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A UNIFIED MODELING APPROACH FOR FLOW OVER POROUS AND ROUGH WALLS IN SCALE-RESOLVING SIMULATION METHODS

Benjamin Krumbein, Suad Jakirlić, Cameron Tropea
Institute of Fluid Mechanics and Aerodynamics
Technische Universität Darmstadt
Alarich-Weiss-Straße 10, 64287 Darmstadt
jakirlic@sla.tu-darmstadt.de

ABSTRACT

In this work, a roughness model suitable for scale-resolving simulation methods is proposed. The model, termed as equivalent-porosity model (EPM) is in principle based on the discrete element method of Taylor *et al.* (1985), whereby the drag, the roughness elements exert on the flow, is represented by a volumetric force term in the momentum equation. In order to appropriately capture the blockage effects associated with rough walls, volume-averaged and ensemble-averaged Navier-Stokes equations originating from the field of porous media flow are adopted, which enables a unified modeling framework for flow over rough walls as well as porous media. The volumetric drag term is modeled based on a modified drag closure for packed beds, where the mean hydraulic diameter in layers parallel to the wall is introduced as a length scale. Two model coefficients associated with viscous and form drag are determined in an *a priori* fashion based on reference DNS data of Forooghi *et al.* (2017) for synthetic, irregular roughness topographies in an open-channel flow configuration at a Reynolds number of $Re_\tau \approx 500$. As a closure for the turbulent sub-scale contributions, the eddy-resolving (ER) ζ-f model, a computationally efficient RANS-based sub-scale model adapted for the volume-averaged computational framework, is employed. Validation results for four roughness topographies with varying skewness exhibit a high level of agreement with the DNS database in terms of mean velocity profiles, friction velocities as well as turbulent intensities. Thus, the geometry-based parametrization of the drag closure in conjunction with volume-averaged equations explicitly accounting for blockage effects, is considered as successfully validated.

INTRODUCTION

Turbulent flows over rough walls can be encountered in a variety of engineering and environmental flows ranging from flow over deposits in exhaust gas systems of internal combustion engines, over aged turbine blades to plant and urban canopies in the atmospheric boundary layer. For most practical applications it is not feasible to geometrically resolve the structure of the roughness due to the associated computational costs. Instead, mostly Reynolds-averaged Navier-Stokes (RANS) models or associated wall functions are modified in order to include the effects the roughness exerts on the flow, based on the so-called equivalent sand roughness height introduced by Nikuradse (1933). Unfortunately a universal relation between the roughness topography and the equivalent sand roughness has to the authors knowledge not been found so far. Thus, usually experiments or direct numerical simulations (DNS) are required in order to determine the equivalent sand roughness of any uncharacterized rough surface. Furthermore, models based on this single parameter for the quantification of the roughness effects can, by design, only reproduce the mean velocity profile in the logarithmic layer, but fail in the direct vicinity of the rough surface. Especially in the context of scale-resolving simulations, such as large-eddy simulation (LES) or DNS, modeling approaches based in principle on the discrete element method (Taylor *et al.*, 1985), where a volume force describing the drag roughness exerts on the flow is introduced into the momentum equation, have been proposed (e.g. Busse & Sandham, 2012; Krumbein *et al.*, 2017). However, blockage effects have often been neglected, in contrast to the original formulation of Taylor *et al.* (1985), and model coefficients have been tuned predominantly to fit friction coefficients, which does not necessarily result in an accurate representation of the turbulent flow in the near wall region.

