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FLOW STRUCTURE AND TURBULENCE MODIFICATION BY EVAPORATING DROPLETS IN A SWIRLING TWO-PHASE CONFINED FLOW

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ABSTRACT

This contribution presents the mathematical model to simulate the swirling turbulent gas-droplet flow in a sudden pipe expansion. The dispersed phase is modeled by the Eulerian approach. The set of 3D steady-state RANS equations for the two-phase flow is utilized. The numerical investigation of the effect of the swirling parameter of the flow and the thermophysical properties of the water, ethanol, and acetone droplets on the mean flow structure, turbulence modification and heat transfer in the droplet-laden flow is carried out. The flow swirling causes an increase in the intensity of heat transfer (more than 1.5 times in comparison with the non-swirling mist flow at other identical inlet conditions). Large particles are located in the near-wall region of the channel due to the action of centrifugal forces. Smaller ones are in the central recirculation zone of the chamber. Droplets that appear in the axial separation region cannot leave it, as the level of turbulence in the shear layer is higher than that of the dispersed phase.

INTRODUCTION

Two-phase droplet-laden flows with separation, swirling and rotation are frequently encountered flow phenomena in various practical applications: gas and coal combustor chambers, flow in turbomachinery parts, centrifugal separators, etc. Turbulent single-phase flows with separation and swirling are characterized by high local gradients of the mean and fluctuational parameters due to the action of centrifugal and Coriolis forces. Interaction between the particles and gas phase turbulence is a very complex process (Fessler and Eaton, 1999). Evaporation of the droplets is one of the ways to achieve flow control method and significant enhancement of heat transfer (Hishida et al., 1995). The presence of flow recirculation and swirling affects the intensity of momentum, heat, and mass transfer and strongly determines the structure of the turbulent flow. Therefore, despite a widespread use of two-phase swirling flows in various practical applications, the processes of turbulent mixing and transport in such flows remain insufficiently studied.

The thermophysical properties of liquid droplet can be affected by the intensity of the mixing, turbulence modification and heat transfer. The latent heat of evaporation differs significantly for various liquids (for example, water and alcohol and acetone) and the droplets diameter of these materials varies markedly due to the evaporation. The particle diameter is one of the important parameter effecting the turbulence modification of the carrier gas phase (Crowe, 2000).

The aim of the present study is to numerically examine the effect of the droplets and swirl number on flow, turbulence and heat transfer in a two-phase mist swirling flow in a pipe with sudden expansion for liquids with various latent heat of evaporation. The current study is a continuation of (Pakhomov amd Terekhov, 2017), in which the SMC model was applied to describe the gas-droplet swirling flow downstream of a sudden pipe expansion.

MATHEMATICAL MODEL

Numerical Model

The dispersed phase is modeled by the Eulerian approach, which treats the dispersed phase (droplets) as a continuous medium (gas) with properties analogous to those of a fluid (Drew, 1983; Derevich and Zaichik, 1988; Reeks, 1991). This technique involves the solution of a second set of Navier–Stokes-like equations in addition to those of the carrier (gas) phase. The flow is treated as a steady-state, incompressible, axisymmetrical and swirling with negligible mass forces. The mean flow is described by the continuity, three-momentum energy equations and the equation of vapor diffusion into the binary gas-vapor mixture. In the present study, the elliptic blending Reynolds stress model (Fadai-Ghotbi et al., 2008) is employed. The back effect of particles on the carrier phase turbulence (two-way coupling) is considering (Derevich and Zaichik, 1988; Zaichik, 1999).

The droplets' behavior in turbulent fluid and their back action on the flow is determined by drag, gravity force, turbulent transport and turbulent diffusion. In the present paper, the dilute droplet-laden swirling flow downstream of a sudden pipe expansion is numerically examined (Pakhomov and Terekhov, 2017a). The volume concentration of the dispersed phase is assumed to be lower than ($\Phi_1 < 10^{-4}$) and sufficiently fine ($d_1 < 100$ μm); therefore, according to (Elghobashi, 1994), the effects of inter-particle collisions are neglected when treating the hydrodynamic and heat and mass transfer processes in the two-phase flow. Break-up or droplet deformation processes are not taken into account. The Weber number We << We_{cr} = 7, shows that this effect can be eliminated. Therefore, neither break-up nor droplet deformation can occur in the flow (Lin and Reitz, 1998).

