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INTER-SCALE ENERGY TRANSFER IN CLUSTER OF DISSIPATIVE STRUCTURE IN TURBULENT FREE SHEAR FLOW

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ABSTRACT

Clusters of the fine scale dissipative structures (coherent fine-scale eddies) in decaying homogeneous isotropic turbulence (HIT) and turbulent mixing layer (TML) at high Reynolds number range up to $Re_\lambda = 290$ is investigated using direct numerical simulation (DNS). Clustering of the fine-scale eddies is evaluated by number density of eddy axes. Despite the universal characteristics of coherent fine-scale eddies, the eddies in TML are found to be clustered more than those in HIT. To elaborate this phenomenon, relation between the inter-scale energy transfer and the clusters are investigated by means of gird scale-subgrid scale (GS-SGS) energy transfer. At the smaller filter size (in the inertial subrange), the conditionally averaged GS-SGS energy transfer rate shows increase with respect to the number density for both flow cases. However, such tendency persists only for TML when the filter size is comparable to the integral length scale. These results imply the possibility of direct energy transfer from the very-large-scale to the clusters of coherent fine-scale eddies when a mean shear exists in the flow field. To further confirm this, number density of intense coherent fine-scale eddies are investigated, and it is found that the intense fine-scale eddies tend to be clustered more in TML than in HIT.

INTRODUCTION

In the conventional turbulence theory, it is believed that turbulent flows are composed of eddies of different scales where the large eddies break up into somewhat smaller eddies, transferring their energies to the smaller eddies successively until they are dissipated by the viscosity. An identification scheme for the smallest scale motion in turbulence, which is responsible for the dissipation of turbulent kinetic energy, was developed by Tanahashi *et al.* (1999). In their further studies, these fine-scale eddies have been observed at various types of flow such as HIT (Tanahashi *et al.*, 2008), channel flow (Kang *et al.*, 2007) and TML (Tanahashi *et al.*, 2001; Wang *et al.*, 2007; Itoh *et al.*, 2018). In their studies, the fine-scale eddies are called coherent fine-scale eddies since the characteristics of the eddies have been found to be independent of the flow configurations or conditions.

In the classical picture of successive energy transfer in turbulence, directional information involved in large-scale motion is lost; hence, small-scale structures become locally homogeneous and isotropic. However, it has been observed from a number of studies that the fine-scale eddies are not randomly distributed but they rather tend to form clusters (Moisy & Jiménez, 2004; Kang *et al.*, 2007; Tanahashi *et al.*, 2008; Leung *et al.*, 2012; Ishihara *et al.*, 2013; Itoh *et al.*, 2018). Since the small-scale structures are believed to be originated from larger-scale structures, receiving energy from them, the relations between the clusters of coherent fine-scale eddies and larger-scale turbulence quantities, including the inter-scale energy transfer, have been studied by Tanahashi *et al.* (2008) for HIT, and by Itoh *et al.* (2018) for TML.

In our previous study, Itoh *et al.* (2018) focused on the relations between the clusters and the energy transfer in inertial subrange of TML. However, a fundamental difference of the free shear turbulence between idealistic HIT, which has been frequently used for the study of the energy transfer (Moisy & Jiménez, 2004; Tanahashi *et al.*, 2008; Leung *et al.*, 2012), lies on the presence of a mean shear producing turbulent energy at very-large-scale. Therefore, the effects of the very-large-scale motions to the energy transfer process must be clarified to understand the turbulence in real world. This research focuses on comparing the two flows in terms of the relations between the inter-scale energy transfer in large-scale and the coherent fine-scale eddy clusters.

DNS DATASETS

In this study, we analyse our DNS databases of HIT and TML. DNSs of decaying HIT in triply periodic box of length 2π were originally conducted by Tanahashi *et al.* (2008). DNSs of temporally developing TML in doubly periodic box had been carried out by Tanahashi *et al.* (2001) and Itoh *et al.* (2018). The domain size is $4\Lambda \times 6\Lambda \times 8/3\Lambda$, where Λ is the most unstable wave length. All of the simulations had been done by Fourier spectral method with full dealiasing. Temporal integration was implemented by third-order Runge-Kutta method. Since both of the flows are not statistically stationary, a single snapshot at fully developed state is used in the analysis. The development of the flow was confirmed by convergence of the longitudinal velocity derivative for HITs, and by mature of sub-harmonic mode for TMLs. Parameters of the flows are listed in Table 1. The maximum Reynolds number considered in this study is about $Re_\lambda = 290$. To author's knowledge, this is the highest Reynolds number to date at which HIT and TML are directly compared using DNS. For all the cases, the integral length and the Kolmogorov length are separated by two order of magnitude, which gives sufficient scale separation to

Table 1. DNS datasets of decaying HIT (a) and TML (b). N, N_x, N_y, N_z: number of grid, l: integral length scale calculated from auto-correlation, λ: Taylor microscale, η: Kolmogorov length scale, $S_{u'}$ & $F_{u'}$: skewness and flatness of longitudinal velocity derivatives. The values at center line of the domain are shown for the cases of TML.