In this work, the effect of roughness is modeled as an equivalent porosity based on a modified Egrun equation for packed beds (see e.g. Bird *et al.*, 2002) in the framework of the volume-averaged Navier-Stokes equations (VANS) according to Whitaker (1996). In this way, a unified computational framework for porous and rough walls is enabled and blockage effects are explicitly accounted for through the volume-averaged equations. A similar approach for rough walls was recently proposed by Kuwata & Kawaguchi (2017) in the framework of DNS with the Lattice-Boltzmann method. The present model, denoted as equivalent-porosity model (EPM), is validated against DNS data of Forooghi *et al.* (2017) for turbulent flow over four synthetic, irregular roughness topographies consisting of randomly distributed roughness elements at $\mathrm{Re}_\tau \approx 500$ in an open channel configuration. In terms of turbulence modeling, a recently proposed scale-resolving unsteady RANS (URANS) model, termed as eddy-resolving (ER) ζ-f model, which has been applied successfully to flow over porous walls in Krumbein *et al.* (2018), is adopted. The model is based on the elliptic-relaxation eddy-viscosity model of Hanjalić *et al.* (2004), which was appropriately transformed in order to utilize a transport equation for the specific rate of dissipation ω $(= \varepsilon/k)$ as the scale-supplying equation. Sensitivity of the model towards resolving turbulent structures has been achieved following Jakirlić & Maduta (2015) by introducing a source term inspired by the scale-adaptive simulation concept of Menter & Egorov (2010) into the ω-equation.

FLOW CONFIGURATION AND ROUGHNESS TOPOGRAPHIES

As reference for roughness modeling and the associated validation, four irregular rough surfaces from the DNS database of Forooghi *et al.* (2017) are adopted. The DNS has been performed in an open-channel flow configuration at friction Reynolds numbers of $\mathrm{Re}_\tau = u_\tau H_{\mathrm{eff}}/\nu \approx 500$, whereby $H_{\mathrm{eff}} = H - \eta_{\mathrm{md}}$ is the effective channel height available to the fluid, i.e. the empty channel height H reduced by the melt-down height η_{md}. Fig. 1 shows a qualitative sketch of the computational domain illustrating the various geometrical quantities. The position of the smooth base wall corresponds to the minimum of the surface elevation map associated with the rough surface. The grey area with the height k_t, i.e. the peak-to-valley height of the rough surface, indicates the maximum extent of roughness elements. For the origin of the wall-normal y-coordinate, the *a priori* definition used by Forooghi *et al.* (2017) is adopted, where the melt-down plane is considered as the virtual wall. While this is a convenient choice, it should be kept in mind that such an *a priori* definition is detrimental for an accurate estimation of the equivalent sand roughness height (see e.g. Raupach *et al.*, 1991). However, this is of no concern in the present work, since equivalent sand roughness height or other quantities associated with the modified log-law for rough walls, such as the roughness function ΔU^+, are not evaluated here.

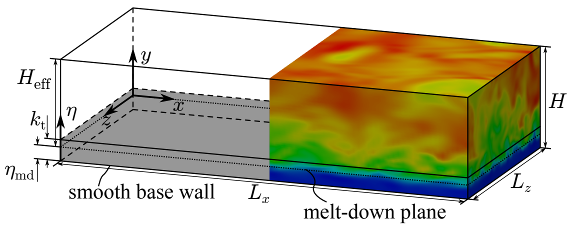


Figure 1. Schematic of the open-channel flow configuration with overlayed instantaneous velocity magnitude (right) obtained with the ER ζ-f model for illustrative purposes.

Samples of the considered roughness topographies are displayed in Fig. 2 for illustrative purposes. The synthetic, irregular roughness topographies have been generated by randomly distributing axisymmetric roughness elements on the smooth base wall in order to achieve predefined surface statistics, whereby roughness elements are allowed to intersect (see Forooghi *et al.*, 2017). The resulting roughness topographies can be considered as macroscopically homogeneous and isotropic. Tab. 1 summarizes statistical properties of the rough surfaces, denoted as RA, RB, RC and RD, as well as associated Reynolds numbers. The bulk Reynolds number $\mathrm{Re_b} = U_\mathrm{b}^\mathrm{s} H_{\mathrm{eff}}/\nu$ is defined in terms of the superficially-averaged bulk velocity based on the empty channel heigh H. The surface statistics of the roughness topographies RA, RB and RC vary mainly in terms of skewness *Sk*, since this quantity has been identified as one of the most influential parameters with respect to the roughness function (Forooghi *et al.*, 2017). The sample RD is similar to RA, with the difference that all roughness elements share the same height (see Fig. 2).

Table 1. Statistical properties of the rough surfaces and Reynolds numbers of the computed flow. η_{md}: melt-down height; k_{rms}: root mean square height; Sk: skewness; Ku: kurtosis; ES: effective slope.