The droplet is the sink of heat and source of steam, and the heat is expended in heating it up leading to evaporation. All computations are performed for monodisperse gas-droplet flow at the inlet cross-section at a uniform wall temperature T_W = 373 K. Then the size of the droplets changes in all directions due to their evaporation. The pipe surface is always dry, so there is no liquid film or spots formed on the wall. This can be used if $T_W - T_{WL} \geq 40$, where T_{WL} is the droplet temperature at the wall conditions.

Numerical Procedures and Boundary Conditions

The mean transport equations for both gas and dispersed phases and the SMC model are solved using a control volumes method on a staggered grid. The QUICK scheme is used to approximate the convective terms, and the second-order accurate central difference scheme is adopted for the diffusion terms. The velocity correction is used to satisfy continuity through the SIMPLEC algorithm, which couples velocity and pressure. The model employs in-house code to simulate droplet-laden turbulent swirling flow in a pipe with sudden expansion.

The results of preliminary calculations for single-phase flow in a pipe with a length of 150R are used for the gas-phase velocity and turbulence on the pipe edge. These conditions are sufficient to achieve fully developed turbulent gas flow. The symmetry conditions are set on the pipe axis for gas and dispersed phases. No-slip conditions are set on the wall surface for the gas phase. Boundary conditions on the wall surface for the dispersed phase correspond to the conditions of an "absorbing surface" [20], under which droplets do not return to the flow after making contact with the solid wall. After their deposition onto the pipe surface, the droplets are assumed to be momentarily vaporised and the wall surface is always dry (Pakhomov and Terekhov, 2017a). At the outlet edge, the computational domain condition $\partial\varphi/\partial r = 0$ is set for all variables.

NUMERICAL RESULTS AND ITS DISCUSSION

The swirling gas-droplet flow is studied in the downward flow regime downstream of a sudden pipe expansion. The computation domain is schematically presented in Fig. 1. The main jet of the mixture of air and droplets (*1*) is supplied into the central channel (2R_1). The swirling single-phase gas flow (*2*) arrives through the annular channel (R_3−R_2). The computation domain geometry is as follows: $2R_1$ = 20 mm, $2R_2$ = 26 mm, $2R_3$ = 40 mm, $2R_4$ = 100 mm, and the step height is H = 30 mm. The computational domain length X = 1 m. The mean mass axial velocity of the main air flow U_{m1} = 15 m/s and its mass flow rate is G_1 = 22.6 g/s. The mass flow rate of the air in the secondary annular jet G_2 = 18 g/s. The flow swirl number varies within S = 0–1. The gas-phase Reynolds number is $\mathrm{Re} = U_{m1}2R_1/\nu = 2\times10^4$. The initial mean axial droplet velocity U_{L1} = 12 m/s. The initial water droplet diameter d_1 = 10−100 μm and their mass concentration varies within M_{L1} = 0−0.1. Then the size of the droplets changes in all directions due to their evaporation. The particle relaxation time τ = 0.3−30 ms. For the conditions of our predictions, the turbulent time scale is $\tau_f = 5H/U_{m1}$ = 10 ms and mean Stokes number $\mathrm{Stk} = \tau/\tau_f$ = 0.03−3. All computations are performed for monodisperse gas-droplet flow at the inlet cross-section at a uniform wall temperature T_W = 373 K. The gas and droplet temperature at the initial cross-section $T_1 = T_{L1}$ = 293 K.

Flow Structure

The effect of droplet thermophysical properties on the distributions of droplets volume fraction along the pipe axis (a) and radial profiles at x/H = 5 (b) in swirling two-phase mist flow is shown in Fig. 2, where Φ_1 is the droplets volume fraction at the inlet. Fine-dispersed droplets Stk = 0.3 (d_1 = 30 μm) are well involved in the recirculation flow and can be presented practically throughout the entire cross section of the pipe. The wall region of the pipe (r/H > 1.2) is practically free of particles due to their evaporation. The dispersed phase accumulates in the near-axis region of the flow due to the action of turbulent migration (turbophoresis force) in swirling flow. The magnitude of water droplets volume fraction has the highest value, while that for acetone is the lowest value. This is due to the significant difference in the latent heat of evaporation of water and acetone.

