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PEAK-TO-MEAN CONCENTRATION RATIO AND FRACTAL SCALING

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ABSTRACT

This paper reports a study of the fractal dimension of concentration time series from a steady point source release in an array of cuboids. The study confirms that these time series are fractal. The fractal scaling is consistent with the peak-to-mean concentration ratio in power law which were obtained from the same concentration time series. This may explain the well-known power law function that is frequently used to predict the peak-to-mean concentration ratio.

INTRODUCTION

It is of great interest to predict peak concentrations from a release in turbulent flow. Numerical simulations and experimental measurements have finite resolution, and are limited in their capabilities to predict extreme concentrations directly. Instead, statistic approaches are used to extrapolate numerical and experimental data, such as the techniques based on Extreme Value Theory (e.g. Xie et al 2007). Another frequently used approach to predict extreme concentration is to estimate the p-value of the peak-to-mean concentration function in power law (e.g. Gifford, 1960; Bartzis et al, 2015; Santos et al, 2019),

$$\frac{\bar{C}(\Delta\tau)_{max}}{\bar{C}(\Delta T)} = \left(\frac{\Delta\tau}{\Delta T}\right)^{-p}, \qquad (1)$$

where ΔT is a time interval long enough to obtain a converged mean $\bar{C}(\Delta T)$, $\Delta\tau$ is a much shorter time interval, typically of duration $\Delta T/n$ where n is the finite number of intervals. $\bar{C}(\Delta\tau)$ is the time-averaged concentration over $\Delta\tau$ and $\bar{C}(\Delta\tau)_{max}$ is its maximum value. A p-value can be obtained by fitting $\bar{C}(\Delta\tau)_{max} \sim \Delta\tau$ with various short time intervals $\Delta\tau$. Santos et al (2019) used this technique for concentration time series from a ground level point source release in an array of cuboids, and found that the exponent p ranges from 0.1 to 0.5.

Peak concentration over an extremely short time interval less than the time resolution of the available data can be obtained by using extrapolation of Eq. (1). However, it is to be noted that the extrapolated extreme concentration in Eq. (1) is not bounded. The concentration cannot exceed the source concentration in applications. Some researchers managed to improve Eq. (1) by considering the upper limit in an equation. However, this is not the focus of this paper, and will not be discussed further.

Eq. (1) is simple and is easy to use in practical applications. Although it has been widely used to predict peak concentration, it has been criticised that it has no obvious theoretical basis (e.g. Santos et al, 2019). Therefore, it remains a question how to understand the mechanism behind it.

Fractal dimension analysis has been used in different applications, such as on the areas of turbulent and non-turbulent interface (Krug et al, 2017), on the cloud shape parameters (Gotoh and Fujii, 1998), and on wind speed time series (Chang et al, 2012). Chang et al (2012) found that the yearly fractal dimension ranges from 1.2 to nearly 2.0. It is to be noted that they analysed serval-year long time series.

To the best of our knowledge, fractal dimension analysis has not yet been used on concentration time series. This paper uses fractal dimension technique to analyse the concentration time series from a ground level source release in an array of cuboids (see more about the time series in Santos et al, 2019), which were generated from large-eddy simulations (LES) (see more of the LES settings in Fuka et al, 2017). Finally, we aim to seek whether there is a link between Eq. (1) and the fractal scaling for such applications, and to understand the mechanism behind Eq. (1).

LES DATA

Figure 1 shows the computational domain and the location of the steady source and the 8 sensors for data sampling. Periodic boundary conditions were used for the velocity field in x and y direction. For the scalar field, zero concentration was set at the inlet. The flow was driven by a body force in the x direction. The Reynolds number based on the block height and the effective friction velocity (which was estimated from the body force and the domain height) was approximately 1,000. A uniform mesh was used with a grid size $h/16$, where h was the block height. The mixed time scale sub-grid scale model was used. The ground level source size was $0.244h$ in diameter. After the flow and scalar field had fully developed, instantaneous concentrations were sampled at the 8 stations for 54,500 time steps which is equivalent to $2{,}100T^*$, where $T^* = \Delta T * U_\infty/h$ with U_∞ the freestream velocity. More details can be found in Santos et al (2019) and Fuka et al (2017).

