EXPERIMENTAL STUDY OF A HIGH REYNOLDS NUMBER TURBULENT BOUNDARY LAYER EVOLVING OVER A ROUGH-TO-SMOOTH CHANGE IN SURFACE CONDITION

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ABSTRACT

The evolution of turbulent boundary layers downstream of a rough-to-smooth transition is investigated at a range of Reynolds numbers. Measurements are performed at friction Reynolds numbers of 4100, 7100, 14000 and 21000 using hotwire anemometry. The wall-shear stress on the smooth surface in each case is measured directly using oil film interferometry. The growth of the internal layer is studied, and a full recovery of all energetic scales in the energy spectrum of the streamwise velocity fluctuations is observed at 80 boundary layer thicknesses downstream of the roughness transition. A comparison of recovery lengths required for various flow statistics (skin-friction coefficient and energy spectrum) is also presented.

INTRODUCTION

Turbulent boundary layers over heterogeneous roughness have a wide range of applications in aerospace and maritime industries, as well as in meteorology. In this study, we consider a simplified two-dimensional case of a rough-to-smooth change in the streamwise direction, as depicted in figure 1. The turbulent boundary layer develops initially over a smooth fetch. At some streamwise location, thereafter referred to as \(x_0\), the surface condition changes to smooth. Following the transition, the new smooth wall condition initially modifies the near-wall region, which then gradually propagates towards the interior of the flow with increasing distance downstream of the transition. The near-wall layer where the flow is modified by the new smooth-wall condition is generally referred to as the internal boundary layer (IBL) with a thickness denoted by \(\delta_i\) (Garratt, 1990).

Despite studies over the past few decades (Antonia & Luxton, 1972; Hanson & Ganapathisubramani, 2016; Ismail et al., 2018), the recovery of the flow following a streamwise rough-to-smooth change is still not fully understood. In particular, the dependence of the flow recovery on the Reynolds number is yet to be systematically examined due to the difficulty in running high Reynolds number simulations and experimental measurements.

Accordingly, this study presents a set of carefully designed experiments to study the evolution of a turbulent boundary layer downstream of a rough-to-smooth transition over a wide range of Reynolds numbers. In this paper, \(x\), \(y\) and \(z\) indicate the streamwise, spanwise and wall-normal directions, respectively. Corresponding mean velocity components are represented by \(U, V\) and \(W\), and the velocity fluctuations are denoted by \(u, v\) and \(w\).

EXPERIMENTAL SETUP

A set of experiments is conducted with varying \(Re_{\infty}\) while holding \(k_s\) constant. \(Re_{\infty}\) is the friction Reynolds number, defined by \(Re_{\infty} \equiv \delta_{99} U_\infty / \nu\) and \(k_s \equiv U_\infty / \nu\) is the roughness Reynolds number. Here \(\delta_{99}\) is the boundary layer thickness (defined as the wall-normal location where the mean velocity reaches 0.99 \(U_\infty\)), \(U_\infty\) is the mean friction velocity, \(\nu\) is the kinematic viscosity of air and \(k_s\) is the equivalent sandgrain roughness. The subscript ‘0’ refers to conditions at the location of the rough-to-smooth transition. The same type of sandpaper is used in all cases, which ensures a constant \(k_s\), while \(x_0\), the downstream location of the roughness transition is varied. The freestream velocity \(U_\infty\) is adjusted to account for the gradual decrease of \(C_f\) with Reynolds number, to maintain a constant \(U_\infty\) at the rough surface immediately upstream of the roughness transition. This will guarantee a constant \(k_s\) for all cases. The variation of \(Re_{\infty}\) is primarily achieved by varying the \(x_0\) location of the transition. Each case is assigned a code in the format of \(Re_{\infty}xxksyy\), where \(xx\approx Re_{\infty}/1000\), and \(yy\approx k_s^*/10\).

THE FACILITY

Most of the experiments are performed in the High Reynolds Number Boundary Layer Wind Tunnel (HRN-
Table 1. Summary of the experimental cases. The friction velocity $U_\tau_0$ employed in calculating $Re_\tau_0$ and $k_+^*$ is obtained over the rough fetch in the immediate upstream of the rough-to-smooth transition. $\delta_0$ is the boundary layer thickness (where the mean velocity reaches $0.99U_\infty$) at the roughness transition, and $l^+$ is calculated using the friction velocity at the most downstream measurement location on the smooth surface.