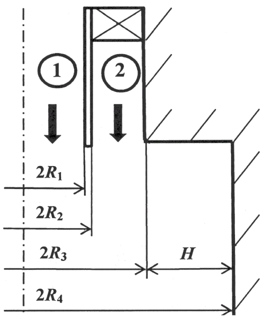

Figure 1. The scheme of the computational domain.

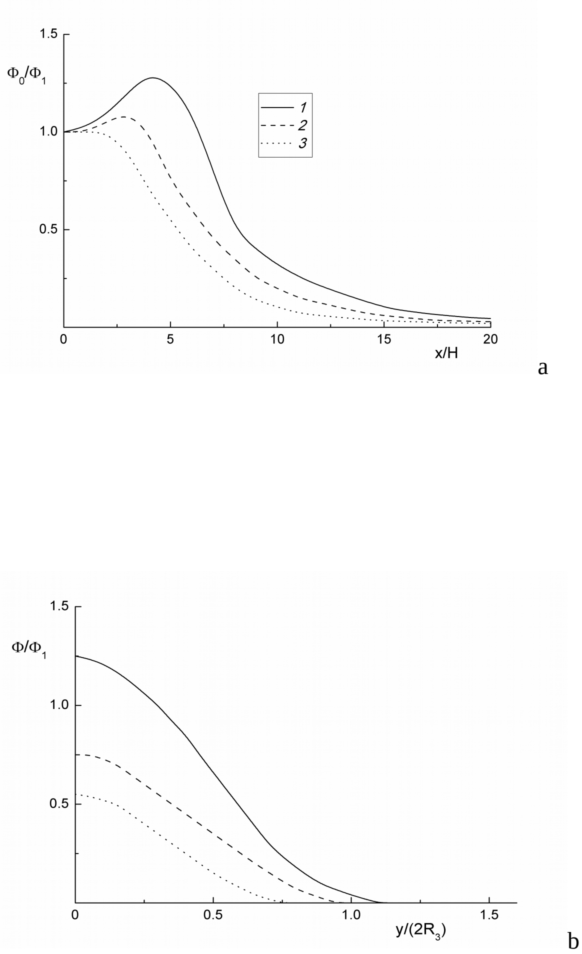

Figure 2. The distribution of droplets volume fraction along the pipe axis (a) and radial profiles at x/H = 5 (b) in swirling two-

phase mist flow for various droplets material. Re = 2×10^4, d_1 = 30 μm, $M_{L1} = 0.05$, $S = 0.5$. *1* – water, *2* – ethanol, *3* – acetone.

It well known the rate of evaporation of droplets has a major effect on the diameter of the particles and, accordingly, on the intensity of heat transfer. It depends on the value of latent heat of the phase transition and also affects the distribution of the fraction of the dispersed phase (see Hishida et al., 1995; Zaichik, 1999; Pakhomov and Terekhov, 2017b).

The distribution of the concentration of the dispersed phase has a key effect on the modification of gas turbulence (Fessler and Eaton, 1999). The numerical results of the effect of the mass fraction of water droplets (*1*), ethanol (*2*) and acetone (*3*) on the maximum values of the modification of the gas phase turbulence $\mathrm{MR}_{\max}$ in the droplet-laden swirling flow show in the Fig. 3. Here $\mathrm{MR}_{\max} = (k_{S,\max}/k_{S,0\max})$, where $k = \langle u_i u_i \rangle / 2$ is the turbulent kinetic energy (TKE) of the carrier phase and subsctipt "*S*" is the swirling flow parameter. An increase in the amount of the dispersed phase leads to a decrease in the level of gas turbulence in a two-phase flow up to 20% at $M_{L1} = 0.1$. The main reason of it the involvement of the fine-dispersed droplets in the turbulent motion of the gas. This agrees qualitatively with the data of (Fessler and Eaton, 1999; Crowe, 2000; Li et al., 2010) for gas-dispersed turbulent flow in a vertical pipe. It is seen the use of acetone has the least effect on the intensity of gas phase turbulence in comparison with the corresponding value for water droplets (up to 10%). An increase in the swirl number leads to an intensification of heat transfer for water droplets. It has been shown in our recent article (Pakhomov and Terekhov, 2017a). This is explained by the fact an increase in the swirl parameter intensifies the process of heat and mass transfer during the evaporation of the droplets and therefore it leads to a decrease in the suppression of the turbulence level of the carrier gas phase. Using ethanol as a coolant has less effect on the intensity of the turbulence of the gas phase as compared with that value for water droplets (up to 10%). Similar results were obtained by the authors of this work for ethanol and acetone droplets.

