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TURBULENT MIXING LAYERS DRIVEN BY TIME-PERIODIC HORIZONTAL ACCELERATIONS

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ABSTRACT

Strong time-periodic accelerations applied tangentially to an infinite horizontal plane layer between two miscible fluids trigger a parametric instability leading to remarkable saw-tooth patterns known as frozen waves. The resulting turbulent mixing zones grow in time and then saturate when the resonance conditions of internal gravity waves are no longer fulfilled. The Floquet analysis of a model equation and direct numerical simulations reveal that the final mixing zone sizes evolve as the square of the forcing amplitude. This suggests that an horizontal forcing mixes more efficiently fluids than a vertical one at large forcing accelerations.

INTRODUCTION

It is known since Faraday (1831) that a parametric instability can result from a time-periodic acceleration applied to the interface between fluids of different densities (see also Miles *et al.* (1990)). In this work, we propose a theoretical and numerical analysis for the special configuration of miscible fluids forced by an oscillating acceleration tangent to the interface. When dealing with miscible fluids, the parametric instability, if strong enough, eventually leads to a turbulent mixing zone. However, turbulence cannot be sustained and irremediably decays as the mean density gradient decreases. The gravity waves propagating inside the layer and responsible for turbulence production have progressively lower frequencies and can no longer be parametrically excited by the forcing. Therefore, the system is expected to converge toward a final saturated mixing layer. This has been indeed observed experimentally by Zoueshtiagh *et al.* (2009) for the vertical forcing case. Gréa & EboAdou (2018) have also evidenced the phenomenon with numerous simulations and have further proposed a prediction criterion for the final mixing layer width using Floquet analysis of gravity wave equations.

Our goal is to conduct a similar analysis for the horizontal forcing case. Still, the problem becomes more complex to study due to the presence of a mean shear rendering the turbulence no longer axisymmetric. The horizontal configuration has been addressed particularly by Gaponenko *et al.* (2015), who in the context of miscible fluids have observed striking elongated structures refereed as frozen waves first observed by Wolf (1969, 2018). Simulations show that these structures even exist in the turbulence regime but remains unexplained (See Fig. 1). Accordingly, we try to elucidate how such structures can appear as well as predicting the turbulent mixing layer dynamics.

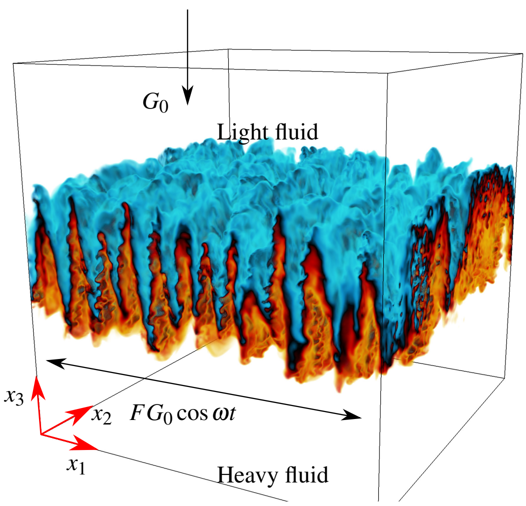

Figure 1. Visualization of turbulent frozen wave patterns inside a mixing zone driven by time-periodic horizontal acceleration extracted from a 1024^3 DNS with $F = 40$. Volume rendering colors indicate the concentration of heavy fluid C inside the mixing layer. Pure fluids are transparent.

THEORY

Basic equations

We start by defining the components G_i of the acceleration vector expressed by:

$$G_i(t) = G_0\left(-\delta_{i3} + F\cos\omega t\,\delta_{i1}\right) \quad (1)$$

with G_0 the mean acceleration value oriented along the $_3$ vertical direction, F the acceleration ratio between the mean and oscillating part along the horizontal $_1$ and the forcing frequency ω (see also Fig. 1). In addition, this problem is also controlled by the Atwood number $\mathscr{A}$, expressing the contrast of density between both fluids.