<table>
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<th>Case</th>
<th>Symbol</th>
<th>Roughness</th>
<th>$Re_\tau_0$</th>
<th>$k_+^*$</th>
<th>$x_0$ (m)</th>
<th>$U_\infty$ (ms$^{-1}$)</th>
<th>$\delta_0$ (m)</th>
<th>$l^+$</th>
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<td>4100</td>
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<td>15.0</td>
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Figure 2. Overview of the experimental setup. The flow is going from the left to the right. The grey shaded surface represents the sandpaper, and the white colour represents the smooth wall. Streamwise locations where a wall-normal hotwire profile is acquired in each case are shown by the corresponding symbols. Note that as Re04ks13 is conducted in a different wind tunnel, the length and width of the working section are different from the rest.

Figure 3. (a) The surface elevation at the rough-to-smooth transition measured using an in-house built laser scanner. The black line in (b) is the spanwise average of the surface elevation, and the red and blue lines are the maximum and minimum of the surface elevation along each spanwise line.

BLWT) with a working section of 27 m at the University of Melbourne. The overview of the experimental setup is depicted in figure 2. $x$ is the distance downstream of the inlet to the working section, $x_0$ is the streamwise location of the surface transition and $\hat{x} \equiv x - x_0$ is the distance downstream of the transition. An upstream portion of the tunnel floor in the test section is covered by P24 grit sandpaper (SP40F, Awuko Abrasives) from the inlet to the location of $x_0$ (as shown by the grey coloured patch in figure 2), while the remaining length is a smooth aluminium surface. To characterise the roughness parameters, a 60mm $\times$ 60mm patch of the rough-to-smooth transition is scanned using an in-house built laser scanner. The resulting surface topography is shown in figure 3a-b.

HOTWIRE ANEMOMETRY

Velocity profiles are obtained by traversing a single-normal hotwire probe over 40 logarithmically spaced wall-normal locations from $z/\delta_0 \approx 0.001$ to 2. The sensing element of this probe has a diameter of $d = 2.5\mu m$ and a length of 0.5 mm. Wall-normal boundary layer profiles are measured at over 10 logarithmically spaced streamwise locations downstream of the rough-to-smooth transition, from $\hat{x} = 12\text{mm}$ to $x = 21\text{m}$. The most downstream measurement is obtained at $\hat{x}/\delta_0 = 120$, 34 and 14 for case Re07ks16, Re14ks16 and Re21ks16, respectively. A reference profile above the rough surface is also acquired at $\hat{x} = -0.1\text{m}$ in each case. In order to accurately measure the wall location, a magnetic linear encoder is incorporated into the hotwire traversing system, while the initial wall offset is determined using a DSLR camera mounted outside the tunnel that captures the location of the hotwire probe and its reflection on the surface using high magnification optics.

OIL FILM INTERFEROMETRY

The wall-shear stress $\tau_w$ is obtained using Oil Film Interferometry (OFI) following a similar process as described in Li et al. (2019). A 1.4 m long glass insert has been installed in the tunnel floor at $x = 5\text{m}$, providing optical access from the underside of the tunnel for $\hat{x}/\delta_0 < 7$ in case Re07ks16. This configuration is similar to the approach described in Li et al. (2019), and a well-resolved fringe pat-
ttern with approximately 50 pixels per wavelength can be achieved. A line of silicone oil is placed along the spanwise direction on the glass surface and spread downstream by the wind shear. The oil film is illuminated by an Imalent DX80 LED torch, and recorded using a Nikon D810 DSLR camera with a Tamron 180mm macro lens. A 532nm bandpass filter with a bandwidth of 10 nm is attached to the camera lens to obtain monochromatic fringe patterns.

For the remaining measurements, a glass insert on the centerline of the working section ceiling provides optical access from above. To improve the fringe quality, the tunnel floor is covered by a piece of black mylar film with a thickness less than 40 µm (equivalent to 2 wall units). The same illumination and imaging system as in the previous configuration is used, but with a reduced resolution of approximately 30 pixels per wavelength due to the 1 m stand-off distance between the camera and oil film. Both OFI configurations (imaging from underneath and above) have been compared at $\delta/\delta_b = 4$ for case Re07ks16, and are shown to give the same result to within 1%.

For both configurations, 100 images are captured with a time interval of five seconds in each measurement. The camera calibration and image processing algorithm are the same as detailed in de Silva et al. (2018).

