INTELLIGENT LAGRANGIAN INTERPOLATION ALGORITHM FOR PIV DATA

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ABSTRACT

A new Lagrangian based interpolation scheme is proposed for recovering velocity field data from within masked regions in particle image velocimetry (PIV) experiments. The proposed method estimates velocity field data within the masked region using a weighted average of past and future snapshot data along a streamline from outside of the masked region. The estimation invokes satisfying the steady-state diffusion equation for the velocity field. This Lagrangian interpolation scheme is compared to linear interpolation using a CFD test case and the results indicate a 3-5 times improvement in the relative reconstruction error.

INTRODUCTION

In PIV experiments, erroneous velocity vectors or gaps in PIV data are a common occurrence. These may arise from experimental conditions such as regions of low laser sheet intensity, optical distortions, strong light reflections from a surface, or solid blockage induced shadowing. To remove shadowing or surface reflections, much more complex experimental designs are employed wherein the laser light illuminates the model from multiple angles and often multi-camera systems are required, e.g., Limacher *et al.* (2019). As noted by Sciacchitano *et al.* (2012), these experimental challenges are coupled with other elements contributing to poor velocity calculations such as camera noise and out-of-focus imaging. Thus, erroneous velocity vectors or gaps in PIV data is an ongoing challenge for experimentalists.

Two important applications of PIV measurements are hindered by gappy or erroneous data (Sciacchitano *et al.*, 2012): (i) the calculation of forces acting over a control volume of interest, and (ii) the calculation of pressure directly from the velocity field measurements. Traditionally, erroneous vectors are replaced by either linear or bi-linear interpolation schemes (Raffel *et al.*, 2007) or convolution with a Gaussian kernel (Agüí & Jiménez, 1987). However, these methods become less accurate as the size of the erroneous velocity region increases. In cases where large regions are occluded (e.g. shadowing), the field must generally be masked entirely. More recently, Venturi & Karniadakis (2004) demonstrated the use of proper orthogonal decomposition for reconstructing velocity fields from gappy data, showing a distinct improvement over the use of local Kriging. Subsequently, Sciacchitano *et al.* (2012) developed and implemented the full governing Navier-Stokes equations to predict the velocity in gap regions. This procedure, while providing excellent quality results, comes with the drawback of substantially increased computational requirements: a finite-volume solver is required with well-posed boundary and initial conditions to achieve a stabilized solution.

In the current work, a new interpolation algorithm for erroneous or gappy PIV data is proposed. The method takes advantage of time-resolved information, by recovering vector information from within masked regions based on Lagrangian structure tracking. The methodology developed is compared directly with bi-linear interpolation methods. Importantly, the influence of the employed technique on the estimation of forces through a control volume approach (e.g., Limacher *et al.* (2019)) is elucidated.

METHODOLOGY

Within a PIV domain Ω, a masked region given by Π_m exists in which there is no velocity data. Given the velocity data $u(x,y,t_k)$ and $v(x,y,t_k)$, the mean velocity fields are given by

$$\begin{aligned} U(x,y) &= \frac{1}{N}\sum_{k=1}^{N} u(x,y,t_k), \\ V(x,y) &= \frac{1}{N}\sum_{k=1}^{N} v(x,y,t_k), \end{aligned} \tag{1}$$

where N is the number of PIV snapshots. For an ergodic stationary process with continuous fields, the mean velocity fields are generally quite smooth and can be assumed to satisfy a steady-state diffusion equation:

$$\begin{aligned}\nabla^2 U(x,y) &= 0,\\ \nabla^2 V(x,y) &= 0.\end{aligned} \tag{2}$$

For a square orthogonal grid of vectors, the diffusion equations can be solved numerically via a simple iterative algebraic expression. For example, the streamwise velocity $U(x,y)$ within the cell (m,n) can be solved as

$$\begin{aligned}U(m,n,t_{k+1}) = &\frac{1}{4}\Big(U(m+1,n,t_k)+U(m-1,n,t_k)\\ &+U(m,n+1,t_k)+U(m,n-1,t_k)\Big).\end{aligned} \tag{3}$$

Equation (3) is only valid for points internal to the masked region with the boundary values being fixed (known) values. A more computationally efficient alternative to the numerical implementation of Equation (3) is to calculate the average value at the point (m,n) as the two-dimensional numerical convolution of the neighbouring $[3x3]$ region with the smoothing kernel

$$\frac{1}{9}J_n, \tag{4}$$

where J_n is a $[3x3]$ unit matrix where every entry has the value of 1. This method utilizes hardware level matrix calculations which significantly reduce the computational time associated with the diffusive filling operation. It is for mathematical efficiency that the 2D convolution function in MATLAB is invoked for this calculation.