(a)

Re_λ	N	l/η	λ/η	$S_{u'}$	$F_{u'}$
175.4	512	396.6	26.0	-0.536	6.25
222.7	640	494.3	29.3	-0.534	6.34
287.6	960	676.9	33.3	-0.564	6.97

(b)

Re_λ	N_x	N_y	N_z	l/η	λ/η	$S_{u'}$	$F_{u'}$
161.6	480	721	320	150.3	24.8	-0.587	8.25
191.5	648	973	432	195.4	24.7	-0.537	8.36
275.5	1728	2593	1152	477.1	34.2	-0.492	7.45

study clustering of the fine-scale eddies.

CLUSTER OF FINE-SCALE EDDIES

Coherent fine-scale eddies are extracted by the identification scheme developed by Tanahashi *et al.* (1999). The scheme identifies the rotation axes of the eddies essentially by searching for local maxima of the second invariant of the velocity gradient tensor Q. Different from conventional identification scheme, such as the one based on thresholding on enstrophy (Moisy & Jiménez, 2004; Leung *et al.*, 2012; Ishihara *et al.*, 2013), the scheme identifies all of the vortical motions irrespective to its swirling intensity. Therefore, it is suitable for the extraction of the fine-scale eddies in inhomogeneous turbulence, including TMLs.

Probability density functions (p.d.f.s) of the diameter and the maximum azimuthal velocity of the fine-scale eddies extracted in HIT and TML are shown in Fig. 1. The most probable diameter and the velocity are approximately 8η and $1.2u_k$ respectively for all the cases, showing that the characteristic length and velocity are well scaled by the Kolmogorov units. Figure 2 shows a p.d.f. of Q_c, the maximum Q along each of the eddy axes. Although TMLs are more populated with weaker eddies below $Q_c < 0.01Q_k$, corresponding to the ones with larger diameter which can bee seen in the tail of Fig. 1(a), the p.d.f.s especially above $Q_c > 0.1Q_k$ collapse with each other. Overall, the properties of the fine-scale eddies are universal with respect to flow type and the Reynolds number.

Figure 3 shows the images of extracted eddies for the both flow at the highest Reynolds number cases. In present Reynolds number range, almost entire domain of the HITs are filled by the fine-scale eddies. At the same time, as can be found in literature (Moisy & Jiménez, 2004; Tanahashi *et al.*, 2008; Leung *et al.*, 2012; Ishihara *et al.*, 2013), the eddies exhibit spatial variation in its population and intensity, despite large-scale coherent motions are absent. Note that our present study use decaying HITs and thus the spatial variations are not the artefacts of external forcing. In the case of TMLs, the flow is fully turbulent only in the center region. This image shows that the eddies are roughly bounded in the region inside of large-scale roller motion, coinciding with the turbulent/nonturbulent interface found in various kind of shear turbulence (da Silva *et al.*, 2014). To take in account for this inhomogeneity, only the subdomain which satisfies $|U(y)| < \Delta U/4$ (red rectangular in Fig. 3) will be considered for the following analysis of TML. Here, $U(y)$ and ΔU denote mean streamwise velocity and the free stream velocity difference respectively. The height of the subdomain at $Re_\lambda = 161.6, 191.5$ and 275.5 correspond to 120η, 144η and 328η respectively. General quantities g averaged in whole domain of HITs, and in the

