BI-GLOBAL STABILITY ANALYSIS ON A NACA 0025 AIRFOIL AT A REYNOLDS NUMBER OF 100,000

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ABSTRACT
A method is developed to solve bi–global stability functions in curvilinear systems which avoids the reshaping of the airfoil or remapping the disturbance flow fields. As well, the bi–global stability functions for calculation in a curvilinear system are derived. The instability features of airfoil flow at $AOA = 5^\circ$ are studied and the most unstable mode was found to be related to the wake mode with a dimensionless frequency close to one, as found experimentally. As the spanwise wavenumber increases, the number of unstable modes increases and then stabilizes. A graphical processing unit (GPU) is employed to speed up the solution of the eigenvalue problem. For large matrices, the calculation time using the GPU was roughly one–tenth the calculation time of a CPU.

Introduction
Airfoil performance is often limited or degraded by flow separation, which is usually associated with loss of lift, increased drag, kinetic energy losses and so on. Thus, many methods of flow control methods have been developed to suppress it or avoid it entirely (Greenblatt & Wygnanski (2000)). Exploiting the instability of separated flow, periodic flow control methods improve on steady control while maintaining the same energy input (Huang, Lu, Zhu, Fu & Wang (2017)). As a precursor to developing control strategies, it is important to understand and quantify the flow stability characteristics of separated flow.

There are mainly three kinds of linear stability analysis methods. Classical stability approaches, often based on the Orr–Sommerfeld equation, assume the basic flow is nonhomogeneous in only one spatial direction. In the past, many studies have investigated flow stability by utilizing the Orr–Sommerfeld equation. This is limited by an assumption of locally parallel flow, which is not altogether applicable to separated flow. Tri–global stability considers the three-dimensionality of the base flow and perturbations (Chomaz (2005); Theofilis (2011)). There is a huge memory requirement to solve the eigenvalue problem limiting its practical use; e.g., for a case with 64 mesh points in each directions, 17.6 Tbytes memory are required (Theofilis (2003)). The Bi–global method considers the non–uniformity of the flow variables in two spatial directions and is a good option for airfoil or straight blades flow as the variation of the flow parameters along the spanwise direction is significantly weaker than the other two directions. Taking advantage of this, the bi–global stability method assumes the perturbation as a wavelike mode in the spanwise direction. Compared with tri–global stability analysis, the solution process of the bi-global stability method is simplified and the required memory is significantly reduced. Despite this simplification, bi-global stability analysis is still computationally expensive, as very large partial-derivative eigenvalue problems (EVP) must be solved. Exploiting the sparsity or developing high-order finite-difference scheme are helpful to speed up the solving process.The high-order finite-difference scheme of order-q (FD-q) method was found to significantly outperform all other finite difference schemes in solving classic linear local, BiGlobal, and TriGlobal eigenvalue problems based on both memory and CPU time requirements (Paredes *et al.* (2013)). With parallel computing, GPU computations are expected to further improve computing speed.

Bi–global stability analysis is used in many areas, e.g., channel flow (Floryan & Asai (2011); Merzari *et al.* (2008); Malik & Hooper (2007)), flat plate (Alizard & Robinet (2011)) and bluff body (Sevilla & Martínez-Bazán (2004)). Bi–glogal stability analysis has also looked been applied to flow past a NACA0012 airfoil at high attack angle at Reynolds numbers ($Re_c = u_\infty c/\nu$) ranging from 400–1000, where u_∞ is the freestream velocity, c is the chord length, and ν the kinematic viscosity (Zhang & Samtaney (2016)). It was found that the near wake and far wake instabilities are the two dominant unstable modes and with increasing wavenumber, the unstable modes are suppressed. Kitsios *et al.* (2009) performed bi–global stability analysis for flow past NACA0015 airfoil at the angle of attack 18 degree and $Re_c = 200$. However, most of the airfoil flows analyzed are at very low Reynolds number around 200–2000 He *et al.* (2017); Kitsios *et al.* (2009). The stability research of Reynolds number around 1×10^5 is still very limited, and is an area important for unmanned aerial vehicles (UAVs), small-scale wind turbines, and low-speed aircraft, where flow separation is often encountered.

