INVESTIGATION OF THE MEAN VELOCITY FIELD IN THE INNER LAYER OF A PLANE TURBULENT WALL JET

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ABSTRACT

The present paper focuses on the characteristics of the inner layer of a plane turbulent wall jet on a smooth surface. The analysis is based on a recent experimental study of the mean and fluctuating velocity fields in a plane turbulent wall jet on a smooth surface using Particle Image Velocimetry (PIV). The wall jet is often regarded to consist of an inner layer and an outer layer, which resemble a turbulent boundary layer (TBL) and free jet, respectively. The inner layer is the focus of the present study, which explicitly compares some characteristics of the mean velocity field in the inner layer to those in a zero-pressure gradient (ZPG) and adverse-pressure gradient (APG) TBL. Based on analysis of the experimental data, the inner layer is characterized by a narrow overlap region which is well approximated by both a canonical logarithmic velocity profile and power law. However, the velocity defect profile for the inner layer does not match that of a ZPG TBL. The thickness of the inner region of the wall jet grows more slowly than a ZPG TBL. Finally, the correlation for the skin friction coefficient developed for an APG TBL provides a better fit to that of the inner layer of the wall jet compared to a ZPG TBL. However, it still significantly under-predicts the skin friction in the turbulent wall jet.

INTRODUCTION

The plane turbulent wall jet is a flow with many important practical applications in industry. It is also of interest from a theoretical viewpoint, i.e. in understanding the effect of a wall on the turbulent flow above. A wall jet is often characterized using a simple structure consisting of an inner layer that closely resembles a TBL, and an outer layer that is similar to that of a free jet. The two layers are separated by a mixing region, which is distinguished by the fact that the location y_m of the maximum mean velocity U_m does not coincide with the location of the zero value of the Reynolds shear stress, implying that the turbulent transport is more complex than could be modeled by an eddy viscosity model closure. Wygnanski *et al.* [1] were among the first to document both the mean and fluctuating velocity fields in a plane turbulent wall jet. Their study implied that the mean velocity in the inner layer region was not consistent with a canonical logarithmic profile. However, subsequent studies by Ericsson *et al.* [2], Tachie *et al.* [3], Rostamy *et al.* [4] and others indicate that the overlap region of the inner layer can be fitted to a logarithmic velocity profile, even though the overlap region is very thin. In a comprehensive study that examined the similarity behavior of the turbulent wall jet, George *et al.* [5], derived a power law form for the mean velocity profile at finite Reynolds number. They also derived a composite profile relation based on the power law, which models the mean velocity from the wall up to the end of the overlap region. Banyassady and Piomelli [6] performed a Large Eddy Simulation of a plane turbulent wall jet, and compared the inner layer of a plane turbulent wall jet to the ZPG TBL. They concluded that the inner layer differs from a conventional TBL in terms of the turbulence structure due to the presence of the outer layer. However, this difference is minimal in terms of the mean velocity field, and in near proximity to the wall, the outer-layer effects are almost negligible. They also confirmed that the mean velocity in the inner region could be matched to the canonical logarithmic profile, however, the constants were weakly dependent on the Reynolds number.

One characteristic that distinguishes the turbulent wall jet from a ZPG TBL is the fact that the maximum velocity of the wall jet decays in the streamwise direction due to the combined effects of mixing in the outer layer and skin friction at the wall. This suggests that for some features, it may be more appropriate to compare the inner layer to an APG TBL, for which the freestream velocity also decreases in the streamwise direction. Note that such a comparison only captures the decay of the maximum velocity in the streamwise direction; it ignores the other significant distinction of the wall jet, i.e. that the flow in the outer layer above is characterized by an additional strong shear layer, which is not the case for a TBL.

The present study will revisit the assumption that the inner layer of a plane turbulent wall jet is similar to a conventional TBL. More specifically, it will examine features of the mean velocity profile and where appropriate compare them to a ZPG and APG boundary layer. Included in the analysis are the following: scaling of the mean velocity field for the inner layer using both the logarithmic velocity profile and the defect law based on inner and outer coordinates, respectively; the streamwise growth rate of the location of the maximum velocity of the wall jet; and the skin friction coefficient.

