ATTACHED FLOW STRUCTURE AND STREAMWISE ENERGY SPECTRA IN A TURBULENT BOUNDARY LAYER

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ABSTRACT

On the basis of (i) Particle Image Velocimetry (PIV) data of a Turbulent Boundary Layer with large field of view and good spatial resolution and (ii) a mathematical relation between the energy spectrum and minimally modeled flow structures, we show that the scalings of the streamwise energy spectrum $E_{11}(k_x)$ in a wavenumber range directly affected by the wall are determined by wall-attached eddies but are not given by the Townsend-Perry attached eddy model's prediction of these spectra, at least at the Reynolds numbers Re_τ considered here which are between 10^3 and 10^4. Instead, we find $E_{11}(k_x) \sim k_x^{-1-p}$ where p varies smoothly with distance to the wall from negative values in the buffer layer to positive values in the inertial layer. The exponent p characterises the turbulence levels inside wall-attached streaky structures conditional on the length

of these structures. A particular consequence is that the skin friction velocity U_τ is not sufficient to scale $E_{11}(k_x)$ for wavenumbers directly affected by the wall.

INTRODUCTION

The present work looks at the basis for the Townsend-Perry k_x^{-1} range in the streamwise energy spectrum from a new perspective. Using Particle Image Velocimetry (PIV) of flat plate turbulent boundary layer and a simple model which can in principle be applied to various wall-bounded turbulent flows, we show how, in the turbulent boundary layer, a power-law spectral range exists but is not a Townsend-Perry k_x^{-1} range and how it can be accounted for by taking only streamwise lengths and intensities of wall-attached structures into account.

A SIMPLE MODEL

Perry & Abell (1977), Perry & Chong (1982) and Perry et al (1986) showed how Townsend's attached eddy hypothesis implies $E_{11}(k_x) \sim U_\tau^2 k_x^{-1}$ in the range $1/\delta \ll k_x \ll 1/y$ (δ is the boundary layer thickness and y is the distance from the wall). Perry et al (1986) also developed a flow structure model for this spectral range in terms of specific attached eddies of varying sizes randomly distributed in space and with a number density that is inversely proportional to size. In this paper we attempt to distill such a type of model to its bare essentials. These bare essentials are that flow structures are primarily objects with clear spatial boundaries. We model these boundaries with on-off functions in the expectation that the spectral signature in the attached eddy wavenumber range is dominated by these sharp gradient, effectively on-off, behaviours. The concomitant expectation is that the additional superimposed velocity fluctuations fill the content of a predominantly higher frequency spectral range. In this section we show that the streamwise energy spectrum's k_x^{-1} spectral range can be captured by simple on-off representations of elongated streaky structures of varying sizes as long as their number density has a space-filling power law dependence on size.

We therefore assume that the attached eddies responsible for the k_x^{-1} spectral range have a long streaky structure footprint on the 1D streamwise fluctuating velocity signals at a distance y from the wall. We also assume that these streaky structures can be modeled as simple on-off functions and that it is sufficient to represent the streamwise velocity fluctuations $u(x)$ at a given height y from the wall as follows

$$u(x) = \sum_{n,m} a_{nm} \Pi(\xi) \tag{1}$$

where $\Pi(\xi) = 1$ if $-1 < \xi < 1$ with $\xi = \frac{x - x_{nm}}{\lambda_n}$ and $\Pi(\xi) = 0$ otherwise. The on-off function $\Pi(\xi)$ is our cartoon model of a streaky structure. Streaky structures of length λ_n are centred at random positions x_{nm} and their intensity is given by the coefficients a_{nm}. For each subscript n, the subscript m counts the spatial positions where cartoon structures of size λ_n can be centred in a given realisation. The sum in (1) is over all structures lengths λ_n and all their positions x_{nm}.