To recreate a velocity vector at the point (x_0,y_0) within the masked region Π_m at the time instance t_k, a streamline of the mean velocity that intersects (x_0,y_0) is created. This step is simple to implement as the mean velocity field is known from the prior diffusive filling operation. The velocity vector at the point (x_0,y_0) is calculated as the weighted average of the velocity vectors at the points (x_f,y_f) and (x_b,y_b). The points (x_f,y_f) and (x_b,y_b) are obtained by propagating forwards and backwards along the streamline until out of the masked region. For example, the coordinates x_1 and y_1 can be obtained by propagating forwards along the streamline from (x_0,y_0):

$$\begin{aligned}x_1 &= x_0 + U(x_0,y_0)\Delta t,\\ y_1 &= y_0 + V(x_0,y_0)\Delta t.\end{aligned} \tag{5}$$

The relative contribution of the velocity vectors at (x_f,y_f) and (x_b,y_b) to the velocity vector at (x_0,y_0) are weighted by the number of snapshots required to propagate out of the masked region. This process is illustrated schematically in Figure 1. Therefore, the velocity at the point (x_0,y_0) is given by

$$\begin{aligned}u_0(t_k) &= \frac{1}{t_f+|t_b|}\Big[(t_f)u_b(t_b)+(|t_b|)u_f(t_f)\Big],\\ v_0(t_k) &= \frac{1}{t_f+|t_b|}\Big[(t_f)v_b(t_b)+(|t_b|)v_f(t_f)\Big],\end{aligned} \tag{6}$$

where the spatial coordinates are replaced with subscripts (i.e. $u(x_0,y_0,t_k)=u_0(t_k)$) for brevity. For each point within the masked region Π_m, the 6-component vector $(x_f,y_f,t_f,x_b,y_b,t_b)$ is stored in a lookup table entry prior to running the interpolation for each time step. It should noted that the interpolation algorithm is accurate to a first-order approximation and thus, higher order schemes may improve the results.

Control Volume (CV) Force Formulation

The conservation of momentum for a stationary, non-deforming CV encompassing a fixed, simply-connected curve (in this case a circular cylinder) can be manipulated to obtain the standard impulse force formulation (Noca, 1997):

$$\mathbf{F} = \rho\frac{d}{dt}\left[-\frac{1}{N-1}\int_V \mathbf{x}\times\boldsymbol{\omega}dV + V_b\mathbf{u}_c\right] \tag{7}$$

where $\mathbf{x}$ is a position vector relative to the centre of the cylinder, $\boldsymbol{\omega}$ is the vorticity, $\mathbf{u}_c$ is the linear velocity of the body evaluated at its centroid, and V_b is the body volume. The first integral on the right-hand side is known as the vortical impulse. Thus, this term describes the influence of circulation growth and convection of vorticity in the wake region. The second term treats the acceleration of the body, which in the present work, does not impact the solution.

A discretized version equation 7, as employed by Limacher (2019), has been applied to data from a numerical simulation of the flow around a two-dimensional circular cylinder (of diameter D) in a steady freestream at a Reynolds number of 150. The solution, obtained on a structured O-grid, was interpolated onto a square grid of $\Delta x = 0.01D$ spacing to mimic a PIV dataset. The vorticity is then calculated from the velocity field using a second-order central differencing scheme.

Evaluation of the outlined Lagrangian interpolation scheme for gappy PIV data was facilitated through direct (intentional) manipulation of the numerically determined velocity field information. A mask width W was artificially imposed at any desired location or region downstream of the cylinder. The velocity field within the masked regions were calculated using a basic linear interpolation scheme as well as the Lagrangian interpolation scheme.