LOW REYNOLDS NUMBER CASE

The lowest Reynolds number dataset Re04ks13 in the present study is obtained in an open return section wind tunnel also at the University of Melbourne using a similar experimental arrangement. Seven boundary layer profiles are acquired using hotwire anemometry at $\delta/\delta_b = 0.1 - 13.4$, as well as a reference profile on the rough wall at $\delta/\delta_b = -0.2$. The wall-shear stress on the smooth wall is measured using OFI through an optical access on the tunnel floor. Readers are referred to de Silva et al. (2018) and Li et al. (2019) for further details. Parameters of all the datasets used in this study are summarised in table 1. Note that a different type of sandpaper with larger and sparser grains is used in Re04ks13, along with a lower freestream velocity which results in a $\delta_b$ that is 20% lower compared to the other three cases.

RESULTS AND DISCUSSION

INTERNAL BOUNDARY LAYER

The extent of flow recovery can be quantified by the growth of the IBL. The IBL height $\delta_b$ at each streamwise location is calculated based on the difference between the $\overline{u^2}/U_∞^2$ profile at the current location and the neighbouring upstream measurement location, i.e. $\delta_b$ is defined as the wall-normal location where $\partial(\overline{u^2}/U_∞^2)/\partial x \to 0$. It is well-known that $\overline{u^2}$ exhibits outer-layer similarity only when normalised by $U_∞^2$, and a dependence on Reynolds number presents if the velocity scale $U_∞$ is used instead. However, the adequacy of this approach can be justified considering that the largest change of Reynolds number between the neighbouring profiles used to compute $\delta_b$ is usually within 10%, and the difference in the Reynolds number is negligible close to the roughness transition. The majority of the measurements are concentrated in this region owing to the logarithmic streamwise spacing employed for these measurements. It has been shown that $\delta_b$ determined from the turbulence intensity profile is comparable with the results from the more conventional methods based on the mean velocity profiles (Pendergrass & Arya, 1984; Rouhi et al., 2019). Here we favour the turbulence intensity approach as the distinction associated with the roughness change is more pronounced in $\overline{u^2}$ compared to $U$ and less subject to small uncertainties in the measurement, resulting in a more robust estimation of $\delta_b$.

Figure 4 illustrates the process of extracting $\delta_b$ from the outer-scaled turbulence intensity profiles. A good collapse presents in the outer layer with no appreciable Reynolds number trend, and the decrease in the turbulence intensity related to the internal layer growth is much more pronounced in comparison. In practice, a threshold of $\Delta(\overline{u^2}/U_∞^2) = 3 \times 10^{-4}$ rather than 0 is selected to account for the noise in measurements and also the weak Reynolds number trend (this threshold is illustrated by the black dashed line in figure 4b).

Using the method described above, $\delta_b$ at various streamwise locations is calculated for all cases and presented in figure 5. $\delta_b$ is normalised by the local boundary layer thickness $\delta_9$, while $\delta$ is normalised by $\delta_b$, the boundary layer thickness at the rough-to-smooth transition. All data points collapse on to a straight line in logarithmic scale with no distinguishable Reynolds number trend. A power-law fit

$$\delta_b/\delta_9 = A(\delta/\delta_b)^b$$

results in coefficients of $A = 0.095$ and $b = 0.73$. We consider an alternative power-law relation $\delta_b/\delta_9 = A_0(\delta/\delta_b)^{b_0}$, which is a better representation of $\delta_b$ growth in physical space. A fit through the current data results in $b_0 = 0.8$.
with comparable quality (blue line in figure 5). This agrees closely with the observations of Bradley (1968) and Mulhearn (1978), where $\delta_{t}$ is defined as the ‘merging point’ in the mean velocity profile. The growth appears to be more aggressive than $\delta_{t} \approx 0.43^{0.43}$ (Antonia & Luxton, 1972), and this discrepancy is likely to be due to the different type of roughness (2D square ribs instead of sandpaper), and also to the different extraction method of finding the inflection point in the $U$ versus $1/t^{1/2}$ plot.

If we assume that the flow within the IBL is in equilibrium with the new wall condition, then a complete recovery is achieved when $\delta_{t} = \delta_{t}^{b}$, which is predicted to be $\delta_{t}^{b} / \delta_{0} = 26.5$ using equation (1). However, we would like to re-emphasise that here we adopt the definition of IBL as the region where the flow is modified by the new wall condition, and the flow inside IBL has been shown to be in non-equilibrium state (see Antonia & Luxton, 1971; Rouhi et al., 2019; Li et al., 2019). This implies that even when $\delta_{t} \rightarrow \delta_{t}^{b}$ (when the internal layer has grown to the full layer height), the boundary layer may still not be in equilibrium with the new wall condition. A complete recovery of the flow to quasi-equilibrium is expected at a longer fetch.