	RA	RB	RC	RD
η_{md}/H	0.072	0.084	0.048	0.056
$k_\mathrm{t}/H_{\mathrm{eff}}$	0.248	0.233	0.222	0.145
$k_{\mathrm{rms}}/H_{\mathrm{eff}}$	0.045	0.045	0.045	0.045
Sk	0.21	−0.33	0.66	0.22
Ku	2.62	2.62	2.62	1.88
ES	0.88	0.88	0.88	0.89
$\mathrm{Re_b}$	3954	4520	3855	4105
Re_τ	498	501	502	501

COMPUTATIONAL METHODOLOGY

The VANS equations of Whitaker (1996) for porous media are employed in a double-averaged form (see e.g. de Lemos, 2006) in order to enable the application of the ER ζ-f model. The continuity and momentum equations in an expanded form following Whitaker (1996) and Breugem *et al.* (2006) read

$$\frac{\partial}{\partial x_j}\left(\varphi \langle \widetilde{U}_j \rangle\right) = 0 \tag{1}$$

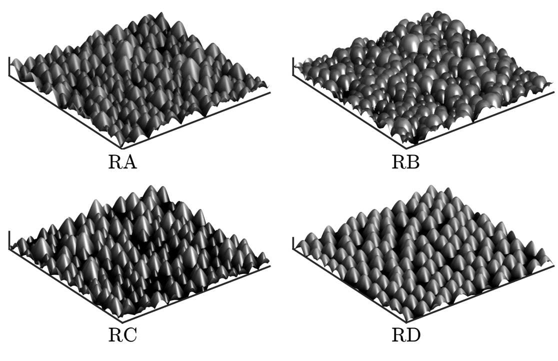


Figure 2. Surface samples of the roughness topographies.

and

$$\frac{\partial\langle\widetilde{U}_i\rangle}{\partial t}+\frac{1}{\varphi}\frac{\partial}{\partial x_j}\left(\varphi\langle\widetilde{U}_i\rangle\langle\widetilde{U}_j\rangle\right)=-\frac{1}{\rho}\frac{\partial\langle\widetilde{P}\rangle}{\partial x_i}$$
$$+\frac{\nu}{\varphi}\left(\varphi\frac{\partial^2\langle\widetilde{U}_i\rangle}{\partial x_j\partial x_j}+\frac{\partial\varphi}{\partial x_j}\frac{\partial\langle\widetilde{U}_i\rangle}{\partial x_j}+\langle\widetilde{U}_i\rangle\frac{\partial^2\varphi}{\partial x_j\partial x_j}\right)$$
$$-\frac{1}{\varphi}\frac{\partial}{\partial x_j}\left(\varphi\langle\widetilde{u_i''u_j''}\rangle\right)+\langle\widetilde{f}_{\mathrm{d},i}\rangle, \quad (2)$$

respectively. Here the $\langle\cdot\rangle$ operator denotes intrinsic volume averaging and φ is the ratio of volume available to the fluid to the total averaging volume, i.e. equivalent to the porosity. The $\widetilde{\cdot}$ represents ensemble averaging, U_i are the velocity components, P is the pressure, ρ the density and ν the kinematic viscosity. Due to the averaging procedures several unclosed terms appear, whereby the term associated with dispersion due to spatial variations in the ensemble-averaged velocity field is neglected based on the arguments of Breugem *et al.* (2006). The closure problem then reduces to the volumetric force $\langle\widetilde{f}_{\mathrm{d},i}\rangle$, representing the drag the roughness exerts on the flow and the macroscopic residual stress tensor (MRST) $\langle\widetilde{u_i''u_j''}\rangle$. In clear fluid regions associated with $\varphi=1$ and $\langle\widetilde{f}_{\mathrm{d},i}\rangle=0$, Eqs. (1) and (2) revert back to the standard single-phase equations, thus enabling their application in the entire computational domain.