EXPERIMENTAL STUDY

The present study utilizes the mean velocity data obtained by the experimental investigation of a plane turbulent wall jet by Tang [7]. The study performed PIV measurements to document the mean and fluctuating velocity fields, on both a smooth and rough-wall ground plane. The inlet streamwise velocity profile was reasonably uniform with a low turbulence intensity level. The experiment was conducted in a finite size tank, so the entrainment to the wall jet was supplied by a relatively weak reverse flow above the outer layer: this was similar to the experimental configuration of Ericsson *et al.* [2]. Three different surface conditions and two different flow rates were considered. The analysis presented in this paper focuses on a smooth surface. The slot Reynolds number for the low flow rate (LFR) and high flow rate (HFR) was $Re = 7{,}190$ and $Re = 14{,}300$, respectively.

MEAN VELOCITY PROFILE

Figure 1 presents the experimental results for the mean velocity profile in the inner layer of a turbulent plane wall jet for the HFR case using inner coordinates. Here x is the streamwise coordinate and H is the slot height. It is evident that the mean velocity profile in the fully developed region is well described by a logarithmic profile, i.e.

$$\frac{U}{u_\tau} = \frac{1}{\kappa}\ln y^+ + B \qquad (1)$$

where u_τ is the friction velocity and the dimensionless wall normal distance is given as $y^+ = yu_\tau/\nu$, where ν is the kinematic viscosity. In this case the conventional values for the coefficients, i.e. $\kappa = 0.41$ and $B = 5.0$ work well. Also shown is the composite profile developed by George *et al.* [5] based on a power law similarity profile for the mean velocity in the overlap region. The composite profile is also in good agreement with the data. Note that unlike the case of a TBL, there is no wake component to the mean velocity profile. Instead, for the wall jet, the velocity profile bends below the logarithmic profile. The maximum velocity U_m occurs at approximately $y^+ = 400$, beyond which the mean velocity begins to decrease as the wall normal distance increases into the outer layer.

DEFECT LAW PROFILE

One of the similarity relations used to assess the mean velocity field in a TBL is the defect law. The defect law is derived as a similarity relation for the mean velocity profile in the outer region of the TBL, which corresponds to the top region of the inner layer of a wall jet. The defect law in outer coordinates is given as:

$$\frac{U_e - U}{U_e} = f\left(\frac{y}{\delta}\right) \qquad (2)$$

where U_e is the free stream velocity and δ is the boundary layer thickness. For the plane turbulent wall jet, U_m and $y_m$were used for U_e and δ in equation (2), respectively.

The defect law profiles based on the mean velocity in the inner layer of a plane turbulent wall jet are presented in Figure 2. For the wall jet, profiles based on the data at two different streamwise sections for two different flow rates are shown. For comparison, defect profiles for a ZPG TBL are also shown based on the measurements of Akinlade [8] and DeGraaff and Eaton [9].

It is evident from Figure 2 that the wall jet profiles collapse well in the outer region of the inner layer. The profiles for the ZPG TBL likewise collapse reasonably well for two different Reynolds numbers based on momentum thickness (Re_θ). However, the defect profile for the turbulent wall jet does not match that of the TBL, and instead sits consistently lower. This suggests that in the outer region, the analogy between the maximum velocity for the wall jet and freestream velocity for the ZPG TBL is incorrect, and that the mechanism accounting for the momentum transport are somewhat different.

COMPARISON OF THICKNESS BETWEEN THE INNER LAYER AND ZPG TBL

Table 1 presents a comparison between the inner layer of a plane turbulent wall jet (based on the HFR results) and a ZPG TBL in terms of the streamwise growth rate of the nominal thickness of each layer. For the TBL, we have used the standard textbook relation based on assumption of a one-seventh power law. Note that both of these thicknesses are relatively fuzzy parameters. For a wall jet, the precise location of the maximum velocity is difficult to determine since the velocity profile has a relatively blunt peak region. On the other hand for a ZPG TBL, the thickness of the boundary layer depends on determining the location where the mean velocity inside the boundary layer is within 1% of the free stream value. Both y_m and δ are characterised by non-linear growth rates, with the wall jet growing more slowly than the ZPG TBL.

Table 1. Comparison of the growth rate of the inner layer of the plane turbulent wall jet with a ZPG TBL.