**SKIN-FRICTION COEFFICIENT**

The skin-friction coefficient $C_{f} \equiv \tau_{\text{w}} / (\frac{1}{2} \rho U_{\infty}^{2})$ over the smooth surface is obtained from the OFI measurements. For the reference profile on the rough wall, $C_{f}$ is calculated using the modified Clauser chart method (Squire et al., 2016). As shown in figure 6a, $C_{f}$ undershoots the expected equilibrium smooth-wall value (shown by the solid lines) immediately downstream of the roughness transition in all four cases, and overall, $C_{f}$ for both rough and smooth surfaces decreases with $Re_{\theta}$. Such a Reynolds number dependence is expected and is similar to that observed for a turbulent boundary layer developing over a homogeneous surface (e.g. Nagib et al., 2007).

To better quantify the state of the recovering boundary layer, we define a reference quantity $C_{fe}$, as the ‘equilibrium skin-friction coefficient’, which is the skin-friction coefficient that an equilibrium turbulent boundary layer at the same Reynolds number (based on momentum thickness) would have. $C_{fe}$ is estimated using an empirical relation obtained from drag balance measurements of a smooth-wall turbulent boundary layer in the same wind tunnel facility (Baars et al., 2016):

$$C_{fe} = 2 \log \left( \frac{Re_{\theta}}{0.38 + 3.7} \right)^{-2},$$

where $Re_{\theta} \equiv U_{in} \theta / v$, and $\theta$ is the momentum thickness computed locally by integrating the measured mean velocity profile. If the flow has fully recovered to the smooth-wall condition, then $C_{f}$ should equal to $C_{fe}$. Therefore, it can serve as an indication of the flow recovery. Figure 6b indicates that $C_{f}$ is approximately 70%—80% of $C_{fe}$ in the immediate downstream of the rough-to-smooth transition for all cases, followed by a quick recovery within $20 \delta_{0}$. The data points overshoot $C_{f} / C_{fe} = 1$ (the black horizontal line) slightly and then reach a plateau at $C_{f} / C_{fe} = 1.03$. This 3% difference is possibly related to the uncertainty in the data and the empirical relationship employed. Regardless, there seems to be little difference between cases in terms of the $C_{f}$ recovery behaviour when scaled by $\delta_{t}$ and $C_{fe}$. After $C_{f}$ reaches its maximum at $\delta_{t} / \delta_{0} \approx 20$ in figure 6a, it decreases gradually further downstream as dictated by the increasing Reynolds number of the flow. When normalised by $C_{fe}$ as shown in figure 6b, to within the experimental uncertainty, beyond $\delta_{t} / \delta_{0} \approx 20$, $C_{f}$ evolves as if the flow were fully in equilibrium with the smooth wall.

Figure 5. IBL thickness $\delta_{t}$ normalised by the local boundary layer thickness $\delta_{t}^{b}$ versus the fetch $\hat{x}$ over the downstream smooth surface scaled by $\delta_{t}^{b}$, the boundary layer thickness at the roughness transition (symbols). The dashed line is equation (1), and the blue line is the best fit to $\delta_{t} / \delta_{0}$ data (data points omitted for clarity).

Figure 6. (a) Skin-friction coefficient $C_{f}$ versus the fetch $\hat{x}$ over the downstream smooth surface scaled by $\delta_{t}^{b}$. The coloured symbols represent OFI measurements on the smooth wall, and the black symbols are obtained from the reference profile over the rough surface. The solid lines are $C_{fe}$ at every streamwise location. (b) $C_{f}$ normalised by its equilibrium value $C_{fe}$. The solid horizontal line is $C_{f} / C_{fe} = 1$, and the dashed line is $C_{f} / C_{fe} = 1.03$. The inset shows a magnified view of the fetch immediately downstream of the transition.
Figure 7. Viscous scaled premultiplied energy spectrum \( \omega \phi_{uu}/U_z^2 \). The colour contours correspond to the rough-to-smooth case Re07ks16, and the white contour lines are interpolated from a reference smooth-wall experimental dataset to matched \( Re_\tau \). Contour levels are chosen at \( \omega \phi_{uu}/U_z^2 = 0 \) to 2 with an increment of 0.25. The vertical black dashed line represents the location of \( \delta \).

Figure 8. The difference between the viscous scaled premultiplied spectrum of Re07ks16 and the smooth-wall reference (matched \( Re_\tau \)) at streamwise locations corresponding to figure 7. The four black contour lines indicate \( \Delta(\omega \phi_{uu}/U_z^2) = 0, 0.25, 0.5, 0.75 \) and 1. The vertical black dashed line represents the location of \( \delta \). The blue line is the difference in \( \phi_{uu}/U_z^2 \) integrated across all wavelengths.