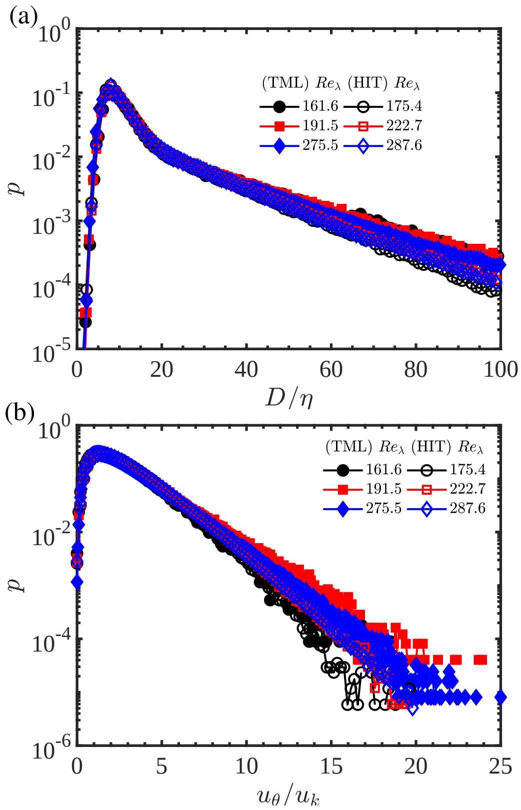

Figure 1. P.d.f.s of the diameter (a) and maximum azimuthal velocity (b) of coherent fine-scale eddies, normalized by Kolmogolov length η and velocity $u_k = \nu/\eta$.

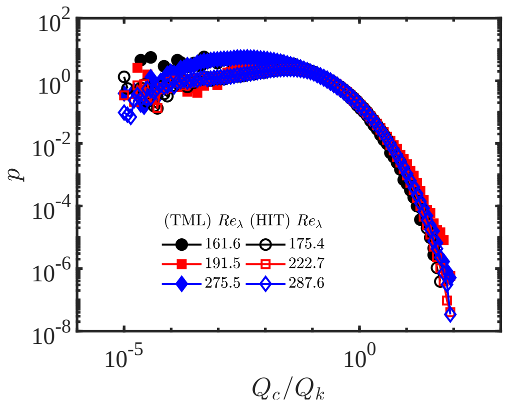

Figure 2. P.d.f.s of Q at the center of the coherent fine-scale eddies normalized by Kolmogorov unit $Q_k = (u_k/\eta)^2$.

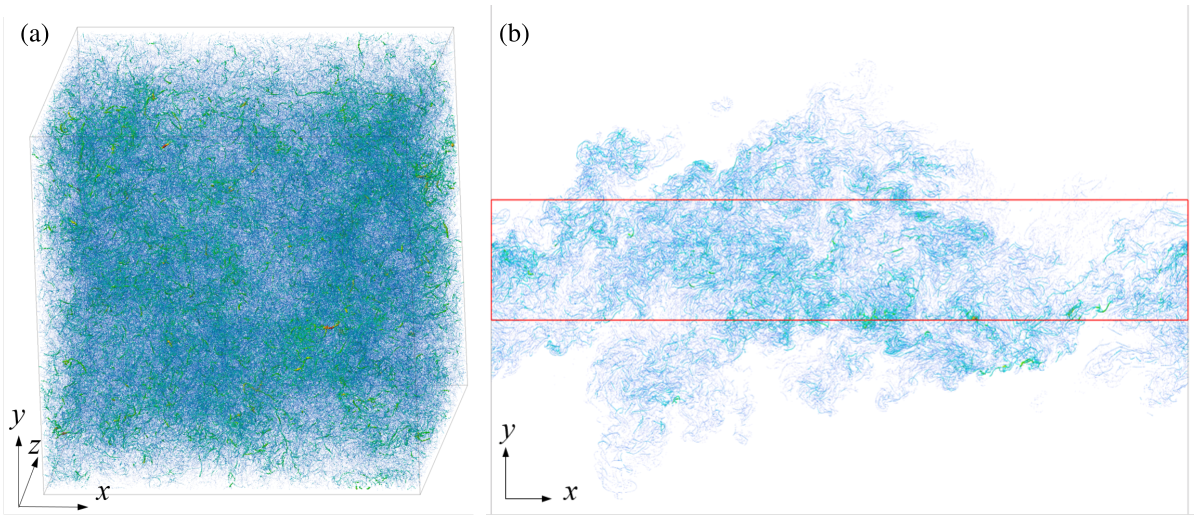

Figure 3. Spatial distributions of the axes of fine-scale eddies in HIT at $Re_\lambda = 287.6$ (a) and TML at $Re_\lambda = 275.5$ (b). Colour represents intensity of Q on axis. Red rectangular in (b) shows subdomain considered for the present analysis.