The energy spectrum of $u(x)$ is $E_{11}(k_x) = \frac{(2\pi)^2}{L_x} \overline{|\hat{u}(k_x)|^2}$ where L_x is the length of the record, $\hat{u}(k_x)$ is the Fourier transform of $u(x)$, and the overbar signifies an average over realisations. The Fourier transform of $\Pi(\frac{x - x_{nm}}{\lambda_n})$ being $\hat{\Pi}(k_x, \lambda_n, x_{nm}) = 2ik_x^{-1} e^{ik_x x_{nm}} \sin(k_x \lambda_n)$, it follows that

$$\hat{u}(k_x) = 2ik_x^{-1} \sum_{nm} a_{nm} e^{ik_x x_{nm}} \sin(k_x \lambda_n) \tag{2}$$

which implies that the energy spectrum is given by

$$E_{11}(k_x) =$$

$$4\frac{(2\pi)^2}{L_x} k_x^{-2} \overline{\sum_{nm} a_{nm} e^{ik_x x_{nm}} \sin(k_x \lambda_n) \sum_{pq} a_{pq} e^{-ik_x x_{pq}} \sin(k_x \lambda_p)}. \tag{3}$$

We introduce two assumptions which were also used by Perry et al (1986) in their more intricate model. The first assumption is that the positions and amplitudes of our cartoon stuctures are uncorrelated and that different positions are not correlated to each other either, i.e. $\overline{e^{ik_x x_{nm}} e^{ik_x x_{pq}}} = \delta_{pn} \delta_{qm}$. As a result, the expression for the energy spectrum simplifies as follows:

$$E_{11}(k_x) = 4\frac{(2\pi)^2}{L_x} k_x^{-2} \sum_{nm} \overline{(a_{nm})^2} \sin^2(k_x \lambda_n). \tag{4}$$

Let us say that there is an average number N_n of cartoon stuctures of size λ_n centred within an integral scale along the x-axis. The expression for $E_{11}(k_x)$ simplifies even further:

$$E_{11}(k_x) = 4\frac{(2\pi)^2}{L_x} k_x^{-2} \sum_n \overline{a_n^2} N_n \sin^2(k_x \lambda_n) \tag{5}$$

where $\overline{a_n^2} \equiv \overline{(a_{nm})^2}$ is the same irrespective of position x_{nm}.

We now consider a continuum of different structure sizes λ rather than discrete length-scales λ_n and the previous expression for $E_{11}(k_x)$ must therefore be replaced by

$$E_{11}(k_x) = 4\frac{(2\pi)^2}{L_x} k_x^{-2} \int d\lambda \overline{a^2}(\lambda) N(\lambda) \sin^2(k_x \lambda) \tag{6}$$

in terms of easily understandable notation. At this point we introduce a generalised form of the second assumption which was also used by Perry et al (1986): we assume a power-law form for $N(\lambda)$ in the range $\lambda_i < \lambda < \lambda_o$ where $\lambda_i \sim y$ and $\lambda_o \sim \delta$, and $N(\lambda) = 0$ outside this range for simplicity. This power law form is $N(\lambda) = (-N_M + N_o(\lambda/\delta)^{-1-D})$ where N_M and N_o are positive dimensionless numbers which increase propotionally to L_x so as to keep number densities constant. The number N_M is introduced to allow for the possibility of an upper bound on streaky structure size given by $N(\lambda_o) = 0$, i.e. $N_M = N_o(\lambda_o/\delta)^{-1-D}$ which should be small given that LSM and VLSM streaky structures have been observed with lengths greater than δ.

Vassilicos & Hunt (1991) proved that, if $0 \leq D \leq 1$, then the set of points defining the edges of the on-off functions $\Pi(\xi)$ is fractal and D is effectively the fractal dimension of this set of points. The case where this fractal dimension is $D = 1$ is the case where these points are space-filling. The population density assumption of Perry et al

(1986) corresponds to $D = 1$ which is also the choice we make in this work. We now show that this choice can lead to $E_{11}(k_x) \sim k_x^{-1}$ in the range $1/\lambda_o \ll k_x \ll 1/\lambda_i$.