Flow	Growth rate
wall jet	$y_m \sim x^{0.8}$
TBL	$\delta \sim x^{0.86}$

SKIN FRICTION

For a smooth wall ZPG TBL, the empirical formula proposed by Osaka *et al.* [10] for the skin coefficient is a popular correlation given by:

$$\frac{1}{C_f} = 20.03\left(\log_{10}\left(\frac{U_e\theta}{\nu}\right)\right)^2 + 17.24\log_{10}\left(\frac{U_e\theta}{\nu}\right) + 3.71 \qquad (3)$$

where U_e represents the freestream velocity, θ is the momentum thickness and $\boldsymbol{\nu}$ is the kinematic viscosity of the fluid. Equation (3) can be reformulated for a plane turbulent wall jet by replacing the freestream velocity U_e and boundary layer thickness δ with the local maximum velocity U_m and height of the maximum velocityy_m, respectively. If we further assume that the mean velocity profile in the inner layer can be approximated by a one-seventh power law, then the relation for the skin friction becomes:

$$\frac{1}{C_f} = 20.03\left(\log_{10}\left(\frac{7}{72}\mathrm{Re}_m\right)\right)^2 + 17.24\log_{10}\left(\frac{7}{72}\mathrm{Re}_m\right) + 3.71 \qquad (4)$$

where Re_m is based on U_m and y_m.

For the case of an APG TBL over a smooth wall, the correlation proposed by Ludweig-Tillmann is given by [11]:

$$C_f = 0.246\left(\frac{U_e \cdot \theta}{\nu}\right)^{-0.268}\left(10^{-0.678H}\right) \tag{5}$$

Using the same change in variables as previously mentioned, equation (3) can be transformed to become:

$$C_f = 0.246\left(\frac{7}{72}\mathrm{Re}_m\right)^{-0.268}\left(10^{-0.678H}\right) \tag{6}$$

where for a one-seventh power law, the shape factor is given by $H = 1.29$. Note that the assumption of a one-seventh power law for the velocity profile is clearly problematic for strong APG flows.

Finally, Bradshaw and Gee [12] proposed the following correlation for the skin friction coefficient for a smooth-wall plane turbulent wall jet:

$$C_f = 0.0315\mathrm{Re}_m^{-0.182} \tag{7}$$

Figure 3 presents Tang's data [7] for the skin friction coefficient in a plane turbulent wall jet for both the LHF and HFR cases, as well as the data of Eriksson et al. [2]. Also shown are the skin friction relations outlined above. Some observations are as follows: the skin coefficient by Bradshaw and Gee [12] matches well with the experimental results, with a slight over-prediction for the HFR case. Based on comparison to the modified correlation of Osaka *et al.* [10], one can observe that the skin friction in the wall jet is approximately 20% greater than that in a ZPG TBL. Finally, the modified Ludweig-Tillmann correlation developed for an APG results in a skin friction profile that is much closer to the experimental data. However, it still tends to under-predict the data for higher values of Re_m by as much as 10%.

CONCLUSION

The present paper documents an analysis of the inner layer of a plane turbulent wall jet on a smooth surface based on a new set of PIV measurements of the velocity field. The wall jet is often regarded as having a hybrid structure consisting of a turbulent boundary layer and a free jet. This assumption was revisited based on comparisons between the inner layer of a wall jet and both ZPG and APG TBL's. According to the analysis, the mean velocity profile in the fully developed region agrees well with the canonical logarithmic profile using the standard set of constants, as well as with the power law relation proposed by George *et al.* [5]. However, the defect law profiles of the inner layer do not agree with those of a canonical ZPG TBL. This suggests that in the outer region of the inner layer, the role of the maximum velocity is somewhat different for a wall jet than for a boundary layer. In terms of the wall normal thickness, the streamwise growth rate of the wall jet is somewhat less that of a ZPG TBL. Finally, for the skin friction coefficient, the correlation developed for the case of an APG TBL does a better job than the correlation for a ZPG TBL in approximating the behavior of the wall jet. This can be explained by the fact that like the wall jet, the APG TBL is characterized by a maximum/freestream velocity that reduces in the streamwise direction. Overall, the correlation of Bradshaw and Gee [12] captures the data well both for both the HFR and LFR cases. In conclusion, although the mean velocity profile in the inner layer of a turbulent wall jet shares some important characteristics with a ZPG TBL, there are also significant differences, including the shape of the defect profile and the value of the skin friction coefficient.