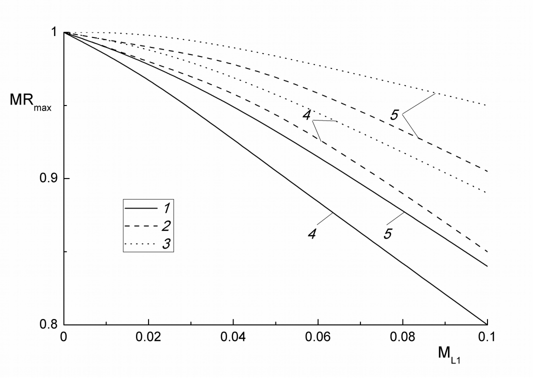

Fig. 3. The effect of droplets thermophysical properties on the maximal turbulence modification ratio $\mathrm{MR}_{\max}$ vs droplets mass fraction. *1* – water, *2* – ethanol, *3* – acetone, *4* – $S = 0.25$, *5* – $S = 0.5$.

Radial distributions of TKE of the gas phase for various Stokes numbers in the mean motion are presented in the Fig. 4. The predicted results are shown for three stations $r/H = 0$, 0.5 and 1. First two stations are situated in the turbulent flow core and the last one is in the near-wall zone (recirculation area). The Stokes number in the mean motion Stk = τ/τ_f is an important dimensionless parameter defining how the particle interact with the mean turbulent flow (Hishida et al., 1995; Fessler and Eaton, 1999), which is given by the ratio of the particle relaxation time τ to the characteristic time of the fluid motion τ_f. Here $\tau = \rho_L d_1^2 (18\mu W)$, $W = 1 + \mathrm{Re}_L^{2/3}/6$ is the deviation from the Stokes power law and $\mathrm{Re}_L = |\boldsymbol{U} - \boldsymbol{U}_L| d_1/\nu$ is the Reynolds number for the dispersed phase and $\tau_f = 5H/U_1$ is the turbulent time scale (large-eddy passing frequency) (Fessler and Eaton, 1999; Li et al., 2010), ρ_L is the density of droplet, d_1 is the initial droplet diameter, $\boldsymbol{U}$ and $\boldsymbol{U}_L$ are mean velocities of the gas and dispersed phases, and μ and ν are dynamic and kinematic viscosity of the gaseous phase. The dispersed phase with small Stokes number Stk << 1 is found to be in velocity equilibrium with the gas phase. Particles are good responsive to gas phase fluctuations and attenuate the turbulent energy of the gas phase throughout the pipe cross-section (Fessler and Eaton, 1999; Li et al., 2010). The particles with large Stokes number Stk >> 1 are found to be no longer in equilibrium with the fluid phase [1, 12–14]. They are unresponsive to gas phase fluctuations and will move unaffected through gas eddies.

A sharp decrease in TKE is characteristic for Stk < 1 in the axial part of the pipe ($r/H = 0$ and 0.5), because in this area the droplets decrease in size slightly due to weak evaporation processes. Then the effect of droplets inertia on TKE decreases due to the large its diameter. A more complex effect of droplets on the turbulence modification is noted in the near-wall region of the pipe at $r/H = 1$. A minimal effect of the presence of an evaporated dispersed phase on carrier gas turbulence is obtained in this region due to intensive processes of droplets heat up and evaporation (up to 6%). Initially the increase in the mean Stokes number (droplet size) causes a decrease in the TKE level and a maximum value for suppressing turbulence for all types of particles. Large particles practically do not penetrate into the region of the separated recirculation flow. The mass concentration of droplets in the separation zone is substantially reduced by the further growth of the Stokes number (Stk > 1). The near-wall region of the pipe ($r/H = 1$) is almost free from particles (see Fig. 4) due to their evaporation (at low Stokes numbers of Stk < 1) and the absence of dispersed phase in the separation zone (for Stk > 1). It leads to the reduction of the suppression level of gas phase turbulence and $k/k_0 \to 1$. The value of the modification ratio is kept as $k/k_0 = 1$ in the whole range of large mean Stokes numbers. The size of evaporating droplets in this zone is reduced by evaporation, so the smaller particles can be entrained by the turbulent motion, whereas the particles without evaporation are not involved in the turbulent motion and therefore the value of the turbulence energy modification ratio is $k/k_0 \approx 1$, and the turbulence of the gas phase increases in this region. These conclusions are obtained for all the droplets materials studied in the work.