Typically, the input for bi–global stability analysis is the steady or time-periodic base flow that is nonhomogeneous in two spatial directions at a given Reynolds number (Taira *et al.* (2017)). The base flow can be stable or in an unstable state. For stable flow, the base flow can be obtained through solving the governing function of the base flow. However, it is difficult to compute the base flow for inherently unstable flows. Normally, selective frequency damping (Åkervik *et al.* (2006)) or the mean flow can be used to obtain the base flow for the analysis of global stability. Sometimes, the selective frequency damping method is unable to identify an unstable steady state (Munday (2017)). Although the use of the mean flow is not strictly correct with respect to the formulation of the linearized Navier-Stokes equations, solving the eigenmodes of perturbations about the mean flow help identify how perturbations can grow or delay with respect to the time-average flow. These growing modes are helpful to aid the design of flow control strategies (Munday (2017)). Bi–global stability function have been found using the mean flow as the base flow, Sun *et al.* (2017)). The results indicate that the choice of the mean flow as the base state allows for the emergence of the wake mode, and the present bi–global stability analysis indicates that the use of the mean flow can identify the wake mode which is observed in the 2D nonlinear simulation. Thus, the time average flow field was chosen as a base flow in this paper.

For the flow past an airfoil, the bi–global stability equations can not be directly discretized. To solve the stability equation in the curvilinear coordinate system, Kitsios *et al.* (2009) used conformal mapping techniques. The mapping between the curvilinear and physical coordinates for an airfoil geometry is very involved, requiring 4 mappings. The first maps the rectangular curvilinear grid to a cylindrical coordinates. A Joukowsky transformation is then used to convert the cylindrical grid to an airfoil shaped mesh. Then it is necessary to select appropriate parameters to best represent the airfoil. This airfoil shaped mesh is then translated, scaled and finally rotated by the angle of attack to align with the finite volume mesh to return the coordinates in physical space. Sometimes, it is difficult to find suitable parameters to best represent the shape of a certain airfoil. There is a mapping relationship between the velocity in physical coordinates and curvilinear coordinates. Thus, after solving the bi–global stability equations in curvilinear coordinates, to show the spatial structure in physical space, a remapping process is needed. To avoid the reshaping of the geometry of airfoil and remapping of the flow field, a new method for solving the stability equation in the curvilinear coordinate system is proposed.

The goal of the present work is to study the instability of flow past NACA 0025 airfoil at an angle of attack of 5 degrees and a chord-based Reynolds number of 100,000. A new way to solve the bi–global stability equations is proposed, followed by a study of the unstable flow features in a frequency range $F^+ = 0-12$ ($F^+ = fc/u_\infty$). The effects of wavelength of spanwise perturbation on the stability of the flow is also investigated. Finally, an investigation of the computational speed-up of the eigenvalue solution using a graphics processing unit (GPU) is introduced.

Base flow computation method

The numerical computations of the base flow were performed using large–eddy simulation (LES). The subgrid scale stress tensor,τ_{ij}, was modeled with an eddy viscosity approach,$\tau_{ij} = -2\upsilon_r \overline{S}_{ij}$, where υ_r is the eddy viscosity and $\overline{S}_{ij}$ is the filtered strain rate tensor. A subgrid scale turbulence kinetic energy model was employed. The temporal and convective terms were discretized using a second order backward implicit time stepping scheme and second order TVD scheme, respectively. An adaptive time stepping scheme was employed to maintain a CFL number of $C_0 < 0.7$ throughout the domain. The PISO algorithm was used for the pressure-momentum coupling. The airfoil surface was defined as a no-slip boundary condition and a periodic boundary condition was applied to the lateral boundaries, spaced c/2 apart, where c is the chord length. The inlet and outlet were assigned laminar inflow and zero-gradient outflow conditions, respectively.

The computations were performed on 64-128 processors using the Blue Gene/Q (BGQ) and General Purpose Cluster (GPC) at Scinet (Loken *et al.* (2010)).A block-structured C-mesh with 32×10^6 cells were employed with mesh refinement concentrated in the wake and around the NACA 0025 airfoil which had a chord length c = 0.3m. For wall-resolved LES, it is well accepted that the required mesh resolution, which has been achieved in all cases presented, is $\Delta x^+ \approx 100$, $\Delta y^+ \approx 2$, and $\Delta z^+ \approx 20$ (Mary & Sagaut (2002); Sagaut (2006)). The full validation of the flow can be found in (Ziadé & Sullivan (2017)).