The volumetric force term is presently modeled in analogy to a porous media drag closure, applying a Darcy term representing viscous drag and a Forchheimer extension describing form drag. Structurally the form of the modified Egrun equation for packed beds (see e.g. Bird *et al.*, 2002; Breugem *et al.*, 2006) is adopted. The original numerical values are replaced by the model coefficients C_{V} for the viscous drag term and C_{F} for form drag. In addition, the mean hydraulic diameter profile $d_{\mathrm{mh}}(\eta)$ is introduced as a length scale, resulting in

$$\langle\widetilde{f}_{\mathrm{d},i}\rangle=-C_{\mathrm{V}}\,\nu\frac{(1-\varphi)^2}{d_{\mathrm{mh}}^2\,\varphi^2}\langle\widetilde{U}_i\rangle-C_{\mathrm{F}}\frac{(1-\varphi)}{d_{\mathrm{mh}}\,\varphi}\sqrt{\langle\widetilde{U}_j\rangle\langle\widetilde{U}_j\rangle}\,\langle\widetilde{U}_i\rangle. \quad (3)$$

Due to the impermeability of the rough surface in wall-normal direction, the drag force for a rough wall is expected to act primarily in wall parallel directions. Thus, the drag force obtained with Eq. (3) is projected on the xz-plane by setting the wall-normal component $\langle\widetilde{f}_{\mathrm{d},y}\rangle=0$, which can be straightforwardly generalized for surfaces not aligned with the main coordinate directions or curved surfaces by making use of the unit normal vector of the smooth base wall. The mean hydraulic diameter d_{mh} of the roughness elements is determined based on the roughness' surface elevation map, as applied in the reference DNS. For real rough surfaces, the surface elevation map can be obtained with an appropriate measurement technique, such as confocal microscopy. For the determination of the mean hydraulic diameter, the wetted perimeter P_{w} and the area $S_{\mathrm{s}}=S-S_{\mathrm{f}}$ blocked by roughness elements is evaluated in wall-parallel layers. Finally, d_{mh} is obtained as

$$d_{\mathrm{mh}}(\eta)=\frac{4S_{\mathrm{s}}(\eta)}{P_{\mathrm{w}}(\eta)}. \quad (4)$$

The parametrization of the drag force according to Eq. (3) implies several assumptions, which limit the applicability of the present EPM formulation. While the equivalent-porosity φ and the mean hydraulic diameter d_{mh} could in principle be considered as scalar fields, presently they are prescribed as a function of the wall-normal coordinate only, which results in a homogeneous forcing in layers parallel to the wall. Hence, spatial variations in the time-averaged flow field, and consequently dispersive stresses, can not be represented. The second important implication stems from the fact that no directional information pertinent to the roughness topographies, e.g. different length scales depending on the flow direction, are applied. Thus, the present EPM is well suited primarily for isotropic roughness. For highly non-isotropic topographies, consisting of e.g. grooves or other high aspect ratio roughness elements, a generalization relying on parameterizations for the permeability and Forchheimer tensors could be constructed.

The model coefficients C_{V} and C_{F} are determined in an *a priori* fashion by fitting the mean volumetric force obtained from the model equation (3) (in other words, the force obtained using the mean intrinsic velocity profile from the DNS instead of $\langle\widetilde{U}_i\rangle$) to the mean volumetric force determined directly from the DNS data set. The mean volumetric force from the DNS is in turn calculated numerically based on mean velocity and Reynolds shear stress profiles using equations (1) and (2), simplified for channel flow, in analogy to the procedure described in Krumbein *et al.* (2017). Fig. 3 shows the normalized mean volumetric force (the $\overline{\cdot}$ operator denotes time averaging) determined in this data-driven fashion from the DNS database, as well as the *a priori* estimate from the EPM with the presently adopted values of $C_{\mathrm{F}}=1.4$ and $C_{\mathrm{V}}=300$ for the roughness topography RA as an example. Accordingly, the mean hydraulic diameter profile $d_{\mathrm{mh}}(\eta)$ and the corresponding equivalent porosity profile $\varphi(\eta)$ for the rough surface RA are applied in Eq. (3). As to be expected, the drag force assumes zero values slightly above the roughness crest at $\eta/k_{\mathrm{t}}=1$. With respect to the procedure for the determination of the model coefficients, the quality of the fit in the upper part of the roughness is prioritized, since the lower momentum content closer to the smooth base wall renders this region less influential. A similar level of agreement as indicated in Fig. 3 is achieved for the remaining roughness topographies, thus enabling the application of a single set of model coefficients for all presently considered roughness topographies.