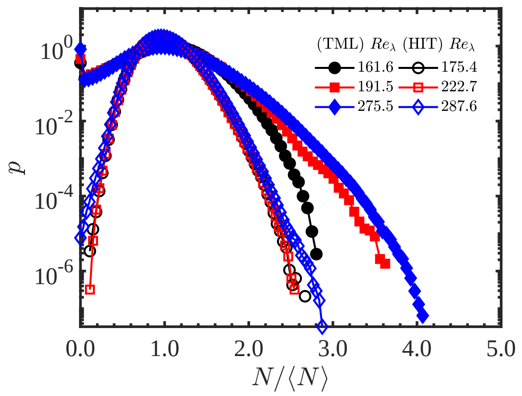

Figure 4. P.d.f.s of the number density of coherent fine-scale eddies, normalised by the spatial mean.

subdomain of TMLs are hereafter denoted by $\langle g\rangle$.

Following our previous study (Itoh *et al.*, 2018), clustering of the fine-scale eddies are quantified by counting the number of the eddies within a sampling box of size $(40\eta)^3$. In Fig. 4, p.d.f.s of the number density, N, normalized by the spatial mean are shown. Despite the universal characteristics of the fine-scale eddies, the tendencies of clustering are different in the two flows: the p.d.f.s of TMLs are higher at extremely low and high number density range, showing that the eddies are tend to be more clustered than those in HITs. Another interesting fact is that the number density of HITs show little Reynolds number dependency, whereas TMLs are more populated with higher number density region as Re_λ increases. This suggests that the inhomogeneity in large-scale is vital for existence and development of the fine-scale eddy clusters.

CLUSTER AND GS-SGS ENERGY TRANSFER

Relation between the inter-scale energy transfer and clustering of the fine scale structures are studied in terms of grid scale-subgrid scale (GS-SGS) energy transfer. By introducing low-pass spatial filter, GS-SGS energy transfer rate E_τ is written as

$$E_\tau = -\tau_{ij}\overline{S_{ij}} = -(\overline{u_i u_j} - \overline{u_i}\,\overline{u_j})(\partial_i \overline{u_j} + \partial_j \overline{u_i})/2 \qquad (1)$$

where $\overline{g}$ denotes filtered (GS) quantity of g. τ_{ij} and $\overline{S}_{ij}$ are called as SGS stress tensor and GS strain tensor respectively. In this study, Gaussian filter are chosen as the filter function. The GS-SGS energy transfer rate can be further decomposed as

$$\begin{aligned} E_\tau &= -\tau_{ij}\overline{S_{ij}} \\ &= -(\overline{\overline{u_i}\,\overline{u_j}} - \overline{u_i}\,\overline{u_j})\overline{S_{ij}} - \overline{u_i'\overline{u_j} + \overline{u_i}u_j'}\,\overline{S_{ij}} - \overline{u_i'u_j'}\,\overline{S_{ij}} \\ &= E_L + E_C + E_R \end{aligned} \qquad (2)$$

where $u_i' = u_i - \overline{u}$ stands for SGS velocity component. E_L, E_C and E_R are called as Leonard term, Cross term and Reynolds term which correspond to GS-GS, GS-SGS and SGS-SGS interactions respectively. Figure 5 shows spatially averaged values of each terms in (2) at various filter width Δ, normalized by energy dissipation rate ε. Under $\Delta \approx 40\eta$, each terms are universal to flow type and the Reynolds number. At around $40\eta < \Delta < 80\eta$, the energy transfer rates show Reynolds number dependency, but are still comparable for different flow type. Above $\Delta \approx 80\eta$, the difference between the flow configuration becomes distinctive. In HITs, all of the terms start to decrease because of lack in energy production which is to be followed by the energy transfer. In contrast, $\langle E_\tau\rangle$ of TMLs continue to increase until the filter size reaches the integral length, due to the energy production by the mean shear. $\langle E_\tau\rangle$ at the range is larger than the energy dissipation rate, reflecting the fact that the flow is temporarily developing. Moreover, $\langle E_C\rangle$ of TMLs predominate $\langle E_R\rangle$ almost until at the inertial subrange, whereas $\langle E_C\rangle$ of HITs are taken over by $\langle E_R\rangle$ at $\Delta \approx 60$. This suggests that the GS-SGS interaction in TMLs persists in larger scale than in HITs, which may lead to the stronger clustering tendency of the fine-scale eddies.

