

ANALYSIS OF THE TURBULENT KINETIC ENERGY BUDGET FOR MEANDERING MOTION APPEARING IN A BACKWARD-FACING STEP FLOW OF VISCOELASTIC FLUID IN HIGH REYNOLDS NUMBER

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ABSTRACT

It is known from our previous study (Ii et al., 2015) that a remarkable meandering motion occurs depending on the Reynolds number and surfactant concentration in a backward-facing step flow of a viscoelastic surfactant solution in high Reynolds number. The present report provides the examination of the meandering motion from the analysis of the turbulent kinetic energy budget. In particular, we capture the characteristic of the streamwise turbulent velocity fluctuation in terms of analysis of production, turbulent diffusion and dissipation terms. We conclude that the meandering motion become prominent in the flow-regime where the production rate surpasses the dissipation rate resulting in onset of instability at large-scale eddies.

INTRODUCTION

Separation and reattachment of turbulent flows occur in many practical systems. There have been many studies by both experiment and simulation (Abe et al., 1994; Kasagi and Matsunaga., 1995; Rani et al., 2007; Khoury et al., 2010). However, these studies have remained the only Newtonian-fluid case. There have been few studies on the complicated flow of viscoelastic non-Newtonian fluid. It is well known that surfactant additives give viscoelasticity to the flow and can suppress turbulence in canonical flows like a straight pipe or channel. On the other hand, in the viscoelastic turbulent channel flow with rectangular orifice, the authors' group reported a complex fluid motion that quasi-streamwise vortices were sustained for a longer period (Tsukahara et al., 2011). This flow with the orifice is fundamentally different as compared to straight channel, most notably in the flow instability due to a sudden

contraction/expansion or in the influence of high shear past an obstruction for the flow and fluid property. Also the flow changed

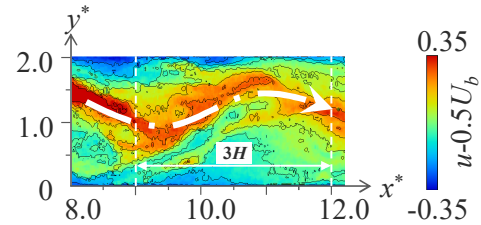


Figure 1. Instantaneous velocity field past a BFS for surfactant solution flow, viewed in the x - y plane. The contour level means the instant streamwise velocity using Galilean decomposition. High-speed region meanders broadly as indicated by a white arrow.

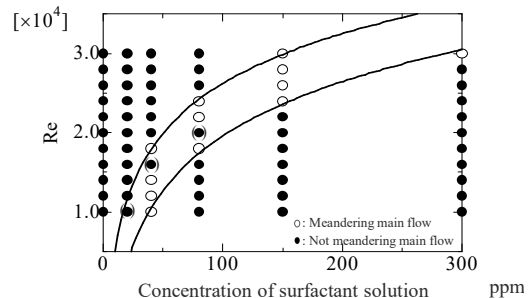


Figure 2. Flow-regime diagram in the map of the Reynolds number and concentration of the surfactant solution. The meandering motion is observed in the area between two curves.

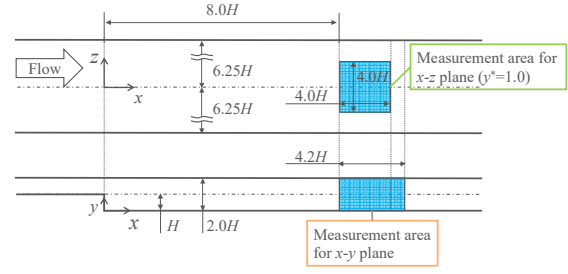
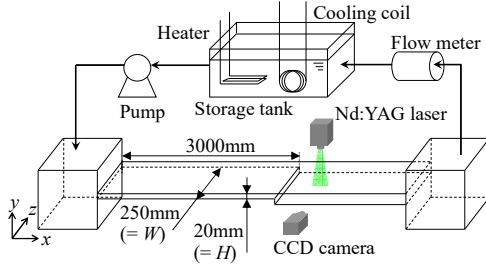


Figure 3. Outline of the experimental apparatus. (a) Closed-circuit channel flow with the backward-facing step of expansion rate of 1:2, (b) the measurement area.

Table 1. Spatial resolution of the PIV experiments using square interrogation area.