A new method to solve the biglobal stability function

The state variables (ϕ) can be decomposed into the base state ($\overline{\phi}$) and the perturbation(ϕ') (Theofilis (2011)).

$$\phi(x,y,z,t) = \overline{\phi}(x,y) + \varepsilon\phi'(x,y,z,t) \quad (1)$$

where $\overline{\phi}(x,y)$ indicates the two-dimensional steady base flow, in this study obtained by three-dimensional large-eddy simulation, and $\phi'(x,y,z,t)$ is the perturbation. The perturbation was assumed to have a form of

$$\phi'(x,y,z,t) = \hat{\phi}(x,y,z)e^{i(\beta z-\omega t)} + \hat{\phi}^*(x,y,z)e^{i(-\beta z+\omega t)} \quad (2)$$

where the * superscript denotes the complex conjugate, the second term is required because $\hat{\phi}$ and ω in general is complex, while ϕ' must be real. β is the wavenumber of the structure of the perturbation in the spanwise direction z. The real part of complex value ω represents the angular frequency and the imaginary part of ω corresponds to the growth/damping rate of the associated amplitude function. A positive value of $\mathrm{Im}(\omega)$ indicates the exponential growth of the perturbation, whereas a negative value of $\mathrm{Im}(\omega)$ corresponds to the damping of the unstable mode. In the context of bi-global stability,the base flow in the spanwise direction z , w, is assumed to be zero. When the above base flow simplifications and modes of perturbation are substituted into the Navier-Stokes equations and higher order terms ($O(\varepsilon^2)$)are neglected, the linearized Navier-Stokes equations are obtained:

$$\hat{u}_x + \hat{v}_y + i\beta\hat{w} = 0 \tag{3}$$

$$-\overline{u}\hat{u}_x - \overline{v}\hat{u}_y - \hat{u}\overline{u}_x - \hat{v}\overline{u}_y - \hat{p}_x + (\hat{u}_{xx} + \hat{u}_{yy} - \beta^2\hat{u})/Re_c = -i\omega\hat{u} \tag{4}$$

$$-\overline{u}\hat{v}_x - \overline{v}\hat{v}_y - \hat{u}\overline{v}_x - \hat{v}\overline{v}_y - \hat{p}_y + (\hat{v}_{xx} + \hat{v}_{yy} - \beta^2\hat{v})/Re_c = -i\omega\hat{v} \tag{5}$$

$$-\overline{u}\hat{w}_x - \overline{v}\hat{w}_y - i\beta\hat{p} + (\hat{w}_{xx} + \hat{w}_{yy} - \beta^2\hat{w})/Re_c = -i\omega\hat{w} \tag{6}$$

where the subscripts denote partial differentiation with respect to the indicated variable. The bi-global stability equations are cast as a partial-derivative eigenvalue problem.

$$A\hat{\phi} = \omega M\hat{\phi} \tag{7}$$

$$\hat{\phi} = (\hat{u}, \hat{v}, \hat{w}, \hat{p}) \tag{8}$$

A matrix based approach is the most used method for bi-global stability analysis (Kitsios *et al.* (2009); Paredes *et al.* (2013)). A is the spatial discretization operator, which is a function of the mesh, base flow, Renolds number (Re_c), β and other variables. Finite difference methods are often used to determine the expression for A. The finite difference method is performed on a set of discrete grid points. When it comes to the problem of flow around the airfoil, the above equations cannot be directly discretized.

To solve the stability equation in the curvilinear coordinate system, Kitsios et al. used conformal mapping to transform the airfoil to a rectangular domain.

In the present work, base flows are generated in physical space and need to be transformed to the curvilinear calculation domain before performing the stability calculation. First, the relationship between the physical coordinate system (x, y) and the calculation curvilinear coordinate system (i, j) is established. These two coordinate systems are orthogonal.

$$i = i(x, y); j = j(x, y) \tag{9}$$

Then, the parameters in the physical coordinate system can be represented as a function of the curvilinear coordinates. For example, considering the first expression of the continuity equation, $\hat{u}_x$ can be expressed in parametric form in the computed coordinate system;

$$\hat{u}_x = \hat{u}_i * i_x + \hat{u}_j * j_x \tag{10}$$

And the bi–global stability equations in the new curvilinear system are

$$\hat{u}_i * i_x + \hat{u}_j * j_x + \hat{v}_i * i_y + \hat{v}_j * j_y + i\beta\hat{w} = 0 \tag{11}$$