The energy transfer rate conditioned on the number density is investigated next. Following analysis was repeated within the inertial subrange, $30\eta \leq \Delta < l$ to guarantee the consistency of the results. The scale larger than the

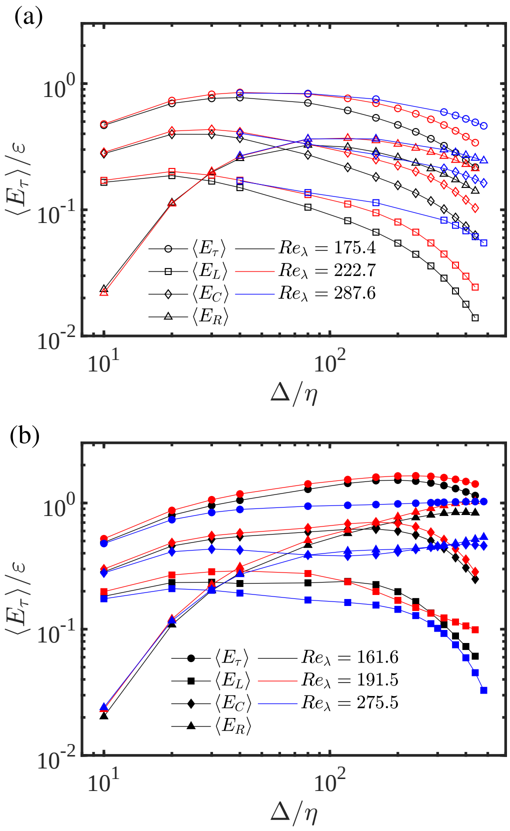

Figure 5. Energy transfer rate and its components at different filter width. (a)HIT, (b)TML.

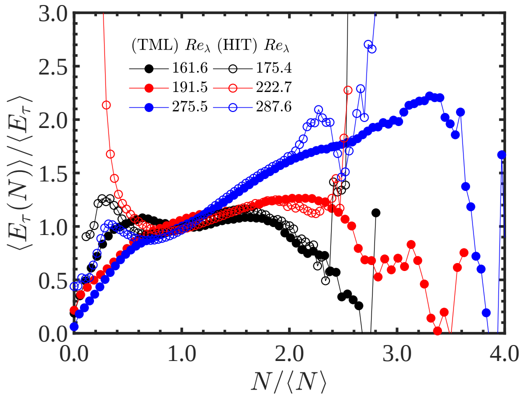

Figure 6. GS-SGS energy transfer rate conditionally averaged on the number density ($\Delta = 80\eta$).

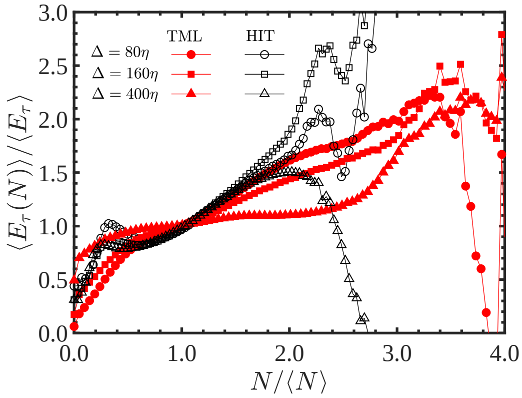

Figure 7. GS-SGS energy transfer rate at several filter width, conditionally averaged on the number density (HIT: $Re_\lambda = 287.6$, TML: $Re_\lambda = 275.5$).

height of the subdomain will be referred as very-large-scale. At TML of $Re_\lambda = 275.5$, the range in $30\eta \le \Delta \le 400\eta$ is considered, and $\Delta > 320\eta$ is regarded as the very-large-scale. Note that $\Delta \approx 160\eta$ is the representative scale in inertial subrange which was used in our previous study (Itoh *et al.*, 2018). Figure 6 shows mean GS-SGS energy transfer rate at $\Delta = 80\eta$ conditioned on the number density, normalized by unconditional value. Consistent with Fig. 5, the relation between the mean energy transfer rate and the number density is qualitatively independent of the flow case when the filter size is below $\Delta \approx 80\eta$. Higher number density is related to higher energy transfer rate, which represents active energy transfer in the clustered region. The tendency of active energy transfer is more pronounced as Re_λ is increased. Difference between the flow type become apparent as the filter width become closer to the very-large-scale. Figure 7 shows the conditionally averaged energy transfer rate at several filter size. Only the largest Reynolds number cases are shown for brevity. At $Re \approx 290$, the energy transfer rate in two type of the flow is clearly correlated with the clustered region until the middle of the inertial subrange, $\Delta \approx 160\eta$. Above $\Delta \approx 320\eta$, a sharp increase of the energy transfer rate in HIT at $N/\langle N \rangle > 2.0$ is lost. In TML, in contrast, the the conditional average still exhibits increase at the clusters ($N/\langle N \rangle > 2.5$), which means that the correlation persists. These results may imply the direct energy transfer from very-large-scale to the clusters when a mean shear exists in the flow field. Although only the largest Reynolds number case is shown in Fig. 7, qualitatively the same results are obtained for other Reynolds number cases.