Measurement plane	Δx [mm]	$\Delta x/H$
x - y	0.33	0.017
x - z	0.85	0.043

significantly in the streamwise direction induces complicated phenomenon on the relationship between a downstream distance from the orifice and a relaxation time of a fluid, reported by Tsukahara et al. (2016).

Recently we have also reported an eye attracting feature of a backward-facing step (BFS) flow in which the instantaneous main stream meanders broadly in the wall-normal direction as shown in Fig.1 (Li et al., 2015). The meandering motion that leads to active fluid mixing occurs depending on the Reynolds number and surfactant concentration, C (see Fig. 2). The purpose of this study is to analyze the meandering phenomenon of the viscoelastic fluid flow in terms of the turbulent kinetic energy budget. If the turbulent dissipation becomes small due to the disappearance of small-scale eddies by the viscoelasticity in the condition where the turbulent kinetic energy is largely produced, large-scale eddies could store the turbulent kinetic energy leading to the increase of its instability. Hence, we try to compare the occurrence condition of the meandering motion to the disequilibrium condition between production and dissipation rates of the turbulent kinetic energy. In this study, the fluid flow of surfactant solution is experimentally investigated in the channel with the BFS. Particle image velocimetry (PIV) measurements are performed around the reattachment zone in order to capture the spatial distribution of velocity fluctuation and estimate the gain and loss of turbulent kinetic energy.

EXPERIMENT APPARATUS AND PROCEDURE

Figure 3(a) shows closed-circuit channel flow with the BFS of expansion rate of 1:2. A channel height H is 20 [mm] in upstream of the BFS. A fully-developed velocity profile was ensured at the expansion geometry 3000 mm downstream. PIV measurements were performed in a streamwise-wall-normal plane (x - y plane) and streamwise-spanwise plane (x - z plane), as shown in Fig. 3 (b). Here, the streamwise direction is $x_1 = x$, the wall-normal direction is $x_2 = y$, and the spanwise direction is $x_3 = z$. The instantaneous velocity field in the respective directions is u , v , w . The PIV measurement area was set near the reattachment zone downstream. A double-pulse laser consisting of a pair of Nd:YAG lasers was used for illumination by laser sheet. The flow

was seeded with nylon particles as tracer particles with a mean diameter of 4.1 μm in the measurements in x - y plane and with polyethylene powder with a mean diameter of 20 μm in the measurements of x - z plane. Turbulent statistics were obtained by averaging 500 images of velocity field. The surfactant solution as viscoelastic fluid was prepared by adding the same mass of cetyltrimethylammonium chloride (CTAC) and sodium salicylate to the tap water. The concentrations of the surfactant solutions in the x - z plane measurements are 0, 40, and 150 ppm. In the x - y plane measurements, cases for C of 20, 80, and 300 ppm were added. Resolutions of the PIV measurements are listed in Table 1. In the table, the resolution was indicated in the outer units, i.e., normalized with the outer variables: the bulk mean velocity U_b obtained by an electromagnetic flow meter and H . The total vector numbers per frame are $x \times y = 253 \times 120$ and $x \times z = 97 \times 97$, respectively. The inlet temperature was controlled at 298.2 ± 0.2 [K]. The Reynolds number ($\text{Re} = U_b H / \nu$) was varied from 1.0×10^4 to 3.0×10^4 . Here, ν [m^2/s] is the kinetic viscosity of water to be the same as that of the surfactant solution. Hereafter, the superscripts “ ’ ”, “ $\bar{}$ ”, “ $\langle \rangle$ ” and “ $\ast$ ” denote fluctuating component, time-averaged value, spatial-averaged value at each wall-normal position, and normalization with the outer units, respectively. Our previous study (Li et al., 2015) provides more information of fundamental statistic and instantaneous velocity fields.

ANALYSIS OF THE TURBULENT KINETIC ENERGY BUDGET

In this study, we especially focus on production and dissipation rates for the streamwise turbulent kinetic energy, $u'u'$. Here, assuming that $\bar{w} = 0$ and $\partial/\partial z = 0$, the expression for the streamwise production of the turbulent kinetic energy, P_{11} , is written as,

$$P_{11} = -\left\langle u'u' \right\rangle \left\langle \frac{\partial \bar{u}}{\partial x} \right\rangle - \left\langle u'\bar{v}' \right\rangle \left\langle \frac{\partial \bar{u}}{\partial y} \right\rangle. \quad (1)$$

