MODELING THE INFLUENCE OF EXTERNAL PERTURBATIONS ON THE IONOSPHERE LAYERS

Abdibekov Ualikhan

Khoja Akhmet Yassawi.
International Kazakh-Turkish University
28, Bekzat Sattarkhanov str. Turkestan, 161200
Uali1@mail.ru

Zhakebayev Dauren

Department of Mathematical
and computer modeling,
Kazakh National University al- Farabi
71, al-Farabi ave., Almaty, 050012, Kazakhstan
Dauren.ZHakebaev@kaznu.kz

ABSTRACT

This work is devoted to the mathematical and numerical modelling the influence of variable concentrations of electrons, depending on the time of day on the turbulence of the ionospheric E and D layers. Solving systems of equations for the concentration of electrons determines the conductivity of the environment, which allows solving of magnetohydrodynamics equation. Numerical simulation of the problem is based on the averaged magnetohydrodynamic equations with the continuity equation and the equation for the concentration of the electrons in the Cartesian coordinates. As a result of scientific research the variation of the kinetic and dissipation energies depending on the time are obtained.

INTRODUCTION

A study of the dynamic mode of the upper layer of the atmosphere - rather complicated process, in view the instability of these layers. Most of the experimental papers are devoted to the study of ionospheric dynamics for altitudes from 200 km to 400 km, in the areas E and F, the heights from 60 to 200 km are insufficiently studied. For information about the atmosphere at ionospheric altitudes require very different machinery measurement application installed on a variety of carriers, from meteorockets - in the lower part, to the satellites - in the upper part and on the surface of the Earth (Danilov, 1987). At the modelling of the ionosphere layer, it is necessary to have an idea of the reliability of the parameters used by the neutral atmosphere, that is, to know the quantitative provision of various high-altitude areas of the experimental data and the difficulties inherent in modern measuring techniques and their accuracy.

The greatest amount of information available on the satellites orbit altitudes, mostly exceeding 200 km. A significant amount of data is less, but also enough in the altitude of $h \leq 60$ km, provided by the meteorological rocket network. In the intermediate data is obtained either from sporadic rocket launches, or by optical and radio- physical observations, but the last one is difficult to interpret in comparison with direct methods. Therefore, the gap space contributes to the use of mathematical and numerical study.

This paper considers the modelling of the external disturbances influence on the generation and evolution of large-scale irregularities in the E and D layers of the ionosphere in the range from 60 km to 110 ~ 120 km, where the effect of the charged particles in the ionosphere plasma is taken into account. Depending on the time of day the concentration is changed, which in turn affects the evolution of large scale inhomogeneity.

Within the framework of the present study considers the cubic area, where the inside of the cube there is an incompressible electrically conductive medium with different concentration of electrons: the upper layer corresponds to an environment with strong electron density, relating the daytime. The lower layer corresponds to the weakly concentrated electronic medium, relating night time of day. The concentration influences on the evolution of large-scale eddy тnonhomogeneities over time.

The theoretical basis of the mathematical model of this problem is presented in the following form: the electron concentration is determined by the solution of a system of differential equations describing the behavior of the concentrations of neutral and charged particles. The solution of the system of the equation for the concentration determines the conductivity of the electrically conductive medium, which makes it possible to solve the full equation of magnetic hydrodynamics. Numerical simulation of the problems is carried out on the basis of the magnetohydrodynamic equation, taking into account the continuity equation, and the equation for the electron concentrations in the Cartesian coordinate system. In the numerical realization of concentration calculations, the following boundary conditions are set: on the lateral sides, periodic boundary conditions, which indicates an infinitely large region. The Neumann condition is imposed on the upper and lower bounds. While modelling dynamics of the concentration of electrons as the initial state is chosen when a day follows night.

In the considered region, the upper half is a highly electronically concentrated medium, which corresponds to the day mode, and the lower half is weakly concentrated by electrons, which determines the night mode accordingly.

FORMULATION OF THE PROBLEM

To evaluate the MHD turbulence decay is necessary to numerically simulate the change of all physical parameters over time at various concentrations of electrons in the ionosphere.