CLUSTER OF INTENSE FINE-SCALE EDDIES

Figure 2 showed consistency of the eddy intensity over the type of flow and the Reynolds number. However, if the fine-scale eddies in the clusters receive energy directly from very-large-scale structures, presumably, then intense fine-scale eddies would be clustered intermittently. Therefore, to support the idea of direct energy transfer, clusters of intense fine-scale eddies are investigated. The intense fine-scale eddies are defined as the eddies that satisfy

$$Q_c \ge Q_{thr} \tag{3}$$

Number density of the intense eddies are then calculated by counting them within the sample box of sized $(40\eta)^3$, in the same manner as was done for the all eddies. P.d.f.s of the number densities of the intense eddies at $Q_{thr} = 1.0Q_k$ are shown in Fig. 8. In this figure, the p.d.f.s of the intense fine-scale eddies in TML show higher deviation from the mean, and higher possibility at high number density range, forming more intermittent clusters than those of HIT. Above result is confirmed to be consistent at least in $0 \le Q_{thr} \le 5.0Q_k$. Again, this evidence may lead to the idea of direct energy transfer from very-large-scale to the clusters – against with the conventional turbulence theory which postulates that the energy is transferred from larger structures to somewhat smaller eddies – when there is a mean shear at the flow.

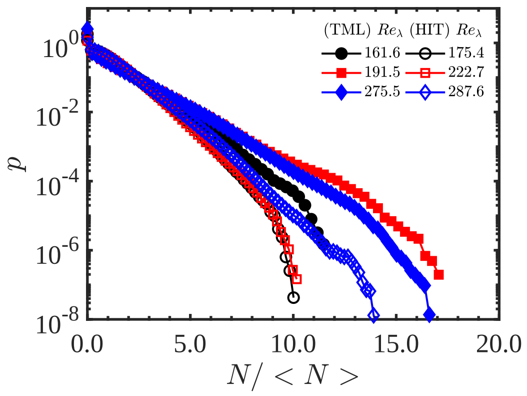

Figure 8. P.d.f.s of the number density of intense coherent fine-scale eddies ($Q_c \geq 1.0 Q_k$).

CONCLUSION

To clarify the relation between large-scale motions and the clustering of fine scale dissipative vortices, DNS datasets of decaying HIT and temporally developing TML up to $Re_\lambda = 290$ are compared. The fine scale filamentary vortices are identified by the method based on local maxima of the second invariant of the velocity gradient tensor. The clustering is evaluated by calculating number density of the fine-scale eddies. Despite properties of individual eddies show universality over different flow type, they are found to be more clustered in TMLs than in HITs.

Relations between the clustering and inter-scale energy transfer is studied by means of GS-SGS energy transfer rate. When the filter size Δ is higher than 80η, only TMLs show high energy transfer rate due to energy production by the mean shear. Comparison between decomposed terms of the energy transfer rate show that GS-SGS interaction of TMLs up to $\Delta \approx l$ predominates SGS-SGS interaction, while the same is not true for HITs above $\Delta \approx 60\eta$. Conditionally averaged energy transfer rate over the number density shows that the energy transfer under $\Delta \approx 80$ is active in the clustered region independent of the flow configurations. However, tendency of the active energy transfer persists only for TMLs when the filter size is increased up to integral length. Those results imply the possibility of direct energy transfer from the very-large-scale to the fine-scale eddies in the clusters when there is mean shear in the flow.

As the direct energy transfer may lead to preferential amplification of the fine-scale eddies, clustering of the intense eddies are evaluated. The intense eddies are extracted by applying threshold on the second invariant of the velocity gradient tensor of the rotation axis, and then their number density is calculated. As a result, the intense eddies are found to be more intermittently clustered in TMLs than in HITs, which support the idea of the direct energy transfer.

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