Figure 4 shows P_{11} at $y/H = 1.0$ in the map of Reynolds number and concentration of surfactant solution. High C and low Re flows show lower P_{11} because the stabilizing action of viscoelastic fluid suppresses the turbulent motions. On the other hand, the stabilizing action becomes weak in low C and high Re flows, and then the flow should be like the Newtonian fluid flow, resulting in moderate P_{11} . In the area between the two curves in Fig. 4 the meandering motion occurred in the flow field drastically increases the Reynolds shear stress, $-\bar{u}'\bar{v}'$. In addition

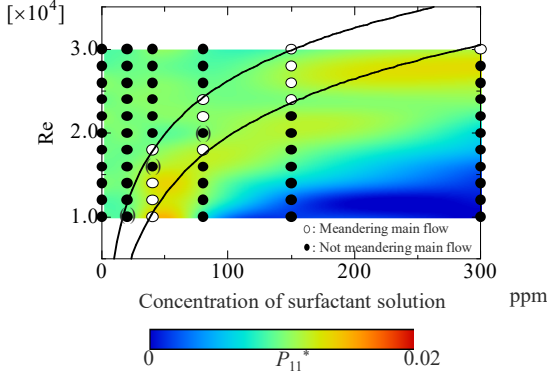


Figure 4. Flow-regime diagram in the map of Reynolds number and concentration of surfactant solution and the contour of the streamwise production rate at $y/H = 1.0$. The meandering motion is observed in the area between the two curves.

to this, the velocity gradient becomes steep over the BFS because drag-reducing state appears in the upstream region, namely, the smooth channel. These should result in an intensive P_{11} in the meandering condition.

Next, we discuss a part of dissipation rate estimated from the x -directional component of velocity fluctuation. The dissipation rate of ε_{11} for the streamwise component is the mean square of velocity fluctuation gradient was averaged in the x -direction, which is defined as;

$$\varepsilon_{11} = \nu \left\langle \left(\frac{\partial u'}{\partial x} \right)^2 + \left(\frac{\partial u'}{\partial y} \right)^2 + \left(\frac{\partial u'}{\partial z} \right)^2 \right\rangle. \quad (2)$$

The dissipation rate for the streamwise component based on the PIV measurement can be assessed with consideration in respect to the spatial resolution in Table 1.

Firstly, in an attempt to provide an overall picture of the instantaneous dissipation for the streamwise component, contour maps of squared fluctuating velocity gradient fields at $Re = 1.0 \times 10^4$ are plotted in Figure 5(a-c). Preliminary, it is confirmed that high-speed region flows with the fluid mixing in the water case, and meanders broadly at 40 ppm case, and flows stably straight in the streamwise direction at 150 ppm case. The comparison of streamwise length scales at $y/H = 1.0$ in the x - y plane and in the x - z plane indicates the following relation: $(\partial u'/\partial x)^2 < (\partial u'/\partial z)^2 \leq (\partial u'/\partial y)^2$. For the water flow, ε_{11} everywhere in the field results from a spatial variation of velocity fluctuation due to high diffusivity. The streamwise-elongated u' -spatial structure appearing in the transitional regime past the BFS leads to the smaller length scales dominant in $(\partial u'/\partial x)^2$. It is also assumed that the high values of $(\partial u'/\partial y)^2$ and $(\partial u'/\partial z)^2$ appearing around this structure persist in the x -direction. Additionally, the inherent fluctuating velocity gradient in the y -direction encourages the magnitude and length scale in $(\partial u'/\partial y)^2$. On the other hand, clearly, overall view of the surfactant solution flows exhibits the disappearance of small scales in the velocity fluctuation gradients. It is also interesting that high-dissipation zones are localized, as shown in Fig. 5(b) and (c). The even larger u' -spatial structure is caused by the stabilization of the turbulent flow field due to the viscoelasticity and it is reminded that ε_{11} in the mixing-layers

emanates from this structure. This argument, therefore, can explain the small value of $(\partial u'/\partial x)^2$ and high-dissipation zones in $(\partial u'/\partial y)^2$. The streamwise fluctuating velocity gradient in the y -direction is observed corresponding to the meandering motion at 40 ppm and to high-speed flow without the fluid mixing at 150 ppm. This plays a key role in the dissipation mechanism in BFS flow of the surfactant solution. Also the high-speed fluid narrowly meanders in the z -direction even though the meandering motion appears (Onishi et al. 2017), and so comparing the surfactant solution and water flows, close similarity is observed in $(\partial u'/\partial z)^2$.