$$\begin{aligned}
-(\overline{u}_i * i_x + \overline{u}_j * j_x) * \hat{u}\\
-(\overline{u}_i * i_y + \overline{u}_j * j_y) * \hat{v}\\
-(\hat{p}_i * i_x + \hat{p}_j * j_x) -\\
\overline{u}(\hat{u}_i * i_x + \hat{u}_j * j_x) - \overline{v}(\hat{u}_i * i_y + \hat{u}_j * j_y) +\\
\{\hat{u}_i * i_{xx} + \hat{u}_j * j_{xx}\\
+\hat{u}_{ii} * (i_x)^2 + \hat{u}_{jj} * (j_x)^2 + 2\hat{u}_{ij} * i_x * j_x +\\
\hat{u}_i * i_{yy} + \hat{u}_j * j_{yy} + \hat{u}_{ii} * (i_y)^2 +\\
\hat{u}_{jj} * (j_y)^2 + 2\hat{u}_{ij} * i_y * j_y - \beta^2\hat{u}\}/Re_c\\
= -i\omega\hat{u}
\end{aligned} \tag{12}$$

$$\begin{aligned}
-(\overline{v}_i * i_x + \overline{v}_j * j_x) * \hat{u} -\\
(\overline{v}_i * i_y + \overline{v}_j * j_y) * \hat{v} -\\
(\hat{p}_i * i_y + \hat{p}_j * j_y) - \overline{u}(\hat{v}_i * i_x + \hat{v}_j * j_x) -\\
\overline{v}(\hat{v}_i * i_y + \hat{v}_j * j_y) + \{\hat{v}_i * i_{xx} + \hat{v}_j * j_{xx}\\
+\hat{v}_{ii} * (i_x)^2 + \hat{v}_{jj} * (j_x)^2 +\\
2\hat{v}_{ij} * i_x * j_x + \hat{v}_i * i_{yy} + \hat{v}_j * j_{yy} +\\
\hat{v}_{ii} * (i_y)^2 + \hat{v}_{jj} * (j_y)^2 + 2\hat{v}_{ij} * i_y * j_y - \beta^2\hat{v}\}/Re_c\\
= -i\omega\hat{v}
\end{aligned} \tag{13}$$

$$\begin{aligned}
-\overline{u}(\hat{w}_i * i_x + \hat{w}_j * j_x) -\\
\overline{v}(\hat{w}_i * i_y + \hat{w}_j * j_y) -\\
i\beta\hat{p} + \{\hat{w}_i * i_{xx} +\\
\hat{w}_j * j_{xx} + \hat{w}_{ii} * (i_x)^2 + \hat{w}_{jj} * (j_x)^2 +\\
2\hat{w}_{ij} * i_x * j_x + \hat{w}_i * i_{yy} + \hat{w}_j * j_{yy} +\\
\hat{w}_{ii} * (i_y)^2 +\\
\hat{w}_{jj} * (j_y)^2 + 2\hat{w}_{ij} * i_y * j_y - \beta^2\hat{w}\}/Re_c\\
= -i\omega\hat{w}
\end{aligned} \tag{14}$$

It should be noted that the value of i_x should be solved through an inverse transformation.

$$x = x(i, j); y = y(i, j) \tag{15}$$

Differentiating, it is possible to obtain

$$dx = x_i * di + x_j * dj \tag{16}$$

$$dy = y_i * di + y_j * dj \tag{17}$$

$$di = i_x * dx + i_y * dy \tag{18}$$

$$dj = j_x * dx + j_y * dy \tag{19}$$

As a matrix, this becomes

$$\begin{bmatrix} i_x & i_y \\ j_x & j_y \end{bmatrix} = \frac{\begin{bmatrix} y_j & -x_j \\ -y_i & x_i \end{bmatrix}}{\begin{vmatrix} x_i & x_j \\ y_i & y_j \end{vmatrix}} \tag{20}$$

and the coefficients in the above equations can be solved. For corresponding points, they share the same value and simplify to

$$u(x_A, y_A) = u(i_A, j_A) \tag{21}$$

The bi–global stability equations can also be cast as a partial-derivative eigenvalue problem,

$$\hat{A}\hat{\phi} = \omega M \hat{\phi} \tag{22}$$

in which $\hat{A}$ is the spatial discretization operator, which is a function of the mesh, base flow, Reynolds number Re_c, $\hat{\phi}$ and spanwise perturbation wavenumber β. The code was validated against the results of Theofilis *et al.* (2004) for Poiseuille flow in a rectangular domain at $Re = 100$ and $\beta = 1$ and showed good agreement.