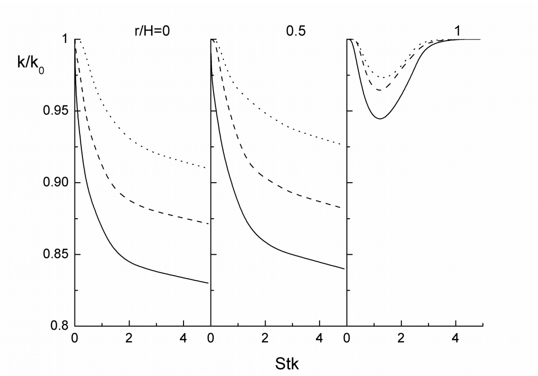

Fig. 4. The effect of Stokes numbers on TKE modification ratio at r/H = 0, 0.5 and 1. M_{L1} = 0.05, S = 0.5, x/H = 5. *1* – water, *2* – ethanol, *3* – acetone

Heat Transfer

The local Nusselt number distributions show along the longitudinal coordinate in the swirling flow behind a sudden pipe expansion for various liquids in the Fig. 5. The local Nusselt number at a constant wall temperature was determined by the difference between the wall temperature and the mean-mass temperature of gas phase in the corresponding section:

$$\mathrm{Nu} = \frac{-\left(\partial T/\partial y\right)_W 2R_4}{T_W - T_m} \; .$$

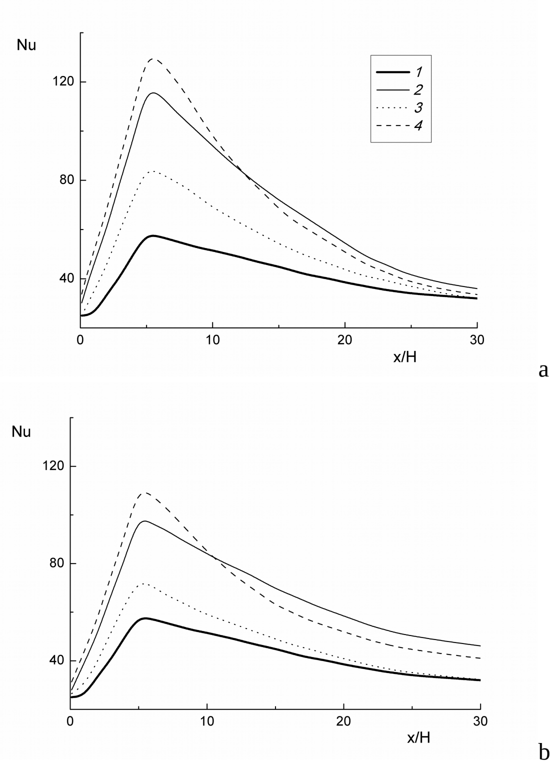

Fig. 5. The distributions of local Nusselt numbers along the pipe length for various types of liquids for inlet droplet diameter d_1 = 10 μm (a) and 30 (b). Re = 2×10^4, M_{L1} = 0.05, S = 0.5. *1* – single-phase air swirling flow, *2* – water, *3* – ethanol, *4* – acetone.