Secondly, we perform the spectrum analysis at $y/H = 1.0$ corresponding to the mixing-layer in order to statistically examine the scale of the localized dissipation according to the meandering motion mentioned above. The one-dimensional premultiplied Fourier spectra, $F_{\varepsilon_{11,j}}$, of $(\partial u'/\partial x_j)^2$ can be computed as follows:

$$\nu \left\langle \left(\frac{\partial u'}{\partial x_j} \right)^2 \right\rangle = \int_0^\infty k_x F_{\varepsilon_{11,j}}(k_x) d(\log k_x), \quad (3)$$

where k_x is the wave number in the x -direction. As shown in Figure 6, the peak in the premultiplied spectra of $(\partial u'/\partial x)^2$ occurs at much higher wavenumbers than the others. Note that it is difficult to quantify the wavelength of the meandering motion in Fig. 6(a), and hence they will not be discussed here. It is now evident that the most distinguishing features of the surfactant solution flow are found in Fig. 6 (b) and (c). The streamwise fluctuating velocity gradients in the y and z directions are dominated by the meandering motion with a low wavenumber at 40 ppm and by the high-speed main flow without the fluid mixing with a low wavenumber at 150 ppm. This is consistent with the agreement discussed previously for the instantaneous velocity field analysis. It should be noted that the high premultiplied spectra of 150 ppm is presumed due to the strong shear at $y/H = 1.0$ (as reported by Li et al., 2015) and the magnitude of the premultiplied spectra at 40 ppm decays by the resulting low diffusivity. Besides, the wavenumber of the peak in the premultiplied spectra of $(\partial u'/\partial z)^2$ occurs in $1.0 < k_x^* < 3.0$, approximately corresponding to that of $(\partial u'/\partial y)^2$. In addition to this, the present study considers the low magnitude of $(\partial u'/\partial z)^2$, and hence ignores $(\partial u'/\partial z)^2$ for the calculation of ε_{11} rate because ε_{11} in validation of our hypothesis can substitute $(\partial u'/\partial y)^2$ for $(\partial u'/\partial z)^2$ in terms of the behavior in the wavenumber space.

Figure 7 shows ε_{11} at $y/H = 1.0$ in the map of Reynolds number and concentration of surfactant solution. The dissipation rates are inversely proportional to the Reynolds number below a critical value in the BFS flows. This is derived from the increase of a streamwise distance scaled with inner variables (the friction velocity and ν) at PIV measurement area. At Re exceeding the critical value, the dissipation rate increases because the increasing shear rate induces the frequent occurrence of high wavenumber eddies involving small velocity fluctuations. Besides ε_{11} in the map provides the complicated dependence on C . Firstly, ε_{11} in moderate C and Re flows indicates a low value that is reflected in the resulting disappearance of small-scale eddies. Secondly, high C case at low Re shows the high ε_{11} because the shortening of the inner-scaled streamwise distance from the BFS involved with the decreased friction velocity. Thirdly, the contour map in Fig. 7 includes the dependence of the wall-normal distance, and especially ε_{11} in high C and Re flows indicates a slightly