With such large–scale partial–derivative eigenvalue problems, it is computationally more efficient to solve for the eigenvalues nearest a certain point in the complex plane using a shift-and-invert method, such as the implicitly restarted Arnoldi method (IRAM). Thus, the above eigenvalue problem is modified as shown, given a complex shift σ:

$$(\hat{A} - \sigma M)^{-1} M \hat{\phi} = \hat{\phi}/(\omega - \sigma) \tag{23}$$

0n the airfoil surface, the boundary conditions $\hat{u} = \hat{v} = \hat{w} = 0$ are imposed on the perturbation velocities and the compatibility condition for pressure of a zero wall normal gradient is employed (Theofilis *et al.* (2004)). Since the properties of the perturbations are not known before solving the EVP, the amplitude functions are linearly extrapolated from within the domain (Kitsios *et al.* (2009)). A code was written to solve this eigenvalue problem using GPU acceleration.

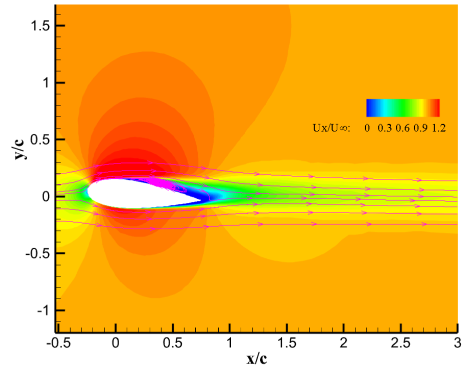


Figure 1. The base flow of the NACA 0025 at AOA=5

Results and discussion

The base flow for an airfoil angle of attack of 5 degrees and $Re_c = 1 \times 10^5$ is shown in Figure 1. There is a small separation bubble on the airfoil surface near the middle, and the flow reattaches at prior to the trailing edge. The time–averaged flow is dominated by a small recirculation zone in this area.

When the Reynolds number is very low (below 2000), the angular frequency is close to 0 and there are only 1 or even no unstable modes in some cases, which means that setting the shift value σ to zero is the proper choice (Kitsios *et al.* (2009)). When the Reynolds number is higher, as in this case, there are many unstable modes and the dimensionless frequency F^+ has a large range from 1 to 10, corresponding to the angular frequency around 0–62.8. To deploy the full power of the Arnoldi algorithm and to avoid computing all the eigenvalues (which requires significant memory) (Groot (2013)), several shift values σ are chosen to capture the eigenvalues with the angular frequency in the range of 1–70.

Figure 2 shows the eigenvalue distribution with the spanwise perturbation wavenumber set to zero, which is used to examine the stability characteristics of the two-dimensional flow. There are fewer growth modes in the high-frequency region compared to the low-frequency region. When dimensionless frequency is larger than 6, there are roughly 8 unsteady modes and the unsteady growth rate is significantly decreased. This means that it is possible to excite the unsteady separated flow at low frequency to fully utilize the instability of flow. The dimensionless frequency of the most unstable modes was about 1 and the spatial structure of this most unstable mode is shown in Figure **??** by the real part of the streamwise and normal velocities. This structure exhibits classical features of the wake type mode. The alternating velocity perturbation originates approximately 2 chord lengths downstream the airfoil.

Monotonically growing modes corresponding to $F^+ = 0$ are also found at this Reynolds number. The spatial structure of this mode, known also as the stationary mode, is shown in Figure 4. While this mode does not fluctuate in time (F+ = 0), it does grow ($\omega_i \approx 0.36$). This kind of stationary mode was also identified in other studies (Kitsios *et al.* (2009)).

Another purpose of this work is to examine actuator spacing as this is an important flow control parameter. The spanwise spacing on the eigenmodes is helpful for preliminary selection of actuator spacing. Large spanwise wavenumber, β, corresponds to small-wavelength perturbation in the three-dimensional flows. In this case, shift

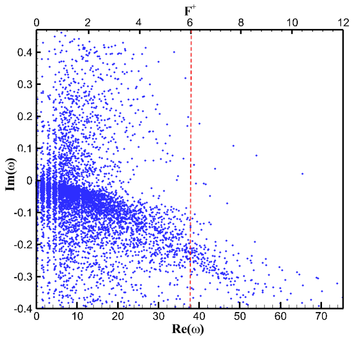
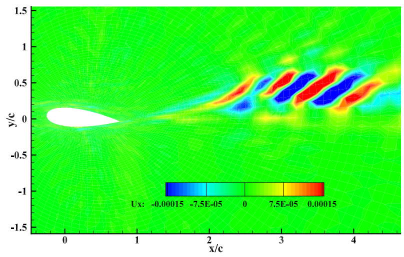
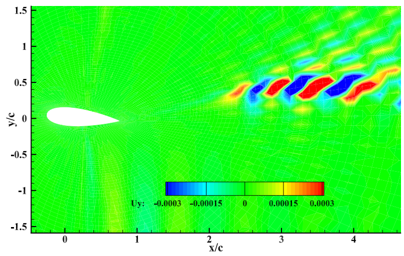