Numerical simulation of the problem is based on the filtered solution of unsteady MHD equations with the continuity equation and the equation for the concentration in the Cartesian coordinate system:

$$\begin{cases} \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial(\bar{u}_i \bar{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{1}{\mathrm{Re}}\left(\frac{\partial^2 \bar{u}_i}{\partial x_j^2}\right) - \frac{\partial \tau_{ij}^u}{\partial x_j} + A\frac{\partial}{\partial x_j}\left(\bar{H}_i \bar{H}_j\right), \\ \frac{\partial \bar{u}_i}{\partial x_i} = 0, \\ \frac{\partial \bar{H}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_j \bar{H}_i - \bar{H}_j \bar{u}_i) = \frac{1}{\mathrm{Re}_m}\frac{\partial^2 \bar{H}_i}{\partial x_j^2} - \frac{\partial \tau_{ij}^H}{\partial x_j}, \\ \frac{\partial \bar{H}_i}{\partial x_i} = 0, \\ where\ \tau_{i,j}^H = \left((\overline{u_i u_j}) - (\bar{u}_i \bar{u}_j)\right) - \left((\overline{H_i H_j}) - (\bar{H}_i \bar{H}_j)\right), \\ \qquad \tau_{i,j}^u = \left((\overline{u_i H_j}) - (\bar{u}_i \bar{H}_j)\right) - \left((\overline{H_i u_j}) - (\bar{H}_i \bar{u}_j)\right). \end{cases} \quad (1)$$

where $\bar{p}$ - pressure, t - time, $\bar{u}_i\ (i=1,2,3)$ - velocity components, τ_{ij}^u, τ_{ij}^H - grid tensor, responsible for the small-scale structure, H - the magnetic field, L - the characteristic height between the mesosphere and the thermosphere, ρ - density of the medium, U_0 - the characteristic velocity of the mean motion in the thermosphere, A - parameter interactions and is defined as

$$A = \frac{\sigma H_0^2 L}{\rho U_0} \quad (2)$$

In our case, all parameters are constant, except for the σ - conductivity, which is determined from plasma physics:

$$\sigma = \frac{e^2 n_e}{m_e v_i} = n_e \frac{e^2 n_{e_0}}{m_e v_i} = n_e \sigma_0 \quad (3)$$

where e - charge of the electron, m_e - mass of the electron, n_e - concentration of electron, v_i - the effective frequency of collisions between electrons in the ionosphere.

Substituting (3) into (2)

$$A = \frac{n_e \sigma_0 H_0^2 L}{\rho U_0} = n_e \cdot N_0 \quad (4)$$

where N_0 - at calculation takes the values of 0.1, 1 and 10.

To determine the concentration of electrons involved in the equation

$$\frac{\partial n_e}{\partial t} + \bar{u}_j \frac{\partial n_e}{\partial x_j} = \frac{\partial}{\partial x_j}\left(v_T \frac{\partial n_e}{\partial x_j}\right) + P_e - L_e n_e \quad (5)$$

where $v_T = C_S \Delta^2 (2\bar{S}_{ij}\bar{S}_{ij})^{1/2}$ - eddy viscosity, $\bar{S}_{ij}$ - rate of deformation, P_e - formation of an electron, $L_e n_e$ - recombination of electrons disappearances.

The initial conditions for concentration are defined as follows, for the electron density:

$$n_e = \begin{cases} 1, & x_3 \ge \frac{1}{2},\ 0 \le x_2 \le L_2, \\ 0.01, & x_3 < \frac{1}{2},\ 0 \le x_2 \le L_2. \end{cases} \quad (6)$$

for each component of the velocity and intensity are specified as functions dependent on wavenumber in phase space (Abdibekov, 2013):

$$\overline{u_i}(k_i, 0) = k_i^{\frac{b-2}{2}} e^{-\frac{b}{4}\left(\frac{k_i}{k_{\max}}\right)^2}; \quad (7)$$
$$\overline{H}_i(k_i, 0) = k_i^{\frac{b-2}{2}} e^{-\frac{b}{4}\left(\frac{k_i}{k_{\max}}\right)^2},$$

where $\overline{u_i}$ - a one-dimensional range of velocity, $i=1$ - longitudinal, $i=2$ and $i=3$ - cross spectrum; $\overline{H_i}$ - one-dimensional range of magnetic field strength, b - the power of the spectrum, k_1, k_2, k_3 - the wave numbers. For this task b - variational parameter and $k_{\max}$ - wave number are chosen, which determine the kind of turbulence.