FRACTAL DIMENSION ANALYSIS

The so-called box-counting dimension is a widely used indicator for the fractal dimension and is defined as

$$\boldsymbol{D} = \mathbf{lim}_{\varepsilon\to 0} \frac{\ln N(\varepsilon)}{-\ln\varepsilon}, \qquad (2)$$

where $N(\varepsilon)$ is the smallest number of square boxes with side $\boldsymbol{\varepsilon}$ to cover the data set. To estimate the fractal dimension of the concentration time series, a modified box-counting method was used (Chang et al 2012):

$$\boldsymbol{D} = \mathbf{2} - \boldsymbol{H}(\emptyset), \qquad (3)$$

where

$$H(\emptyset) = \lim_{\Delta\tau\to 0} \frac{\ln\emptyset(\Delta\tau)}{\ln\Delta\tau}\ , \quad (4)$$

and

$$\emptyset(\Delta\tau) = \sum_{i=1}^{n-1} |c(t_i + \Delta\tau) - c(t_i)|\Delta\tau. \quad (5)$$

Here $c(t_i)$ is the instantaneous concentration at time t_i, $\Delta\tau$ is a time interval equal to $\Delta T/n$, ΔT is the total time interval of the entire time series, n is the number of sectors of evenly partitioned shorter time series.

Eq. (3) is not practically suitable for estimation of the fractal dimension. Instead, we re-arrange Eqs. (3) - (5) and get,

$$D = \lim_{\Delta\tau\to\mathbf{0}} \frac{\ln[\emptyset(\Delta\tau)/\Delta\tau^2]}{\ln(1/\Delta\tau)}\ . \quad (6)$$

For a time series with a finite length, by using a set of different interval $\Delta\tau$, we use least-squares linear regression to fit the data set to the following equation,

$$\ln[\emptyset(\Delta\tau)/\Delta\tau^2] \cong D\ln(1/\Delta\tau) + \lambda, \text{ as } \ \Delta\tau \to 0\,. \quad (7)$$

Here, D is the slope of the fitted linear line and λ is the residual of regression.

Re-arranging Eqs. (4-5), we get,

$$\sum_{i=1}^{n-1} |c(t_i + \Delta\tau) - c(t_i)| = \Delta\tau^{-(1-H)} = \Delta\tau^{-P}, \text{ as } \ \Delta\tau \to 0, \quad (8)$$

where the exponent $P = 1 - H$, and

$$P = D - 1. \quad (9)$$

Eq. (1) and Eq. (8) are similar in format. Now the question is whether there is a correlation between the two exponents p and P.

RESULTS AND DISCUSSION

Santos et al (2018) shows that $\bar{\boldsymbol{C}}(\Delta\boldsymbol{\tau})_{\boldsymbol{max}}$ is nearly constant for $\Delta\boldsymbol{\tau} < \mathbf{10}\boldsymbol{\delta t}$, where $\boldsymbol{\delta t}$ is the time step in the LES. Therefore, the smallest time interval $\Delta\boldsymbol{\tau}$ for the first set of short time series was chosen to be $5\boldsymbol{\delta t}$ for the analysis of fractal dimension. The time interval $\Delta\boldsymbol{\tau}$ of the 2nd set was $\mathbf{1.5}$ times of the first, and the time interval $\Delta\boldsymbol{\tau}$ of the 3rd set was $\mathbf{1.5}$ times of the 2nd, and so on. 10 sets data per sensor were processed with the largest $\Delta\boldsymbol{\tau}$ much greater than the integral time scale of this flow.

Figure 2 plots $\boldsymbol{ln}[\emptyset(\Delta\boldsymbol{\tau})/\Delta\boldsymbol{\tau}^{\mathbf{2}}]$ against $\boldsymbol{ln}(\mathbf{1}/\Delta\boldsymbol{\tau})$ for the 1st , 2nd and 3rd sensors of the right column in Figure 1 and the linear regression results. The fittings for the other sensors are in similar performance as these three sensors.

Figure 3 shows an overall comparison of the estimated exponents between the peak-mean ratio (Eq. 1) and the fractal dimension analysis (Eq. 8). We have observed the following points. Firstly, both of the two techniques yield much greater exponents for the sensors of the right column than for those of the left column (Figure 1). Note the sensors in the right column are closer to the edge of the plume where the concentration time series are more random, and yield greater exponents as we expect. Secondly, the exponents estimated from the two techniques are highly correlated with a maximum discrepancy 25%. Thirdly, uncertainties of estimation of the exponents are worth further investigation.