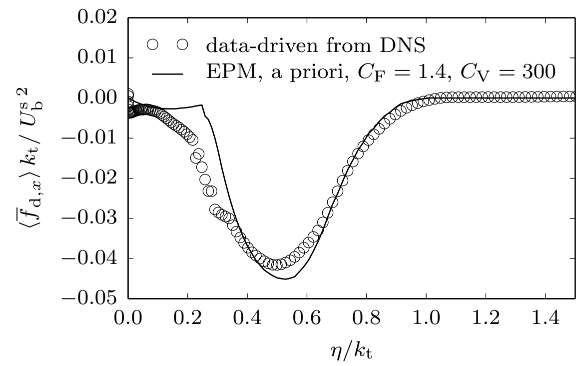


Figure 3. Non-dimensionalized mean streamwise drag force obtained in a data-driven fashion from DNS compared to an *a priori* estimate of the respective force profile obtained with the EPM for roughness topography RA.

The MRST $\langle \widetilde{u_i'' u_j''} \rangle$ is closed with the recently proposed RANS-based sub-scale model, the ER ζ-f model (Krumbein *et al.*, 2019), which was appropriately extended for the additionally volume-averaged mathematical framework by including the porosity φ in the model equations. This model extension has previously been validated successfully by computing turbulent flow over packed beds of different porosities and comparison with respective reference DNS data of Breugem *et al.* (2006) in Krumbein *et al.* (2018). The ER ζ-f model is based on the elliptic-relaxation eddy-viscosity model of Hanjalić *et al.* (2004), which comprises of differential equations for the turbulent kinetic energy k, its dissipation rate ε and the variable $\zeta (= \overline{v^2}/k)$, representing a measure for the wall-normal turbulent intensity, as well as the elliptic function f. In the ER model formulation, the equation for the dissipation rate was transformed to a transport equation for the specific dissipation rate $\omega (= \varepsilon/k)$ and a newly formulated production term inspired by the scale-adaptive simulation concept of Menter & Egorov (2010) was introduced in order to enable the model to adapt modeled sub-scale quantities to resolved turbulent fluctuations. The MRST is expressed as proposed by Antohe & Lage (1997), in analogy to the Boussinesq eddy-viscosity correlation, as

$$\varphi \langle \widetilde{u_i'' u_j''} \rangle = -2\nu_t \langle \widetilde{S}_{ij} \rangle + \frac{2}{3} \varphi \langle k_u \rangle \delta_{ij}, \tag{5}$$

where ν_t is the sub-scale eddy-viscosity presently predicted by the ER ζ-f model, $\langle k_u \rangle$ the sub-scale turbulent kinetic energy and $\langle \widetilde{S}_{ij} \rangle$ the strain-rate tensor given by

$$\langle \widetilde{S}_{ij} \rangle = \frac{1}{2} \left[\frac{\partial}{\partial x_j} \left(\varphi \langle \widetilde{U}_i \rangle \right) + \frac{\partial}{\partial x_i} \left(\varphi \langle \widetilde{U}_j \rangle \right) \right]. \tag{6}$$

It can be expected, that other sub-scale models adapted for the volume-averaged framework or DNS without a sub-scale model can be employed in conjunction with the present roughness modeling approach, provided the applied computational grid is fine enough for the respectively associated resolution requirements. For example the performance of the dynamic Smagorinsky of Lilly (1992) was comparatively assessed with the ER ζ-f model in Krumbein *et al.* (2018) for flow over porous beds. Since the ER ζ-f model exhibited the overall best performance, it is applied exclusively in this study.

