissipative and scaling ranges. To carry out the analysis, we measured the terms of the SBS energy budget transport equation on the centerline of the TSL. The results showed that while, as expected, the large scale motion is the sole contributor of energy at large scales, it also contributes at scales of the order of the Taylor microscale. This contribution appears to result from a balance between the production, the turbulent diffusion and correlation velocity-pressure at these small scales. The energy contribution of large sale to the small scale is similar to that observed in a turbulent boundary layer.
It is clear that further measurements and analysis on and off the centreline of the TSL are require to unravel the actual physical mechanism by which the large scale motion affect the smaller ones. Further, while the actual physical mechanism is yet to be determined, the present results illustrate the impact the mean shear can have on the small scale motion. This may have important implications for the development of control/management strategies of turbulent flows.

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