We calculate the energy spectrum by carrying out the integral in (6). This requires a model for $\overline{a^2}(\lambda)$ which, in this section, we chose to be as simple as possible and therefore independent of λ in the relevant range, i.e. $\overline{a^2}(\lambda) = A^2/\delta$ for $\lambda_i < \lambda < \lambda_o$ where A^2 is a constant. Using our models for $N(\lambda)$ and $\overline{a^2}(\lambda)$ and the change of variables $\lambda k_x = l$, (6) becomes

$$E_{11}(k_x) = A^2\delta(C_o(k_x\delta)^{-2+D} - C_M(k_x\delta)^{-2}) \tag{7}$$

where

$$C_o = 4(2\pi)^2 N_o \frac{\delta}{L_x} \int_{\lambda_i k_x}^{\lambda_o k_x} dl \sin^2(l) l^{-1-D}$$

and

$$C_M = 4(2\pi)^2 N_M \frac{\delta}{L_x} (k_x\delta)^{-1} \int_{\lambda_i k_x}^{\lambda_o k_x} dl \sin^2(l)$$

which is bounded from above by $\frac{N_M}{L_x}\frac{\lambda_o - \lambda_i}{\delta}$. In the attached eddy range $1/\lambda_o \ll k_x \ll 1/\lambda_i$, $C_o \approx 4(2\pi)^2 \frac{N_o}{L_x} \int_0^\infty dl \sin^2(l) l^{-1-D}$ which means that C_o is approximately independent of k_x in this range.

Substituting the value $D = 1$ in equation (7), we get $E_{11}(k_x) = A^2(C_o k_x^{-1} - C_M \delta^{-1} k_x^{-2})$ which is well approximated by

$$E_{11}(k_x) \approx C_o A^2 k_x^{-1} \tag{8}$$

for wavenumbers $k_x\delta \gg C_M/C_o$ (i.e. $C_o k_x^{-1} \gg C_m \delta^{-1} k_x^{-2}$). Note that C_M/C_o is much smaller than 1 because N_M is much smaller than N_o and that (8) is valid in the range $1/\lambda_o \ll k_x \ll 1/\lambda_i$ where λ_o scales with but is much larger than δ. For a good correspondence with the scalings of the Townsend-Perry attached eddy model one needs to take $\lambda_i \sim y$ and $A^2 \sim U_\tau^2$.

We now generalise this model by assuming that $\overline{a^2}(\lambda)$ is not constant but varies with λ in the range $\lambda_i < \lambda < \lambda_o$, for example as $\overline{a^2}(\lambda) = (A^2/\delta)(\lambda/\delta)^p$ where p is a real number with bounds which we determine below. The pervious arguments can be reproduced till equation (6) which now becomes

$$E_{11}(k_x) = A^2\delta[c_o(k_x\delta)^{-2+D-p} - c_M(k_x\delta)^{-2}] \tag{9}$$

where

$$c_o = 4(2\pi)^2 N_o \frac{\delta}{L_x} \int_{\lambda_i k_x}^{\lambda_o k_x} dl \sin^2(l) l^{-1-D+p}$$

and

$$c_M = 4(2\pi)^2 N_M \frac{\delta}{L_x} (k_x\delta)^{-1-p} \int_{\lambda_i k_x}^{\lambda_o k_x} dl\, l^{+p} \sin^2(l)$$

which is bounded from above by $\frac{N_m}{(1+p)L_x}[(\frac{\lambda_o}{\delta})^{1+p} - (\frac{\lambda_i}{\delta})^{1+p}]$. In the attached eddy range $1/\lambda_o \ll k_x \ll 1/\lambda_i$, $c_o \approx 4(2\pi)^2 \frac{N_o}{L_x} \int_0^\infty dl \sin^2(l) l^{-1-D+p}$ which means that c_o is approximately independent of k_x in this range if $0 < D - p < 2$.

Substituting the value $D = 1$ in (9), we obtain the following leading order approximation in the parameter range $-1 < p < 1$:

$$E_{11}(k_x) \approx c_0 A^2 \delta (k_x\delta)^q \tag{10}$$

where

$$p + q = -1 \tag{11}$$

for wavenumbers $k_x\delta \gg (c_M/c_o)^{\frac{1}{1-p}}$. Note that c_M/c_o is much smaller than 1 if p is not too close to 1 because N_M is much smaller than N_o.