The flow solver based on the volume-averaged and ensemble-averaged equations originating from the field of porous media modeling as well as the ER ζ-f model were implemented in the open-source continuum mechanics library OpenFOAM. Computations are performed on a computational grid with dimensions $L_x/H = 4$ and $L_z/H = 2$ (see Fig. 1) consisting of $80 \times 64 \times 100 (= 512000)$ cells in streamwise, wall-normal and spanwise directions, respectively. This corresponds to non-dimensional grid spacings in viscous units of $\Delta x^+ = 25$, $\Delta y^+ = 2...20$ and $\Delta z^+ = 12$, whereby the cells are refined towards the smooth base wall. Periodic boundary conditions are applied in streamwise and spanwise directions and a no-slip boundary is prescribed at the smooth base wall. At the top boundary, shear-free, non-permeable conditions are enforced. The flow is driven by a superimposed pressure gradient in streamwise direction, which is adapted in each time step to maintain a constant bulk Reynolds numbers corresponding to Tab. 1. In order to limit numerical diffusion, the second-order accurate central differencing scheme is used for the convective term in the momentum equation. For time marching the second-order backward differentiation formula is applied with adaptive time stepping ensuring a maximum Courant number of 0.7. The instantaneous velocity field is averaged over a span of at least 130 flow-through times $(130 L_x/U_b^s)$ in order to ensure converged first and second statistical moments.

RESULTS AND DISCUSSION

In the following, the EPM is validated by comparison to the reference DNS database of Forooghi *et al.* (2017) in terms of mean velocity and turbulent intensity profiles as well as the friction velocities u_τ for the four rough surfaces. The DNS data has been evaluated in terms of superficial averages in wall parallel layers, which is adopted here. Thus, use of the relation $\langle \Phi \rangle^s = \varphi \langle \Phi \rangle$ for an arbitrary scalar Φ is made to obtain the respective superficially-averaged quantities from the computed data. Fig. 4 shows the velocity

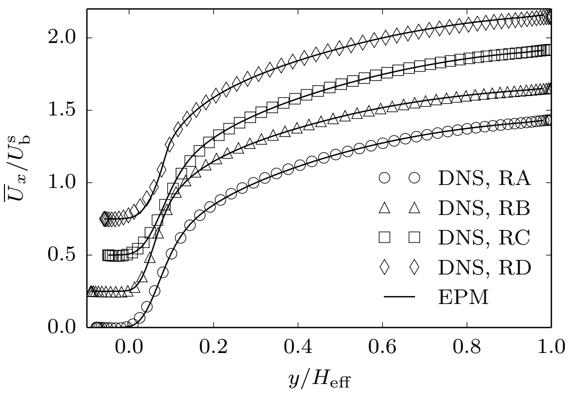


Figure 4. Mean velocity profiles normalized by the bulk velocity U_b^s for all roughness topographies. DNS data taken from Forooghi *et al.* (2017).

profiles in streamwise direction averaged in time as well as superficially in layers parallel to the wall, denoted as $\overline{U}_x$ for brevity. The profiles normalized by the respective superficial bulk velocity U_b^s exhibit a high level of agreement with the DNS reference data. Merely for the roughness topography RD, minor deviations in the shape of the profile are noticeable below the roughness crests in the area $0 < y/H_{\text{eff}} < 0.1$. This indicates that the geometry based parametrization of the volumetric force profiles, which represent the drag the roughness elements exert on the flow, is valid.

In order to evaluate the corresponding velocity profiles in inner scaling, the wall friction velocity u_τ has to be determined. Since the solid-fluid boundary associated with the rough wall is not geometrically resolved, u_τ is conveniently estimated based on a global momentum balance for the present open-channel flow configuration. The resulting expression relying on the mean driving pressure gradient reads

$$u_\tau = \sqrt{-\frac{H_{\text{eff}}}{\rho} \frac{\overline{\partial \langle \widetilde{P} \rangle}}{\partial x}}. \tag{7}$$