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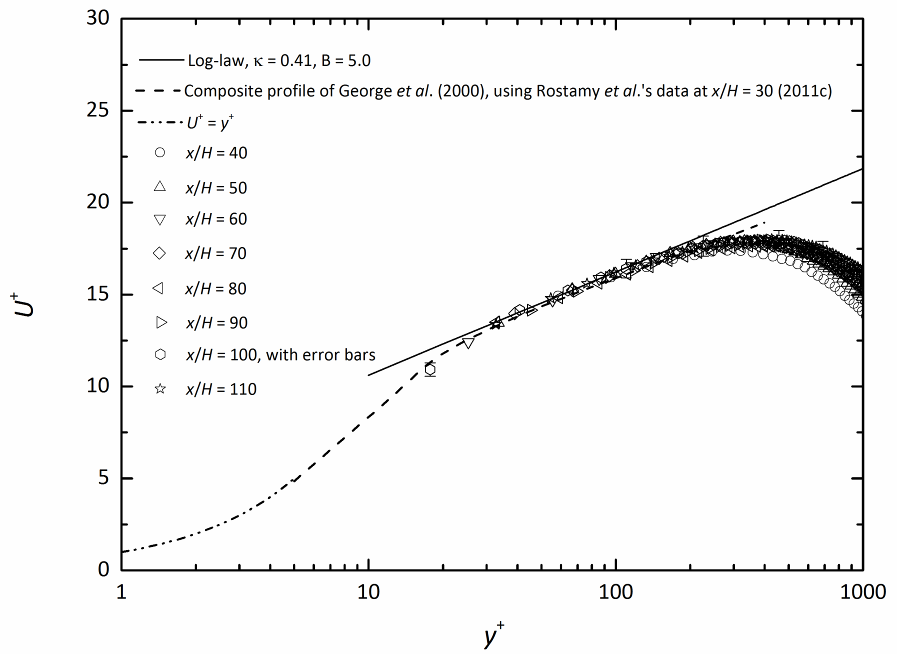

Figure 1. Mean velocity profile using inner coordinates for the HFR case [7].

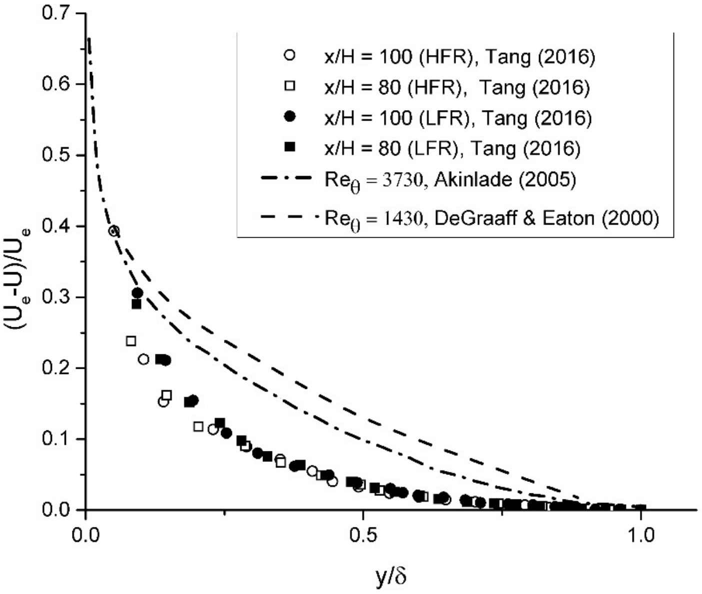

Figure 2. Defect velocity profiles for HFR and LFR case.

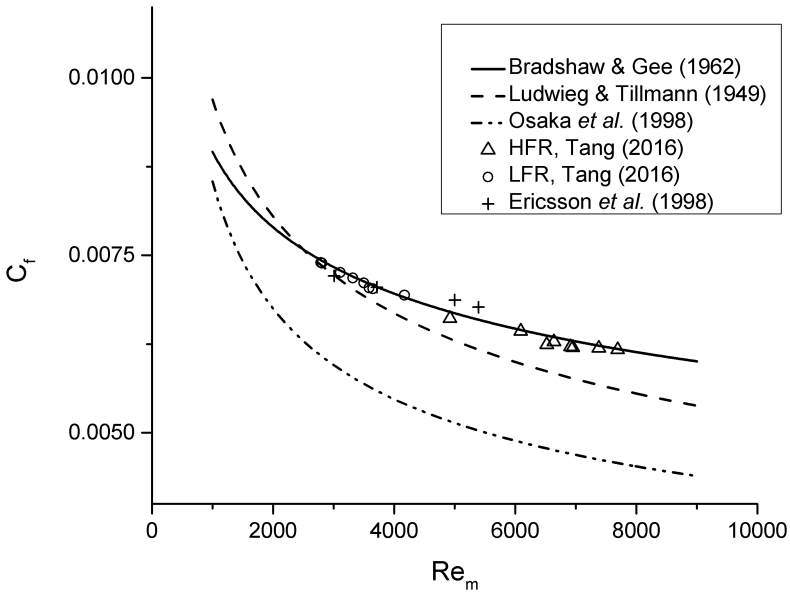

Figure 3. Variation of skin friction coefficient with local Reynolds number.