The spectral shape (10) is potentially significantly different from what the classical Townsend-Perry attached eddy model predicts. We emphasize that in this and the previous sections we have developed a simple model based on on-off functions representing long streaky structures which returns a wavenumber dependency of $E_{11}(k_x)$ which is either identical to the Townsend-Perry spectral shape if $p = 0$, or different but in some ways comparable if $p \neq 0$. In the remainder of this paper we present experimental evidence in support of $D = 1$ and (10)-(11) rather than (8), with p as function of y^+.

Note that $\overline{a^2}(\lambda) = (A^2/\delta)(\lambda/\delta)^p$ is the mean square fluctuating velocity obtained by averaging over wall-attached flow structures of size λ at a certain distance y from the wall. The definition of these wall-attached flow structures is straightforward and minimal, as it is based on identifying their boundaries. This definition was applied to our PIV data to extract wall-attached structures.

EXPERIMENTAL SET-UP

An experiment was performed in the boundary layer wind tunnel at the Lille Fluid Mechanics Laboratory (LMFL) having a test section 2m wide, 1m high and 20.6m long. The tests were conducted at two free stream velocities of 3m/s and 10m/s corresponding to Reynolds numbers $Re_\theta = 8100$ ($Re_\tau = 2700$) and $Re_\theta = 20600$ ($Re_\tau = 7200$) respectively. To capture the large streamwise wall-normal field, four 12 bits Hamamatsu cameras having a resolution of 2048x2048 pixels were installed in series to observe a region between 19.26m and 20.42m from inlet which is 1.16m long ($\approx 3.36\delta$ and 3.85δ, for $Re_\theta = 8100$ and 20600 respectively) and 0.3m high ($\approx 0.86\delta$ and 1δ) for $Re_\theta = 8100$ and 20600 respectively). Nikon lenses of 50mm focal length were set on the cameras and the magnification obtained was 0.05. The Software HIRIS was used to acquire the images of the four cameras simultaneously. A total of 22500 and 29500 samples were recorded at the highest and lowest Reynolds numbers respectively. The flow was seeded with $1\mu m$ Poly-Ethylene glycol and illuminated by a double pulsed NdYAG laser at 400mJ/pulse. The modified version by LMFL of MatPIV toolbox, was used under Matlab to process the acquired images from the 2D2C PIV. A multipass software was used with a final pass of 28x28 pixels (with a mean overlap of 65%) corresponding to 4mm x 4mm i.e. 33x33 wall units for $Re_\theta = 8100$ and 100x100 wall units for $Re_\theta = 20600$. Image deformation was applied at the final pass. The final grid had 766 points along the wall and 199 points in the wall-normal direction with a grid spacing of 1.5mm corresponding to 11 wall units and 35 wall units for the test cases at $Re_\theta = 8100$ and $Re_\theta = 20600$ respectively.

RESULTS

We obtained well-resolved PIV data of a flat plate turbulent boundary layer in a large field of view at two Reynolds numbers, $Re_\theta = 8100$ and $Re_\theta = 20600$. As shown by Srinath et al (2018), these data support the concept of elongated streaky structures which are part of attached eddies and can be modeled as simple on-off functions. Srinath et al (2018) also explain how the raw fluctuating streamwise velocity fields to lead to binary fields which we use in our statistical analysis.

To obtain statistics of wall-attached elongated streaky structures represented as on-off functions in our model and as binary structures in the final stages of our structure eduction method, we first label the connected components of the binary images using image processing tools. Then we compute the streamwise length λ of each labelled structure at a distance y from the wall, i.e. the difference between the smallest and the largest values of streamwise coordinate x in this labelled structure at height y. Finally we compute the average value α of the streamwise fluctuating velocity component u inside this labelled structure at height y. Thus we obtain a pair (λ, α) for each labelled structure at each height y considered. See Srinath et al (2018) for more details.

The model in the previous sections assumes that the number of wall-attached elongated streaky structures of size λ has a decreasing power-law dependence on λ in a certain range of λ values. Following Perry et al (1986), we expect the spatial distribution of such structures to be space-filling, which implies that the exponent of this power law should be -2. We evaluate the probability distribution function (PDF) of lengths λ at various wall distances and find that the most probable length λ lies between 0.3δ and 0.5δ and lengths λ longer than 3.5δ occur very rarely.