Alternatively, the friction velocity can be determined by extrapolating the total shear stress to the position of the virtual wall, as described in Forooghi *et al.* (2017). Thus, given the well represented mean velocity profile, which is decisive for viscous stresses $\nu \partial \overline{U}_x / \partial y$, the quality of the friction velocity prediction relies primarily on the accurate representation of the Reynolds shear stress profile $\overline{u'v'}$ in the outer layer. Tab. 2 summarizes the relative deviation Δu_τ of the obtained friction velocities from the corresponding DNS reference values. With a maximum deviation of $\Delta u_\tau = 3.4\%$ it can be concluded that the friction velocity as well as the friction factor, $C_\mathrm{f} = 2\,(u_\tau / U_\mathrm{b}^\mathrm{s})^2$, is well predicted applying the EPM. Accordingly, the velocity profiles in inner scaling, presented in Fig. 5, exhibit a similarly high level of agreement with the reference data, whereby a slight underprediction in the log-region is obtained for the topography RB, due to the slight overestimation of the corresponding friction velocity. In any case, the results demonstrate that the effect of varying skewness is captured well with the present geometry based parametrization of the drag force.

Table 2. Relative deviation of the wall friction velocity ($\Delta u_\tau = |u_\tau - u_{\tau,\mathrm{DNS}}| / u_{\tau,\mathrm{DNS}}$) for the various roughness topographies.

	Δu_τ
RA	0.3%
RB	3.4%
RC	0.2%
RD	0.8%

Fig. 6 shows the resolved turbulent intensity components based on Reynolds stresses (e.g. $u'^+_{x,\mathrm{rms}} = \sqrt{\overline{u'_x u'_x}} / u_\tau$; dispersive stresses are not considered) for the four roughness topographies, again normalized with the friction velocity u_τ. In the outer layer, reasonable agreement with a slight underprediction of all turbulent intensity components is achieved for the four roughness topographies. Below the roughness crest, turbulent fluctuations are suppressed to a higher extent, likely due to the damping properties of the

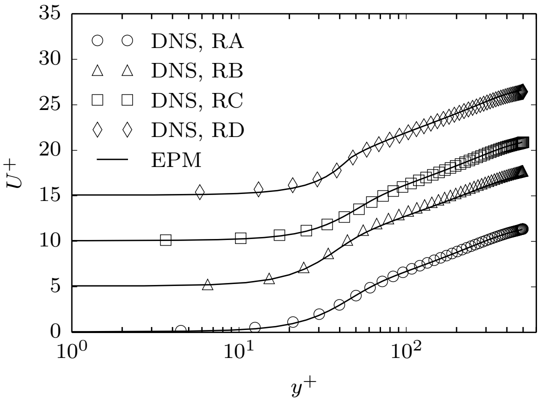
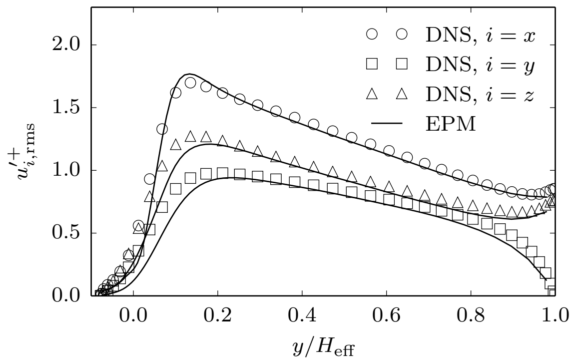
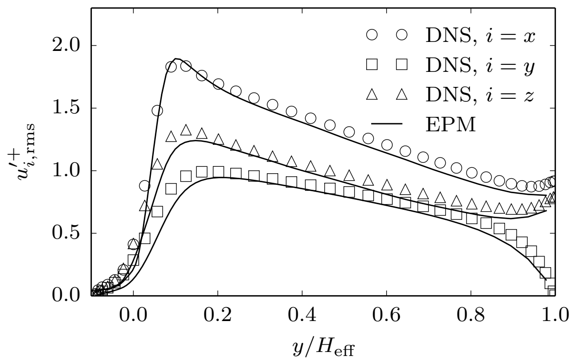


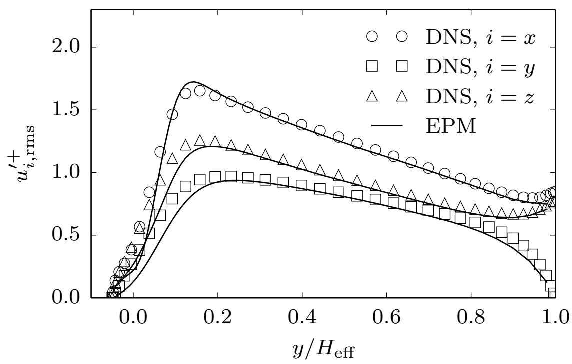
Figure 5. Mean velocity profiles normalized by friction velocity u_τ for all roughness topographies. DNS data taken from Forooghi *et al.* (2017).