Miscible fluids of small density difference can be well described by the incompressible Navier-Stokes equations for the velocity $U_i(\mathbf{x},t)$ and the concentration of heavier fluid $C(\mathbf{x},t) \in [0\ 1]$ using classically the Boussinesq approximation:

$$\partial_t C + U_j\partial_j C = \mathscr{D}\partial_{jj}C, \quad (2)$$

$$\partial_t U_i + U_j\partial_j U_i = -\partial_i P + 2\mathscr{A}G_i C + \nu\partial_{jj}U_i, \quad (3)$$

$$\partial_j U_j = 0. \quad (4)$$

The flow can be assumed statistically homogeneous in the horizontal direction. It is then convenient to introduce the horizontal mean and use the Reynolds decomposition to separate mean (noted with $\overline{*}$ symbol) and fluctuating quantities (thereafter noted with small letter). Within the mixing layer, the mean concentration gradient can be assumed constant and related to the size L of the mixing layer as $\partial_3\overline{C} = -1/L$. Time periodic horizontal forcing induces also a mean shear $S = \partial_3\overline{U}_1$ which is the destabilizing force for turbulent quantities. In the limit of strong forcing parameter F, the shear is equal to $S = N^2F\frac{\sin\omega t}{\omega}$, introducing the buoyancy frequency $N = (2\mathscr{A}G_0/L)^{1/2}$.

Similarly to Gréa (2013), we write the rapid acceleration equations for the vertical velocity component $u_3(\mathbf{k},t)$ and concentration perturbation $c(\mathbf{k},t)$ inside the layer. Here, the mixing zone is assumed sufficiently developed such that the wave vector $\mathbf{k}$ has a modulus $k \gg 1/L$ and we can assume quasi-homogeneity. This gives the system of equations:

$$\dot{c} = \tfrac{1}{L}u_3, \quad (5)$$

$$\dot{u}_3 - 2\tfrac{k_1k_3}{k^2}\mathscr{S}u_3 = -2\mathscr{A}G_0\left(P_{33} - F\cos\omega t P_{13}\right)c, \quad (6)$$

$$\dot{k}_3 = -Sk_1, \quad (7)$$

introducing the projector $P_{ij} = \delta_{ij} - k_ik_j/k^2$. Note the presence of a distortion equation for the vertical component of the wave vector due to the shear.

Stability analysis

The system of equations (5-6) can be put on the form (having replaced $t \to \omega t$):

$$\ddot{c} + 2XY\sin t\,\dot{c} + X(1 + Y\cos t)c = 0 \quad (8)$$

$$X(\theta,\phi) = \sin^2\theta\frac{N^2}{\omega^2} \quad (9)$$

$$Y(\theta,\phi) = \cot\theta\cos\phi F \quad (10)$$

where θ and ϕ are the spherical angles characterizing the wave vector direction. Here, the distortion is neglected corresponding to the limit of large F. This equation constitutes our primary model to study the stability of a layer driven by time-periodic horizontal forcing.

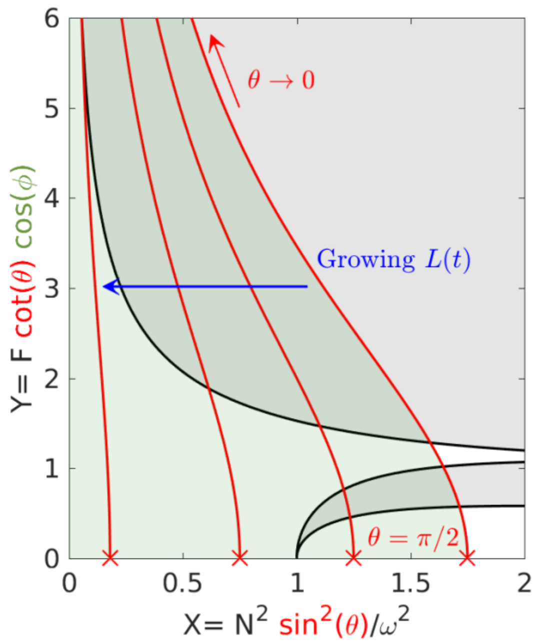

Figure 2. Stability diagram for Eq. 8 as a function of parameter X and Y obtained from Floquet analysis and showing the unstable harmonic regions. Starting from an initial condition, the final layer width is determined such that all the modes are outside the unstable regions as L grows. The green areas delimited by the red curves correspond to the modes which are excited inside the layer.

The solutions for Eq. (8) can be determined by a Floquet analysis (see figure 2) as in Kumar & Tuckerman (1994). As a first result, it is shown that the response of the perturbation is harmonic contrary to the vertical forcing configurations where sub-harmonic modes prevail. Indeed, Eq. (8) can be rigorously turned into a Mathieu equation through the classical change of variable $y = c\exp[-XY\cos t]$. In the new system, the resulting driving term oscillates at twice the frequency of the forcing. This evidences that shear inhibits sub-harmonic modes through the $\dot{c}$ term of Eq. (8).