We also find a power law dependence on λ between about 0.5δ and 2δ with power law exponent -2, i.e. $D = 1$, in all cases. Given the form of $N(\lambda)$ hypothesised in the previous section's model (see Srinath et al 2018 for details), we fit the PDF of λ/δ with a functional form $-C_1 + C_2(\lambda/\delta)^{-2}$. This fit is effectively the same for both Reynolds numbers and all values of y^+ in the mean flow's approximate inertial region, and it is in very good agreement with the hypothesis made in the on-off model briefly sketched in the previous section. It is worth noting that the lower bound of the range where the PDF of λ/δ is well approximated by $-C_1 + C_2(\lambda/\delta)^{-2}$ seems to increase slightly with increasing y^+. These results are obtained for wall-attached structures with negative streamwise fluctuating velocities but identical results can also br obtained for positive such velocitis except that C_1 and C_2 are slightly different. A more detailed account can be found in Srinath et al (2018).

A direct inspection of log-log plots of the streamwise energy spectrum would suggest $E_{11}(k_x) \sim U_\tau^2 k_x^{-1}$ in the range $2\pi/(4\delta) < k_x < 0.63/y$. However, a closer look assisted by relation (10)-(11) reveals a significantly different behaviour. This relation introduces a specific data analysis which involves the extraction of wall-attached elongated streaky structures from PIV data. The concurrent analysis of streamwise energy spectra and of the relation between the turbulence levels inside streaky structures and the length of these structures offers strong support for (10)-(11) over a significant range of wavenumbers and length-scales (see figures 1, 2 and 3 where all the exponents are plotted with the 95% confidence intervals for these fits). This range covers LSMs and is comparable to the range where one might have expected the Townsend-Perry attached eddy model spectra to be present. Even though k_x^{-1} spectra are not validated by our data, the streaky structures which account for the scalings of $E_{11}(k_x)$ do need to be wall-attached for relation (10)-(11) to hold. Our conclusions agree with the experiments of Vallikivi et al (2015) which actually suggest that the Townsend-Perry k_x^{-1} spectrum cannot be expected even at very high Reynolds numbers. The revised streamwise energy spectral form (10)-(11) with $p = p(y^+)$ given by figure 1 has a range of validity over an extended range of wall distances and returns spectral exponents $q(y^+)$ which agree with the Hot Wire Anemometry (HWA) data of Tutkun et al (2009) and the Direct Numerical Simulation (DNS) data of Lee & Moser (2015) (see figure 2).

CONCLUSIONS

We obtained well-resolved PIV data of a flat plate turbulent boundary layer in a large field of view at two Reynolds numbers, $Re_\theta = 8100$ and $Re_\theta = 20600$. A direct inspection of log-log plots of the streamwise energy spectrum would suggest $E_{11}(k_x) \sim U_\tau^2 k_x^{-1}$ in the range $2\pi/(4\delta) < k_x < 0.63/y$. However, a closer look assisted by relation (10)-(11) reveals a significantly subtler behaviour. This relation introduces a specific data analysis which involves the extraction of wall-attached elongated streaky structures from PIV data. The concurrent analysis of streamwise energy spectra and of the relation between the turbulence levels inside streaky structures and the length of these structures offers strong support for (10)-(11) over a significant range of wavenumbers and length-scales. This range covers LSMs and is comparable to the range where one might have expected the Townsend-Perry attached eddy model spectra to be present. Even though k_x^{-1} spectra are not validated by our data, the streaky structures which account for the scalings of $E_{11}(k_x)$ do need to be wall-attached for relation (10)-(11) to hold. Our conclusions agree with the experiments of Vallikivi et al (2015) which actually suggest that the Townsend-Perry k_x^{-1} spectrum cannot be expected even at very high Reynolds numbers. The revised Townsend-Perry streamwise energy spectral form (10)-(11) with $p = p(y^+)$ given by figure 1 appears to extend the validity of the attached eddy concept and its revised consequences to a wider range of Reynolds numbers and a wider range of wall distances.