CONCLUDING REMARKS

It is evident that the exponents estimated from the fractal dimension analysis are highly correlated with those estimated from the peak-mean ratio analysis. Therefore, this strongly suggests that Eq. (1) arises because the concentration time series is fractal. Now a new question arises - what is the difference of the exponent p in Eq. (1) and the exponent P in Eq. (8), or they are identical? The following argument provides some insight into why p and P are so similar in value. Eq. (1) can be rewritten as

$$\frac{\bar{C}(\Delta\tau_1)_{max}}{\bar{C}(\Delta\tau_2)_{max}} = \left(\frac{\Delta\tau_1}{\Delta\tau_2}\right)^{-p}, \quad (10)$$

where $\Delta\tau_1$ and $\Delta\tau_2$ are two different time intervals. Eq. (10) reflects a similarity across the two scales. We rewrite Eq. (8) as

$$\frac{\sum_{i=1}^{n-1}|c(t_i+\Delta\tau_1)-c(t_i)|}{\sum_{j=1}^{m-1}|c(t_j+\Delta\tau_2)-c(t_j)|} = \left(\frac{\Delta\tau_1}{\Delta\tau_2}\right)^{-P}. \quad (11)$$

Eq. (11) reflects a similarity across the two scales too. Note that $\bar{C}(\Delta\tau)_{max}$ and $\sum_{i=1}^{n-1}|c(t_i + \Delta\tau) - c(t_i)|$ in Eqs. (10) and (11) respectively are both first order statistics. As suggested earlier that the concentration time series is fractal and a similarity across scales exists. Therefore the ratio of $\bar{C}(\Delta\tau)_{max}$ to $\sum_{i=1}^{n-1}|c(t_i + \Delta\tau) - c(t_i)|$ must be constant which is independent of the time scale $\Delta\tau$, given it is small enough. This then implies that indeed the exponent p in Eq. (1) and the exponent P in Eq. (8) are identical.

It is worthwhile to see whether it would be possible to provide more rigorous arguments for why p and P are so similar in value. Apart from this, several questions remain, such as the link between the fractal dimension of the concentration time series and that of the area of the concentration cloud surface, and the upper limit of the peak concentration.

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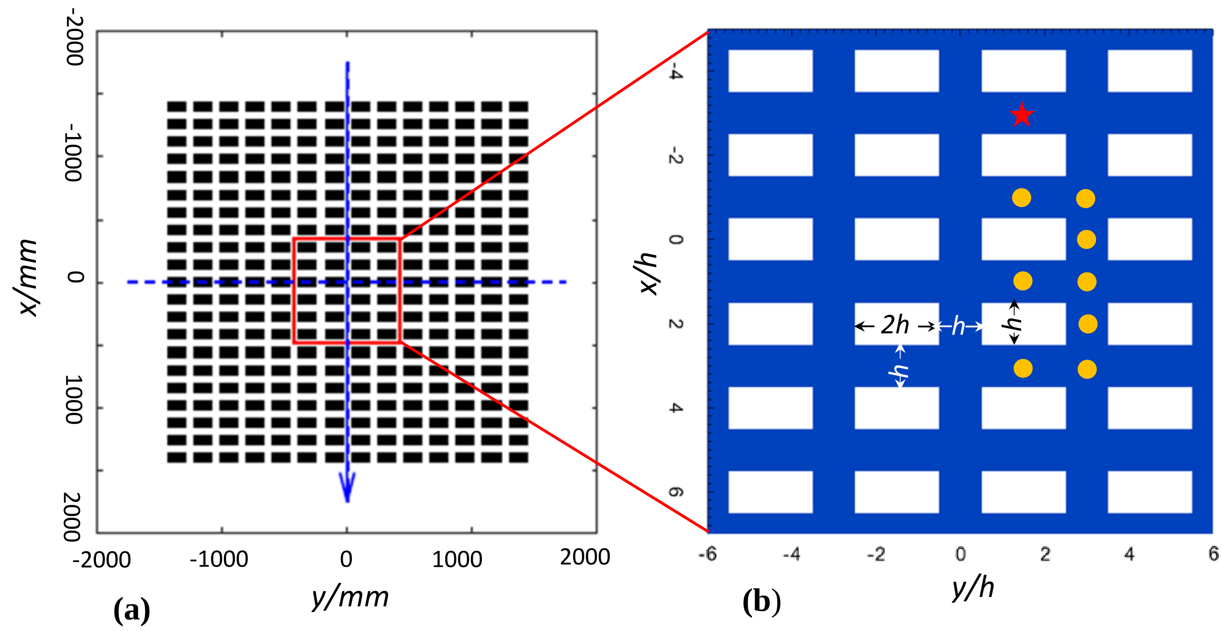