RESULTS

To illustrate qualitatively the utility of the Lagrangian interpolation algorithm outlined in the previous section, Figure 2 shows a masked raw vorticity field obtained from Bingham *et al.* (2018) and the corresponding unmasked vorticity field calculated using the Lagrangian interpolation algorithm. The results pertain to experimental PIV data obtained in the wake of a circular cylinder at $Re = 4000$. The algorithm is able to capture the magnitude and the distribution of vorticity associated with the red counter-clockwise vortex that is passing through the masked region.

To quantify the reconstruction error, the Lagrangian interpolation scheme was compared against linear interpolation in an artificially masked region with known velocity data. The test case was obtained from a two-dimensional CFD simulation of laminar vortex shedding in the wake of a circular cylinder of diameter D at a Reynolds number of $Re = U_\infty D/\nu = 150$ and consists of 2000 snapshots of data. The masked region began roughly $3.5D$ downstream from the rear tangent of the cylinder and its width was varied between $0D$ and $5D$. The original field as well as the estimated fields using bi-linear and Lagrangian interpolation are shown in Figure 3 for a specific mask width of $2.5D$. Since the coherent structures within the mask are of smaller spatial extent than the mask itself, they are almost entirely lost in the linear interpolation reconstruction. The recreation of structures which are completely obscured by a mask is made possible with the inclusion of temporal information from prior and future snapshots, and is not possible with interpolation schemes such as local Kriging.

Figure 4 shows the median relative error in the reconstructed velocity fields as compared to the raw data (the absolute value of the difference relative to U_∞), as well as the upper 95% and lower 5% error bounds. At the lowest bound of the mask width W, the performance of the two interpolation schemes is comparable. However, as the mask width increases, the relative error in the linear interpolation scheme grows rapidly and reaches a peak median error of approximately 13% while the median error of the Lagrangian interpolation scheme grows slowly and reaches a peak error of approximately 3% at a mask width of $5D$. A similar trend can be observed in the error bounds where the upper 95% error bound in the Lagrangian scheme remains below 10% error for all tested mask widths, while reaching well over 30% error using the linear interpolation scheme.

Following validation of the Lagrangian interpolation technique on the numerical data shown in Figs. 3 and Figure 4, we consider the influence of interpolation on the control volume estimation of forces acting on the circular cylinder. A measure of the error in lift and drag coefficient estimation is expressed through the average percentage error as follows:

$$
\begin{aligned}
C_{DR} &= \frac{100}{N}\sum_{j=1}^{N}\frac{|C_D(t_j)-C_{Dg}(t_j)|}{\overline{|C_D|}},\\
C_{LR} &= \frac{100}{N}\sum_{j=1}^{N}\frac{|C_L(t_j)-C_{Lg}(t_j)|}{\overline{|C_L|}},
\end{aligned}
\qquad (8)
$$

where C_{Dg} and C_D are the estimated drag coefficients with and without masked PIV data, and C_{Lg} and C_L are the estimated lift coefficients with and without masked PIV data, respectively. $\overline{|C_D|}$ and $\overline{|C_L|}$ are time average absolute value of the drag coefficient and lift coefficent, respectively. Figure 5 shows the % error in drag and lift using a standard bi-linear interpolation as well as Lagrangian interpolation scheme. The results indicate that for small mask widths less than about 0.3D, the error in drag is less than 0.1% for both the bi-linear and Lagrangian schemes. However, as the mask width increases, errors in the bi-linear scheme continue to increase and reach a peak value of 6%. On the other hand, the error exhibited by the Lagrangian scheme does not exceed more than 0.6%. A similar trend is observed for the lift coefficient: estimation using the Lagrangian interpolation scheme is less than 1.0% and represents a substantial improvement over the bi-linear interpolation scheme whose errors can exceed 200%.

The presented results for the developed Lagrangian interpolation scheme indicate a significant improvement over basic bi-linear interpolation as the velocity field fluctuations and associated dynamical features of the wake are preserved both qualitatively and quantitatively. In considerations of the alternative methodology of Sciacchitano *et al.* (2012) involving the full numerical solution to the Navier-Stokes equations, it is acknowledged that the interpolation scheme presented in the present work is a compromise regarding the computational expense. Moreover, it is not clear under what circumstances the Navier-Stokes solution would provide a better fit to the data.