Finally, we stress that relation (10)-(11) is predicated on these wall-attached streaky structures being space-filling, i.e. $D = 1$ in the notation of Srinath et al (2018). The pdf of the streamwise length of the educed streaky structures does indeed follow a power law with exponent $-1 - D = -2$ over the range of scales which corresponds to the one where (10)-(11) holds.

Our work sheds some new light on the streamwise turbulence spectra of wall turbulence by revealing that some of the inner structure of wall-attached eddies is reflected in the scalings of these spectra via $p(y^+)$. An important implication of this structure is that the friction velocity is not sufficient to scale the spectra. Future work must now further probe the inner structure of wall-attached eddies, attempt to explain it and extend our analysis to higher Reynolds numbers so as to establish with certainty the ranges of the power laws (exponents p and q in (10)-(11)) discussed in this paper. When this will be done, a complete picture of streamwise energy spectra will also need to integrate the spectral model of Vassilicos et al (2015).

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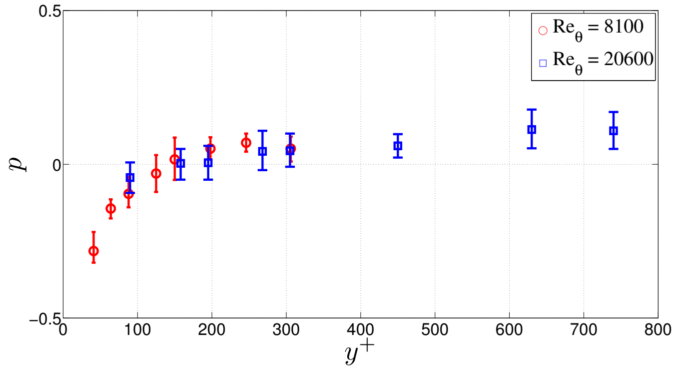

Figure 1. Exponents p obtained from the best power-law fit of $\overline{a^2} \sim (\lambda/\delta)^p$.

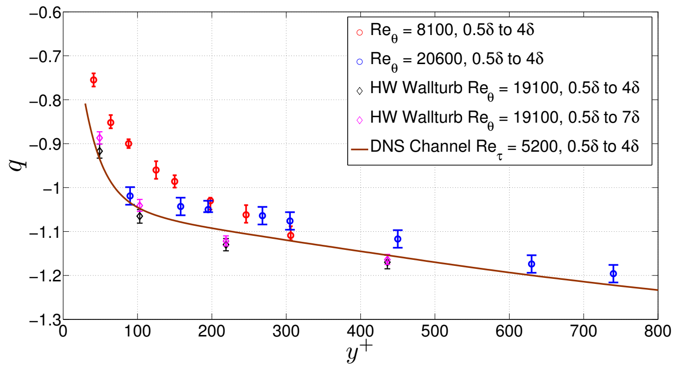

Figure 2. Exponents q obtained from the best power-law fit of $E_{11} \sim k_x^q$ for the present PIV data, the HWA turbulent boundary layer data of Tutkun et al (2009) and the DNS of turbulent channel flow data of Lee & Moser (2015).

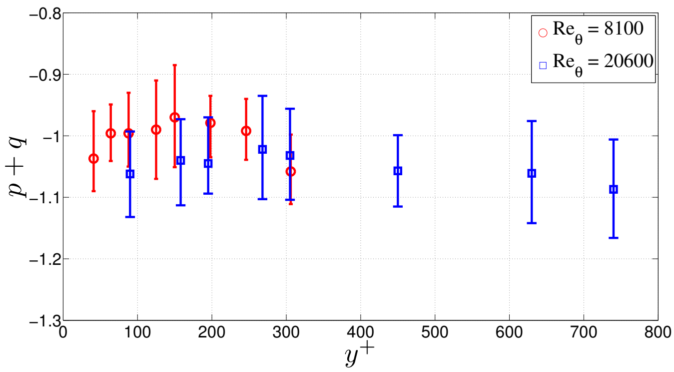

Figure 3. $p+q$ versus y^+.