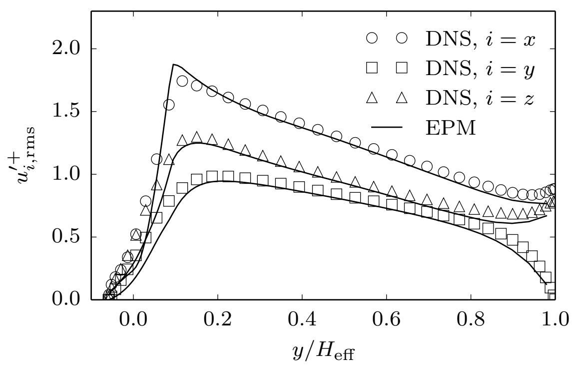
(a) sample RA



(b) sample RB



(c) sample RC



(d) sample RD

Figure 6. Turbulence intensity components normalized by the friction velocity u_τ for different roughness topographies, compared to DNS databased on Reynolds stresses (e.g. $u'^+_{x,\mathrm{rms}} = \sqrt{\overline{u'_x u'_x}} / u_\tau$). DNS data from Forooghi *et al.* (2017).

volumetric forcing term. In any case, it seems unlikely that an accurate representation of the turbulent structures can be achieved below the roughness crest with a spatially homogeneous forcing approach, due to the associated small scales and complex geometry of the roughness, which is not resolved in the computations. Instead, the main goal here is to achieve a correct mean velocity field and provide grounds for an accurate representation of the turbulent state above the roughness crests. In this regard, the presently applied volume-averaged equations explicitly accounting for the mean blockage of the roughness elements are believed to be important to capture modified pressure redistribution effects (compared to a smooth wall).

CONCLUSION AND OUTLOOK

A roughness model for scale-resolving simulation methods, based in principle on the discrete element method of Taylor *et al.* (1985), was proposed. The model, denoted as equivalent-porosity model (EPM), accounts for viscous and form drag roughness elements exert on the flow by means of a volumetric force term in the momentum equation. In order to appropriately capture blockage effects, volume-averaged governing equations, originally proposed in the context of porous media flow, are adopted. The volumetric force is parameterized based on a modified drag closure for packed beds, which relies on geometrical details of the roughness topography as well as model coefficients for viscous and form drag. The model coefficients have been determined in an *a priori* fashion based on DNS data. As closure for turbulent sub-scale contributions, the eddy-resolving (ER) ζ-f model, a computationally efficient RANS-based sub-scale model, is applied, which was appropriately modified for the volume-averaged mathematical framework.

For validation purposes, four synthetic, irregular roughness topographies in an open-channel flow configuration at a friction Reynolds number of $\mathrm{Re}_\tau \approx 500$ were considered, for which reference data from geometry-resolving DNS of Forooghi *et al.* (2017) is available. The roughness topographies vary primarily in terms of their skewness Sk, which has been identified as one of the most influential geometry-related parameters with respect to the roughness function. The results obtained with the EPM, applying a single set of model coefficients, exhibit a high level of agreement with the reference DNS database in terms of mean velocity profiles as well as friction velocities. Turbulent intensities above the roughness crest are captured similarly well. Overall, the suitability of the geometry-based parametrization in conjunction with the volume-averaged equations accounting for blockage effects is demonstrated for the present irregular roughness topographies.

For future applications of the EPM, the model coefficient for viscous drag should ideally be recalibrated based on DNS data at lower Reynolds numbers, since for the present roughness topographies and flow conditions, form drag constitutes the dominant contribution to the overall drag. In addition, a generalization of the model for non-isotropic roughness would be a useful extension towards a more general roughness closure for realistic rough surfaces.

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