Figure 1. Arrays of aligned cuboids with dimensions 2*h* (length)× *h* (width)× *h* (height). All spacings between the cuboids are *h*, with *h* = 70mm. a, wind-tunnel model. b, Numerical model in a computational domain 12*h* (L_x) × 12*h* (L_y) × 12h (L_z). The star marks the ground level source, which is at the middle of the long street. The filled dots mark the sampling sensors at a height 0.5_h. The 3 sensors in the left column are at the middle of the long street, and are 2*h* apart from each other. The 5 sensors in the right column are either at the middle of the short street or at the intersection, and are *h* apart from each other.

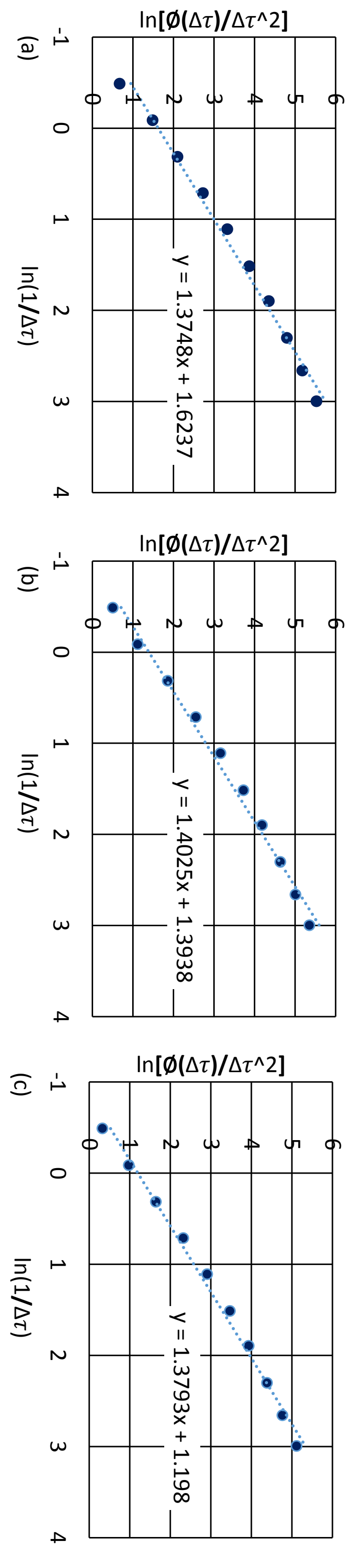

Figure 2. Examples of estimation of fractal dimension using least-squares linear regression (Eq. 7). (a) (b) and (c) are data sampled respectively at the 1st , 2nd and 3rd sensors of the right column in Figure 1.

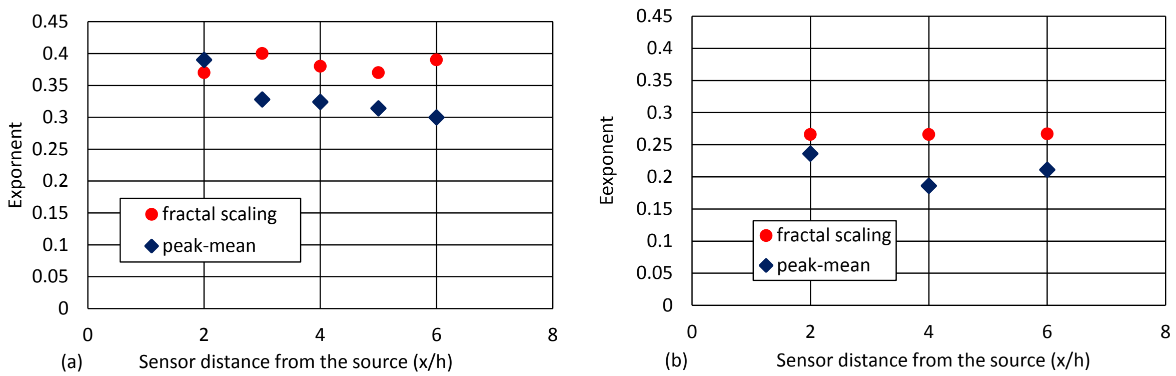

Figure 3. A comparison of the exponents obtained using fractal scaling analysis and peak-mean ratio technique. (a) data of the sensors of the right column in Figure 1; (b) data of the sensors of the left column in Figure 1.