The addition of evaporating droplets (lines *2–4*) leads to an almost twofold increase of heat transfer compared to that of a single-phase air swirling flow (line *1*). The highest value of heat transfer rate is revealed for the ethanol droplets, and the heat transfer enhancement for ethanol droplets is higher than that one for water droplets (up to 10%). This effect is visible at a short distance from the inlet cross-section ($x/H \leq 10$). Further, the heat transfer rate sharply decreases due to the fast ethanol droplets' evaporation and tends to the corresponding value in the single-phase swirling flow. The lowest value of heat transfer rate is obtained for the acetone droplets, and the heat transfer enhancement for them is lower than that one for water droplets (up to 50%). Acetone droplets have the lowest value of the latent heat of evaporation, and they evaporate much faster than droplets of water and ethanol. The region of existence of the two-phase flow using ethanol (*3*) and acetone (*4*) as a coolant decreases significantly in comparison with using water droplets (*2*). The positions of maximum heat transfer value roughly coincide with the reattachment point as well as in single-phase flow and two-phase flow.

The variation of droplets inlet diameter has a complex effect on heat transfer in a swirling flow. Initially, with an increase in the size of the droplets, a significant intensification of heat transfer occurs due to evaporation of the dispersed phase in a swirling flow (see Fig. 5a). This effect is noticeable at a short distance from the flow separation section ($x / H \leq 5$). Further, the heat transfer intensity decreases sharply and tends to the corresponding value in a single-phase swirling flow.

The droplets evaporation depends strongly on latent heat of vaporization. It determines mainly the heat transfer. The effect of adding water (*1*), ethanol (*2*) and acetone (*3*) droplets on the change in the maximum value of heat transfer enhancement ratio $\mathrm{ER}_{\max} = (\mathrm{Nu}/\mathrm{Nu}_0)_{\max}$ in the swirling flow is presented in Fig. 6. Here Nu is the Nusselt number in swirling flow and subscript "0" is the single-phase swirling flow at the same other conditions. Evaporation of water or alcohol droplets leads to a significant increase in the heat transfer intensity in a swirling two-phase flow in comparison with a single-phase swirling flow with other identical conditions (more than 2.5 times). The heat transfer enhancement ratio for ethanol droplets is higher than for water droplets (up to 20%). The region of existence of the two-phase flow using ethanol as a coolant decreases significantly in comparison with that for water droplets. Acetone droplets have the lowest value of the latent heat of evaporation. They evaporate much faster than droplets of water and ethanol. Therefore, the effect of evaporation of acetone droplets on the turbulence and heat transfer of the carrier gas phase will be minimal.

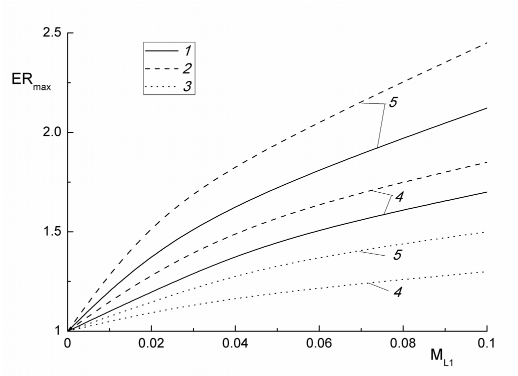

Figure 6. The effect of droplets thermophysical properties on the maximal heat transfer enhancement ratio ER_{max} vs droplets mass fraction. *1* – water, *2* – ethanol, *3* – acetone, *4* – S = 0.25, *5* – S = 0.5.

The distribution of the maximum Nusselt number for various the Reynolds number is presented in the Fig. 7. The Reynolds number is based on the step height H and the initial velocity of the gas phase. Simulations are performed as well as for non-swirling two-phase flow (open symbols, S = 0), and swirling droplet-laden flow (closed symbols, S = 0.5). The maximum Nusselt number increases with the growth in the Reynolds number. These are characteristic for the non-swirling two-phase flow and the swirling one. The values of the heat transfer coefficient for the swirling flow are noticeably higher than those for the non-swirling flow regime (approximately 1.5 times). This is characteristic of all materials studied in the work of the droplets. The highest value of heat transfer intensification is obtained for ethanol droplets (2), the smallest one is predicted for acetone particles (3).