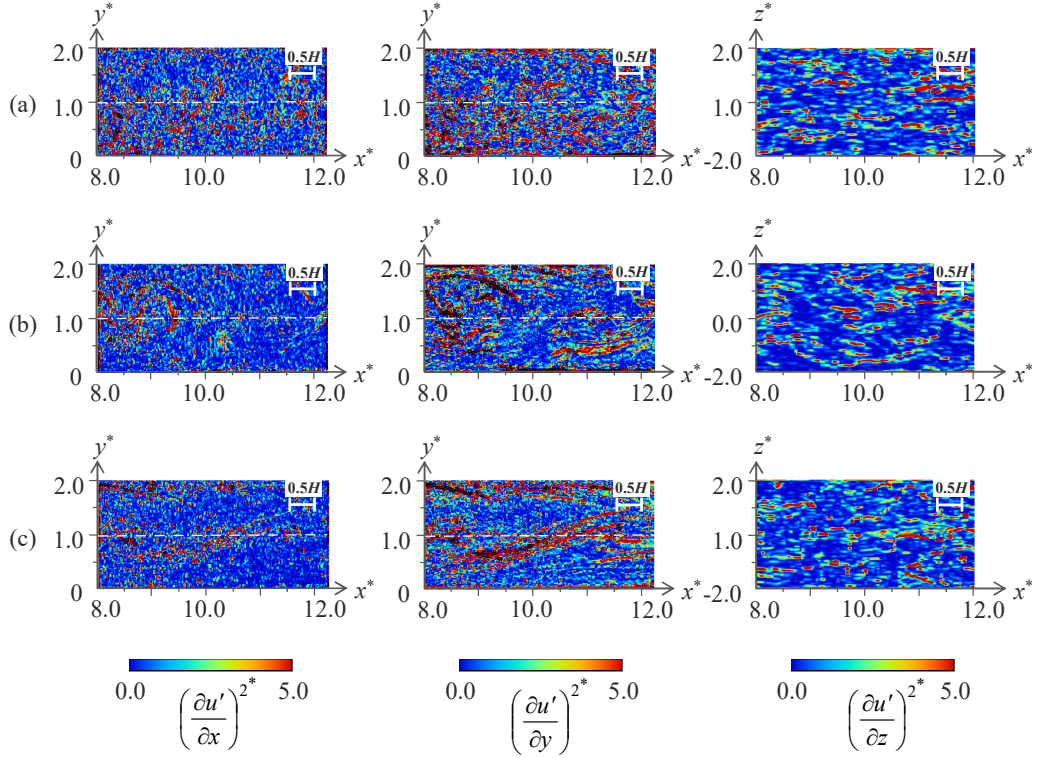


Figure 5. Squared streamwise fluctuating velocity gradient fields past the BFS at $Re = 1.0 \times 10^4$. (a) Water, (b) 40 ppm, (c) 150 ppm. White dashed line shows $y/H = 1.0$.

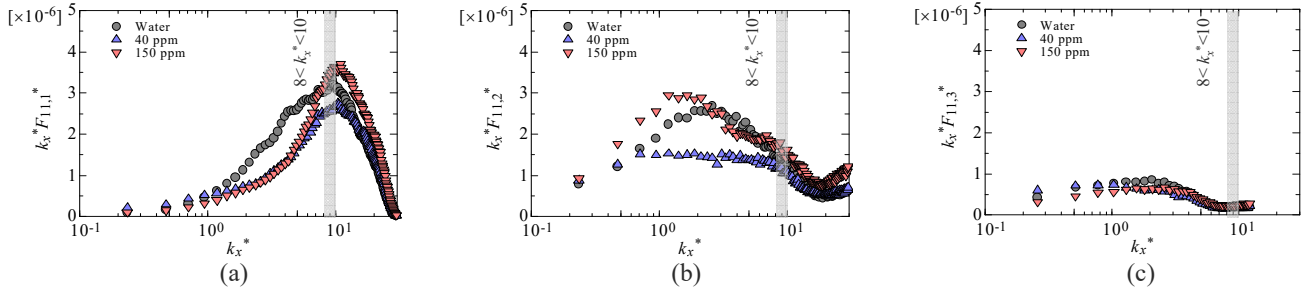


Figure 6. Premultiplied one-dimensional Fourier spectrum of each term in the streamwise dissipation at $y/H = 1.0$ for $Re = 1.0 \times 10^4$. (a) First term, (b) Second term, (c) Third term.

high value because the increasing shear rate at $y/H = 1.0$ is caused by change in the mean velocity profile. Incidentally, since the contour map in Fig. 7 confirm that the measurement area in the water flow is clearly at further downstream from the BFS and that high C cases have the almost zero Reynolds shear stress in the upstream, we can exclude the memory effect for the consideration of ε_{11} . Therefore, the consideration of the shear rate (as the indicator for small eddies) allows us to explain ε_{11} , which summarizes the influence of Re , the inner-scaled streamwise distance from the BFS, and the wall-normal distance. Here, the memory effect indicates the difference in time scale between the occurrence of the turbulent eddies causing the Reynolds shear stress and the relaxation of shear rate in the relaxation process from the BFS.