CONCLUSIONS

A novel Lagrangian interpolation scheme was proposed for reconstructing velocity field data from within masked PIV regions. The method was compared to basic linear interpolation using a CFD test case and was shown to provide a 3-5 times improvement in the relative velocity reconstruction error. Force estimates using the standard impulse formulation of (Noca, 1997) show similar levels of improvement, though they are significantly more sensitive to the mask width as well as the location of the mask relative to the vorticity generation and advection through the domain.

The method was presented with the aim of recovering velocity data from within masked regions in PIV experiments caused by experimental effects like shadowing or optical distortions. However, it may also be a promising technique for replacing erroneous vectors during the vector calculation process.

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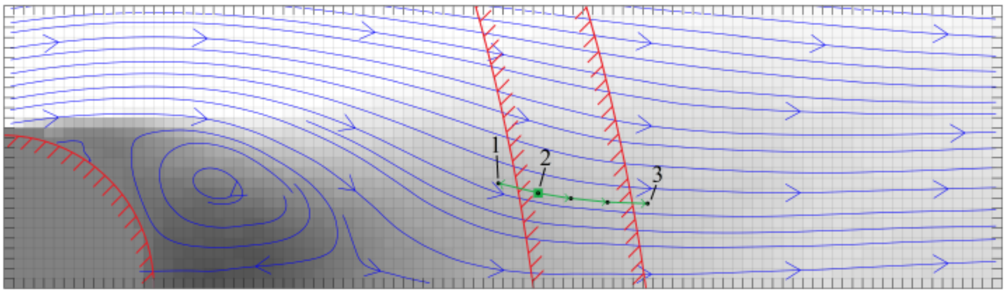

Figure 1. The calculation of $[x_f, y_f, t_f, x_b, y_b, t_b]$ for a point (x_0, y_0). In this case, point 1 corresponds to (x_b, y_b), point 2 corresponds to (x_0, y_0) and point 3 corresponds to (x_f, y_f). The red lines denote the boundaries of the masked region Π_m, while the blue lines represent the mean-field streamlines and the green line is the streamline from (x_0, y_0) projected forwards and backwards in time until it is out of the masked region. The black dots each denote one timestep, and therefore in this example, $t_f = 3$ and $t_b = -1$.

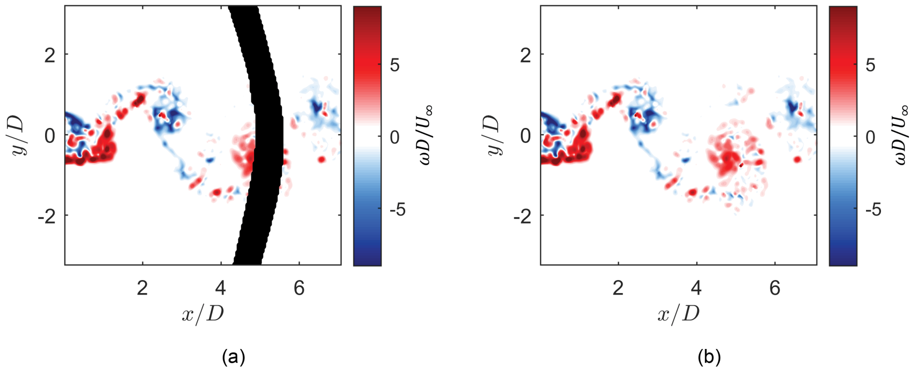

Figure 2. Raw masked (a) and unmasked (b) vorticity fields from experimental PIV data highlighting the output of the Lagrangian interpolation algorithm.