Figure 2. Eigenvalue spectrum for $\beta = 0$

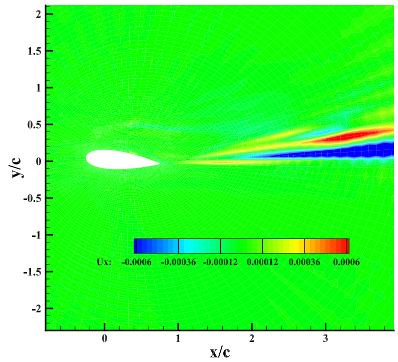


(a) Eigenfunction of $Re(\hat{u})$

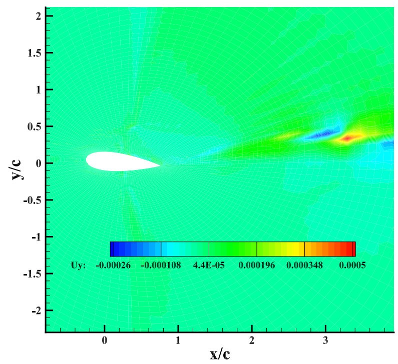


(b) Eigenfunction of $Re(\hat{v})$

Figure 3. Spatial structure of most unstable mode($F^+ \approx 1$)



(a) Eigenfunction of $Re(\hat{u})$



(b) Eigenfunction of $Re(\hat{v})$

Figure 4. Spatial structure of stationary mode($F^+ = 0$)

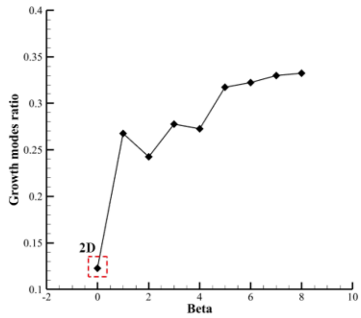


Figure 5. The effects of β on the number of unstable modes

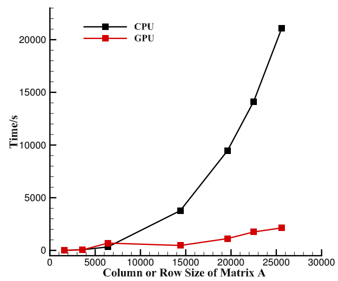


Figure 6. The comparison of computing time between CPU and GPU

values were chosen as 0 to initially study the effect of three-dimensional span spacing on the instability of flow field. Compared with the two–dimensional separated flow $\beta = 0$, adding a three-dimensional perturbation increases the number of unstable modes. Generally, as the spanwise wavenumber increases, the number of unstable modes also increases until it reaches a stable range. For flow control, it is necessary to excite the separated flow at small actuator spacing for low angles of attack to fully exploit the instability of separation.

Implementation of graphics processing unit (GPU)

To minimize time to solution, a Tesla K80 GPU was used to test the performance of GPU speed up. Comparing CPU and GPU performance (figure 6) when the size of Matrix A is less than 7000×7000, CPU calculation speed is better than GPU. As the dimension of the matrix A increases, the advantage of using GPU calculation becomes more and more apparent. When the size of matrix A is 25600×25600, the time required for CPU calculation is ≈ 21100 seconds, while the GPU only requires ≈ 2150 seconds. The calculation time using the GPU is approximately one–tenth that of the CPU.

Conclusions

A new method is developed to solve the bi–global stability equations in curvilinear coordinates which avoids the reshaping of the airfoil or remapping the disturbance flow fields, and the bi–global stability function in the calculation curvilinear system are derived. With the validated code, the instability features around an airfoil at an angle of attack of 5 degrees and $Re_c = 1 \times 10^5$ are investigated. At this angle of attack, there is a small separation bubble in the airfoil surface. The most unstable mode was found to be related to the wake mode and the dimensionless frequency is close to 1. The dimensionless frequency of most of the unstable modes are below 6. As the spanwise wavenumber increases, the number of unstable modes also increases and then remains stable in a range for this case. Compared with tri–global stability, the memory that bi–global stability required is reduced significantly. However, it is still time consuming, especially when the matrix becomes large. A GPU was employed to speed up the solution of the eigenvalue problem. For large matrices, the calculation time using the GPU was roughly one–tenth that of the CPU.

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