Considering change in the amplitude of velocity fluctuations in large scale to sustain turbulence, as mentioned in introduction, we should discuss the dissipation rate associated with the meandering motion. Here, we treat the length scale in the x -direction for the meandering motion as $3H$ from an observation in the instantaneous velocity field (, as highlighted in Fig. 1). Hence, the dissipation rate based on large scale components, ε_L , and that of small scale components, ε_s , are defined as;

$$\varepsilon_{11} = \underbrace{\int_0^{1/3H} F_{\varepsilon_{11}}(k_x) dk_x}_{\varepsilon_L} + \underbrace{\int_{1/3H}^{\infty} F_{\varepsilon_{11}}(k_x) dk_x}_{\varepsilon_s}, \quad (4)$$

where $F_{\varepsilon_{11}}(k_x)$ is the one-dimensional Fourier spectra of ε_{11} . Figure 8 shows $\varepsilon_L / \varepsilon_{11}$ and $\varepsilon_s / \varepsilon_{11}$ in the map of Reynolds number and concentration of surfactant solution, respectively. The values are divided by ε_{11} in order to confirm contribution of large-scale

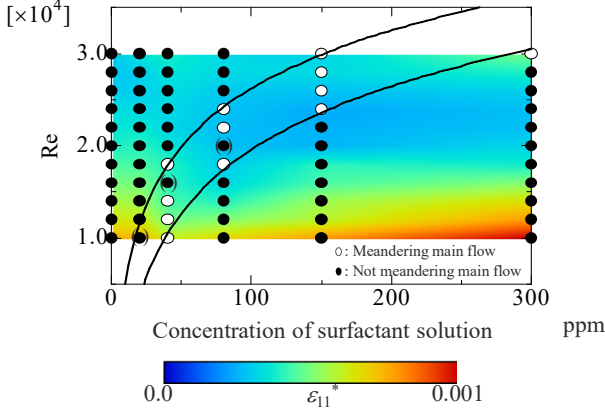


Figure 7. Flow-regime diagram in the map of Reynolds number and concentration of surfactant solution, and the contour of the streamwise dissipation rate at $y/H = 1.0$. The meandering motion is observed in the area between the two curves.

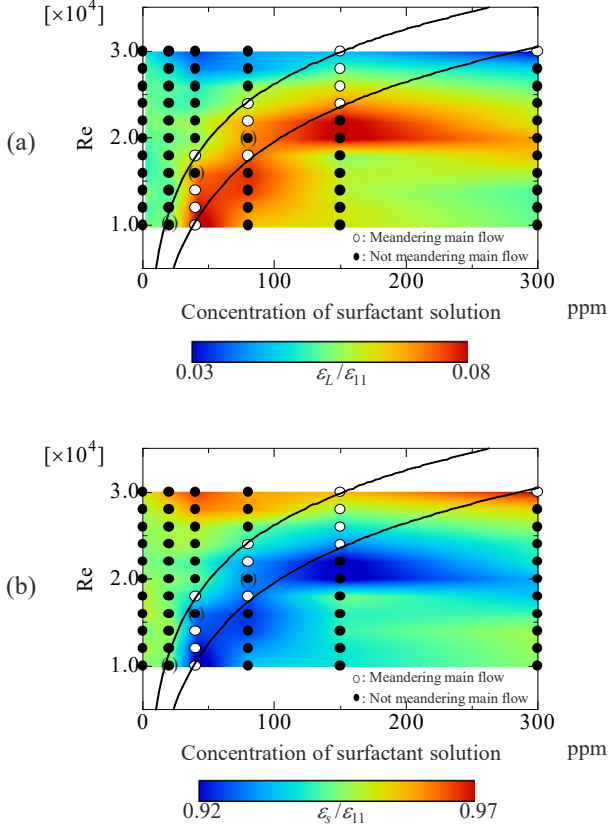


Figure 8. Flow-regime diagram in the map of Reynolds number and concentration of surfactant solution at $y/H = 1.0$. The meandering motion is observed in the area between the two curves. (a) Ratio of the streamwise dissipation rate based on large scale components to ε_{11} , (b) Ratio of the streamwise dissipation rate based on small scale components to ε_{11} .

eddies for ε_{11} in the meandering condition. It is interesting to note that the dissipation in large scale components becomes active and that in small scale components is attenuated only in the meandering condition. The results presented above suggest that the meandering motion occurs for the dissipation of the turbulent kinetic energy supplied by high shear over the BFS, while the dissipation in small scale components is suppressed.