RECOVERY OF THE ENERGY SPECTRUM

Most previous laboratory measurements only cover a downstream fetch of approximately 20\( \delta \), or less, where usually, the flow still has not reached full equilibrium with the new wall condition. In the present study, we are able to measure up to 120\( \delta \) for case Re07ks16, which enables us to study the recovery of the flow in the far field.

Accordingly, the premultiplied energy spectrum \( \omega \phi_{uu}/U_z^2 \) of case Re07ks16 is shown in figure 7, where \( \omega = 2\pi f/T \) is the angular frequency, \( T \) is the time period (corresponding to the wavelength in spatial domain), \( \phi_{uu} \) is the energy spectrum of the streamwise velocity fluctuation \( \int_{-\infty}^{\infty} \phi_{uu} \cos \omega \) \( d\omega \), and \( U_z \) is the friction velocity measured from the OFI experiments. The spectrograms presented are computed from hotwire time series data. Further, since the flow is heterogeneous in \( x \), we refrain from converting the spectrum from temporal to the spatial domain, which has been shown to have limited accuracy in rough-wall flows (Squire et al., 2017). The coloured contours are the current rough-to-smooth data, and the white contour lines are interpolated from a reference smooth-wall experimental dataset (Marusic et al., 2015; Squire et al., 2016) to matched \( Re_\tau \), which ensures that the energy diminishes at the same wall-normal height in viscous units in both rough-to-smooth case and the smooth-wall reference. To further elucidate this behaviour, figure 8 shows the difference between the rough-to-smooth spectrum and the reference smooth-walled spec-
A complete recovery of the energy spectrum is achieved at $\hat{x}/\delta_i = 78.7$, as shown in figure 7f and figure 8f. Note that the complete recovery of the energy spectrum is expected between 39.4$\delta_i$ to 78.7$\delta_i$ downstream of the roughness transition, as there is no measurement location in between. Regardless, it takes a longer fetch downstream for the energy spectrum to relax completely to the smooth-wall state than for the IBL to outgrow the original boundary layer. As limited by the length of the tunnel working section, no measurement is available beyond $\hat{x}/\delta_i = 40$ and $\hat{x}/\delta_i = 15$ for case $Re_{14k}{\delta_i}$ and $Re_{21k}{\delta_i}$, respectively. Nevertheless, the difference in the pre-multiplied energy spectrum at the most downstream location in each case is significant by the power-law $\delta_i$ locations, suggesting that the downstream fetch required for a full recovery in energy spectrum may have little Reynolds number dependence when scaled by $\delta_i$.

Comparing figure 6 and figure 8, it appears that within experimental uncertainty, $C_f$ achieves the complete recovery from the roughness transition in a shorter fetch ($20\delta_i$) compared to the energy spectrum ($40\delta_i$–$80\delta_i$). Similar observations have been reported by Rouhi et al. (2019), Ismail et al. (2018) and Sridhar (2018) in their numerical studies.

The streamwise location where $C_f$ has reached the complete recovery seems to coincide with the location where the energetic large-scale footprint in the near-wall region vanishes. Although at $\hat{x}/\delta_i = 15.7$ (figure 8f), this footprint is already becoming very weak for $z^+ < 100$.

CONCLUSIONS

In this study, a high Reynolds number campaign of hotwire and OFI measurements in a turbulent boundary layer developing downstream of a rough-to-smooth surface transition is presented. $C_f$ on the smooth fetch appears to recover to its equilibrium value $C_{fe}$ (the skin-friction coefficient of an equilibrium smooth-wall boundary layer at matched Reynolds number) at $10\delta_i$ to $20\delta_i$ downstream for all cases. No discernible Reynolds number trend is observed in the non-dimensional internal layer thickness $\delta_i/\delta_{99}$ or $\delta_i/\delta_1$ versus the downstream fetch $\hat{x}/\delta_i$, and a power-law fit results in $\delta_i \propto \hat{x}^{0.5}$, which is in close agreement with previous studies. The fetch where the original boundary layer is completely replaced by the developing internal layer ($\delta_i/\delta_{99} = 1$) case $Re_{7sns16}$ at matched $\hat{x}/\delta_i$ locations occurs between $20\delta_i$ and $30\delta_i$. Finally, a full recovery in all energy scales at all wall-normal locations is observed to fall somewhere between $40\delta_i$ and $80\delta_i$.

ACKNOWLEDGEMENT

The financial support of the Australian Research Council is gratefully acknowledged.

REFERENCES