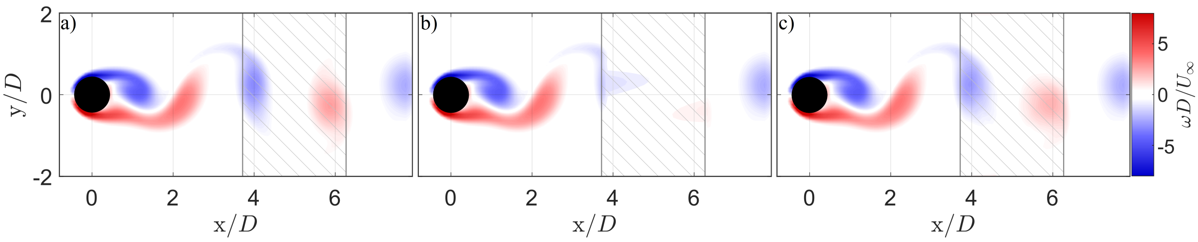

Figure 3. A CFD data set where a mask is artificially applied to the hatched grey region. (a) is the original field, (b) is the recreation using linear interpolation, and c) is the recreation using the Lagrangian interpolation scheme

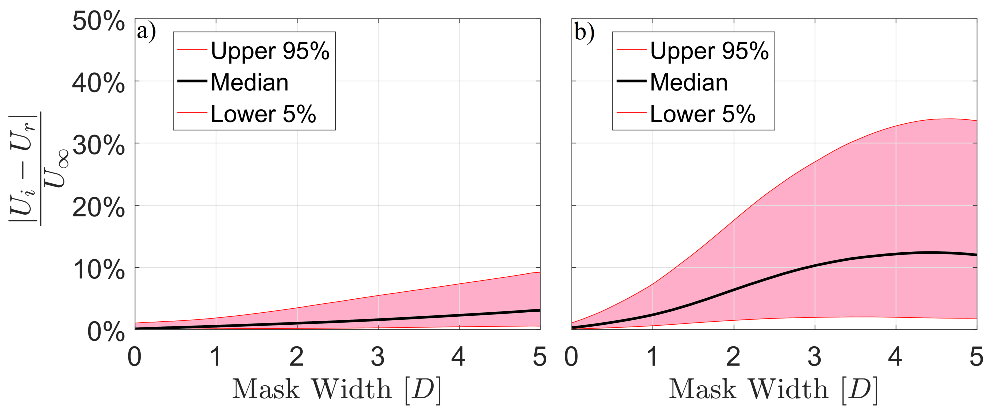

Figure 4. Median, 95%, and 5% absolute relative velocity error of reconstructed fields in the region Π_m using the Lagrangian interpolation scheme (a) and linear interpolation (b).

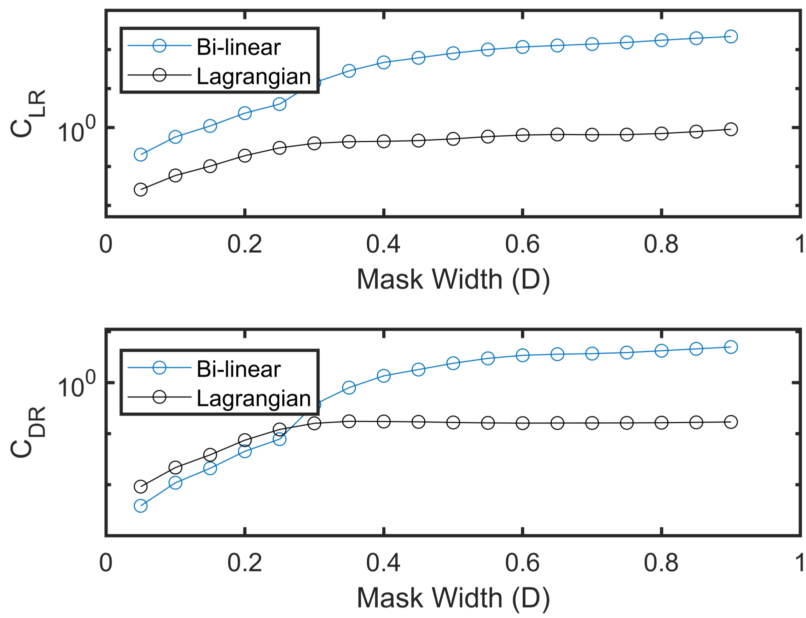

Figure 5. Mean % error in estimation of lift (a) and drag (b) coefficients employing the standard impulse formulation using the Lagrangian interpolation scheme and Bi-linear interpolation.