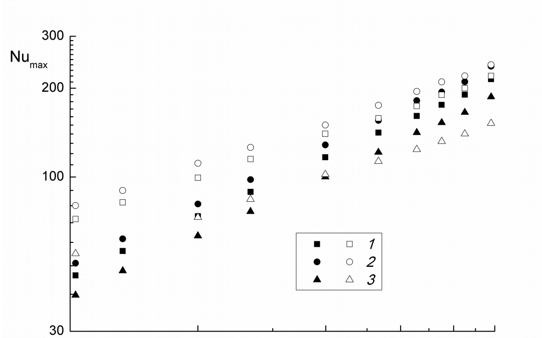

Fig. 7. The dependence of maximal Nusselt numbers vs Reynolds numbers in swirling droplets-laden confined flow for S = 0 (open symbols) and 0.5 (closed symbols). d_1 = 30 μm, M_{L1} = 0.05. *1* – water, *2* – ethanol, *3* – acetone.

COMPARISON WITH RESULTS OF OTHER AUTHORS

The results of the experiments of (Sommerfeld and Qiu, 1998) and LES simulations of (Apte et al., 2009) were used for comparative analysis. The results of measurements of (Sommerfeld and Qiu, 1998) were obtained using the phase Doppler anemometer at atmospheric pressure. The two-phase flow of air and isopropyl alcohol droplets were studied in a downward flow downstream of the pipe sudden expansion without swirling (S = 0). The radial profiles of droplets axial droplet mass flux are shown in Fig. 8 for the five cross-sections. It should be noted that all stations are located into the recirculation region ($x \approx 10H \approx$ 680 mm).

Droplets move downstream of the pipe sudden expansion and entrained into the gas phase motion and their behavior show the same trends as that fir the gas phase. In the two first stations the maximal value of mass flux does not located on the pipe axis (see Fig. 8). The peak spreads radially roughly a 60^0 cone angle. It is obviously visible as well as in experimental and LES data and author's predictions. Further downstream due to the expansion of the spray and mixing with ambient medium and air annular jet the maximum of the droplet mass flux moves towards the pipe axis. The values of droplet mass flux downstream of x = 200 mm is significant decreased due to the evaporation and droplets scattering over the pipe cross-section.

It is shown a good agreement with the experimental of and LES results of (Sommerfeld and Qiu, 1998; Apte et al., 2009) . Accurate predictions of the mean characteristics of the gas and particles velocities and r.m.s values of the gas in two-phase swirling flow in the pipe sudden expansion represents evidence of the accuracy of the numerical Eulerian model with SMC.

CONCLUSION

Large particles are located in the near-wall region of the channel due to the action of centrifugal forces. Smaller ones are in the central recirculation zone of the pipe. The numerical study of the effect of the swirling numbers of the flow and the thermophysical properties of the water, ethanol, and acetone droplets on the mean flow structure, turbulence modification and heat transfer in the droplet-laden flow is carried out. The flow swirl leads to an increase in the intensity of heat transfer (more than 2.5 times in comparison with the single-phase swirling mist flow at other identical conditions).

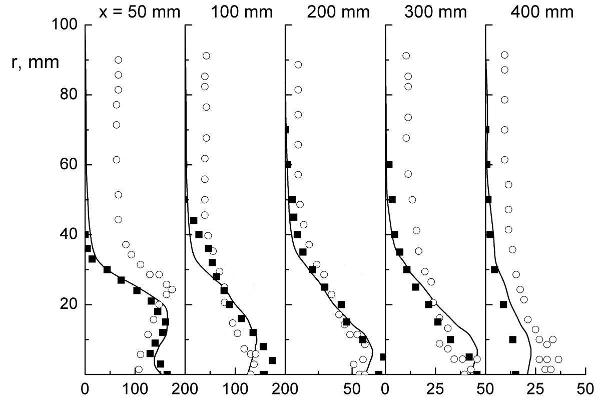

Fig. 8. Radial profiles of droplets axial mass flux in mist flow in a pipe sudden expansion without swirling. U_{m1} = 15.47 m/s, G_1 = 28.3 g/s, Re = $2R_2U_{m1}/\nu$ = 2.11×10^4, T_1 = 373 K, G_{L1} = 0.443 g/s, T_{L1} = 313 K, $2R_1$ = 40 mm, $2R_2$ = 64, $2R_3$ = 200 mm, S = 0. *1* – experimental results of [39], *2* – LES [40]; *3* – authors' simulations.

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