The final size of the mixing layer is derived such that for all θ and ϕ, Eq. (8) is stable. As L grows (N^2 decreases) fewer modes are indeed excited and one can show that the latest ones correspond to $\theta = 0$ justifying to neglect distortion effects in the theory. This result is only valid for $F > \sqrt{2}$ and leads to a criterion for the final value of the mixing layer:

$$L_{sat} = \frac{\mathscr{A}G_0}{\omega^2}F^2 \quad (11)$$

SIMULATIONS

We perform direct numerical simulations (DNS) (around 30) using a classical pseudo-spectral method in order to investigate parametric instabilities developing in miscible fluids forced by horizontal accelerations (see Briard *et al.* (2019) for details). A large range of parameters $F \in [0.5\ 40]$ is explored in order to assess the theory. The frequencies ω, Atwood numbers $\mathscr{A}$ and mean accelerations

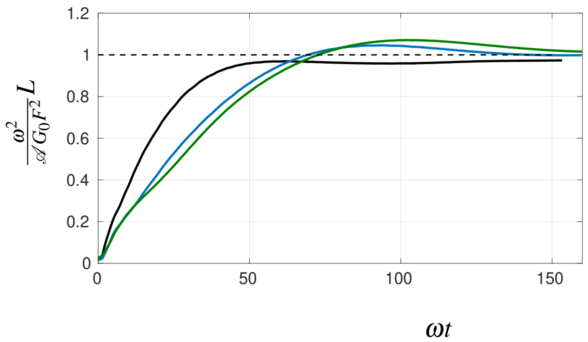

Figure 3. Time evolution of the mixing zone size L evaluated from the mean concentration profiles from 1024^3 DNS at $F = 8, 16, 20$. Dashed line corresponds to the theoretical prediction of the saturation.

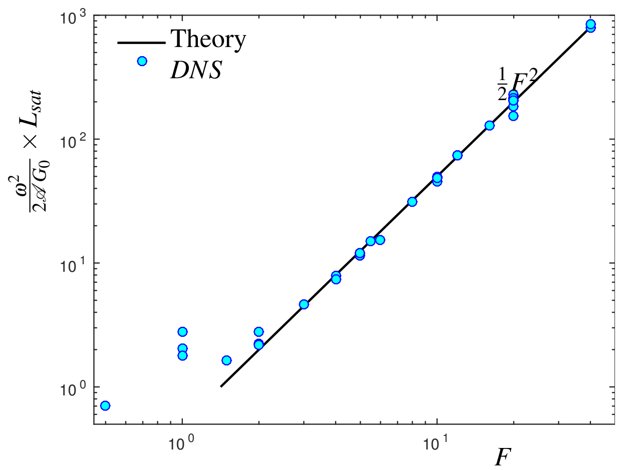

Figure 4. Values of final mixing zones size L_{sat} as a function of the acceleration ratio F and derived from DNS (symbols). Theoretical predictions are shown in plain lines.

G_0 are chosen such that the predicted final size of the layer corresponds to half the size of the computational domain.

In Figure 3, typical time evolutions of the mixing zone are presented for various forcing parameters at $F = 8$, 16, 20 evaluated from DNS with

$$L = 6\int_{-\infty}^{+\infty} \overline{C}(1-\overline{C})dx_3. \tag{12}$$

The layer monotonically grows until reaching saturation as expected. No oscillations on the mixing zone size are observed contrary to the classical vertical problem, although turbulent fluctuations exhibit a harmonic response. At large F, frozen wave patterns emerge at large scales which can be understood as an equilibrium between buoyancy forces and the horizontal shear.

The predictions of the final size of the layer are remarkably well verified at large parameter F as shown in Figure 4. In addition, the threshold $F = \sqrt{2}$ is also confirmed bringing strong support to the theory. However at smaller F the main assumptions used to derive the model equation are no longer valid (distorsion neglected). This is why the predictions are no longer expected to be valid.

CONCLUSION

A new purely inviscid theory consisting in Floquet analysis of gravity waves equations is proposed in order to study the dynamics of turbulent layers driven by time-periodic horizontal forcing. This allows for the prediction of mixing layers final size as long as for a criterion on the acceleration parameter F for the appearance of frozen wave structures. These findings have been successfully compared to numerous direct numerical simulations. This suggests that horizontal forcing is more efficient at producing mixing than vertical one, for which the final mixing layer size evolves only linearly with the forcing amplitude as shown in Gréa & EboAdou (2018).

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