From a comparison between the production and dissipation rates in the map of Reynolds number and concentration of surfactant solution, the Newtonian-like state at low C and high Re shows moderate P_{11} and slightly low ε_{11} and the stable flow at high C and low Re exhibits low P_{11} and high ε_{11} . Furthermore, the meandering condition corresponds to the disequilibrium condition where high P_{11} and low ε_{11} are observed, and then exhibits the active dissipation in large scale components and the attenuated dissipation in small scale components.

In the complex flow with separation, the viscous and turbulent transport terms would make a significant contribution to determine the distribution of velocity fluctuations. However, the viscous diffusion term is not shown here because of being around zero value. Here, we especially focus on the turbulent transport term expressed by

$$T_{11} = \left\langle \frac{\partial}{\partial x} \left(-\frac{1}{2} \overline{u'u'u'} \right) \right\rangle + \left\langle \frac{\partial}{\partial y} \left(-\frac{1}{2} \overline{u'u'v'} \right) \right\rangle. \quad (5)$$

Figure 9 shows the turbulent transport distribution of representative cases. For the water flow, the kinetic energy is transferred from $y^* = 0.5-1.2$ to other locations in different heights y^* because the velocity gradient becomes large due to the velocity difference the main flow and the recirculation zones. Besides, the meandering condition of 40 ppm exhibits the additional appearance of negative T_{11} area from $y^* = 1.5-1.9$. This is derived the high- P_{11} at the second recirculation zones appearing exclusively in the meandering condition (please refer to Li et al., 2015). The integral of the turbulent diffusion term at water and 40 ppm flows is almost zero, and so this term does not contribute to the total turbulent kinetic energy in the flow field. Note that a detailed examination of two terms in the right-hand side of eq. (5) proves the dominant second term in T_{11} for water and 40 ppm cases. It is also interesting that the 150 ppm case persists in the negative T_{11} for $y^* = 1.2-1.7$ and in the almost zero T_{11} for the other heights (see Fig. 9). This suggests that the largely suppressed Reynolds shear stress in the transient regime starting from the BFS results in the almost zero second term in eq. (5), and the sensitive first term for the streamwise variation impacts on T_{11} . This behavior implies the steep increase and gradual decrease in the x -direction of the streamwise velocity fluctuation. From an examination for the result where our previous study (Li et al., 2015) performed PIV measurement at three areas in the x -direction, we captured that the integral of the first term in the x -direction is almost zero, and then the influence of this term is not critical for our hypothesis mentioned above Introduction.

The present study examined the turbulent diffusion term in the BFS flow of the viscoelastic fluid. In comparison with DNS of viscoelastic turbulent channel flow with rectangular orifice (Tsukahara et al., 2011), both flows at low- Re with low-Weissenberg number corresponding to low C can be explained in terms of the interference between the main flow including the meandering motion and the recirculation zones. Based on the result of the turbulent kinetic energy budget in the turbulent flow

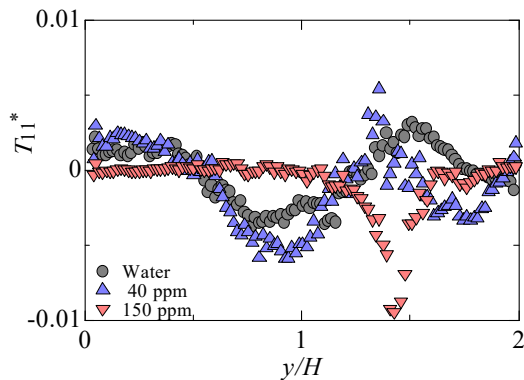


Figure 9. Turbulent transport distribution for $Re = 1.0 \times 10^4$.

with rectangular orifice, it is expected that the other uninvestigated terms in the turbulent kinetic energy budget might be trivial for the magnitude of u' . Therefore, the present analytical result in the meandering condition supports the hypothesis. It will also be necessary to calculate viscoelastic flows by DNS at high- Re and high-Weissenberg number in near future.

CONCLUSION

It is known from our previous study (Ii et al., 2015) that a remarkable meandering motion occurs from combination of the Reynolds number and surfactant concentration in a backward-facing step flow of a viscoelastic surfactant solution in high Reynolds number. The present report provides the examination of the meandering motion in terms of the analysis of the turbulent kinetic energy budget. In particular, we capture the characteristic of the streamwise turbulent velocity fluctuation from analysis of production, turbulent diffusion and dissipation terms. We conclude that the meandering motion become prominent in the flow-regime where the production rate surpasses the dissipation rate resulting in onset of instability at large-scale eddies.

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