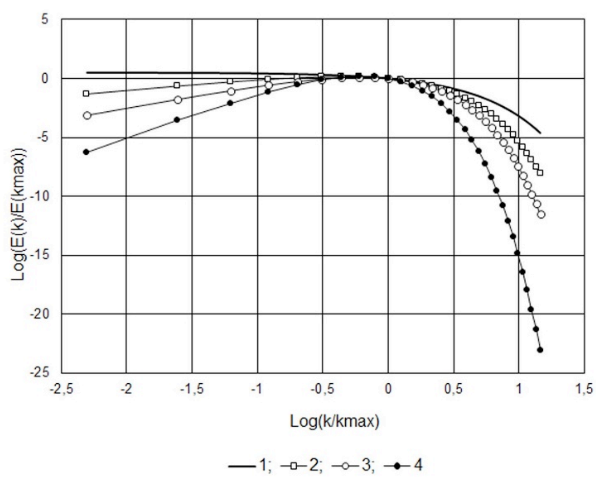


Figure 2. - Equation of the initial level of turbulence as a function of the fixed wave number and the variation parameter b : 1) b = 2; 2) b = 4; 3) b = 6; 4) b = 8.

The boundary conditions of the electron concentration, magnetic field strength and velocity components of this problem in considered area are selected values

$$\left.\frac{\partial n_e}{\partial x_3}\right|_{\substack{x_3=0,\\ x_3=L.}} = 0;\ \left.\frac{\partial H_i}{\partial x_3}\right|_{\substack{x_3=0,\\ x_3=L.}} = 0;\ \left.\frac{\partial u_i}{\partial x_3}\right|_{\substack{x_3=0,\\ x_3=L.}} = 0;\ i = 1, 2, 3. \quad (8)$$

on the upper and lower sides of the cube. For the other walls of the cube: periodic boundary conditions n_e, U_i, H_i.

NUMERICAL SIMULATION

In this paper a three-dimensional non-stationary Navier-Stokes equations for modelling of isotropic turbulence decay with using finite-difference and spectral methods with high order of accuracy and the efficiently algorithm for parallelization at different Reynolds numbers is developed.

For solving the Navier–Stokes equation, we use a splitting scheme by physical parameters that consist of five stages.

At the first stage, the equation of motion is solved, without taking pressure into the account. For approximation of the convective and diffusion terms of the intermediate velocity field the finite-difference methods in combination with cyclic three-diagonal matrix is used (Navon, 1987). The cyclic three diagonal matrixes are solved by using the explicit scheme of Adams-Bashforth for convective terms and implicit scheme of Crank Nicolson for the diffusion members (Kim, and Moin, 1985).

$$\frac{\vec{u}^{*} - \vec{u}^{n}}{\tau} = -\left(\vec{u}^{n}\nabla\right)\vec{u}^{*} + A\left(\vec{H}^{n}\nabla\right)\vec{H}^{n} + \frac{1}{\mathrm{Re}}\Delta\vec{u}^{*} - \nabla\tau^{u}$$

The resulting intermediate velocity field does not satisfy the continuity equation. The exact expression for the new velocity field is obtained by adding to the intermediate field the term corresponding to the pressure gradient.

At the second stage the spectral method in combination with the inverse Fourier transform is performed to obtain the solution of the Poisson equation, which is satisfies the continuity equation with considering the velocity field from the first stage (Zhakebayev, 2014).

$$\Delta p = \frac{\nabla\vec{u}^{*}}{\tau},$$

The boundary conditions are taken as periodic for Poisson equation. The obtained pressure field with using fast Fourier transform is translated from the phase space to the physical space and used at the third stage to obtain the final velocity field.

$$\frac{\vec{u}^{n+1} - \vec{u}^{*}}{\tau} = -\nabla p.$$

At the fourth stage the found velocity field equation is used for the solution of the magnetic field components.

$$\frac{\vec{H}^{n+1} - \vec{H}^{n}}{\tau} = -rot(\vec{u}^{n+1} \times \vec{H}^{n+1}) + \frac{1}{\mathrm{Re}_m}\Delta\vec{H}^{n+1} - \nabla\tau^{H}$$

At the final step the equation of concentration is solved in a whole domain.

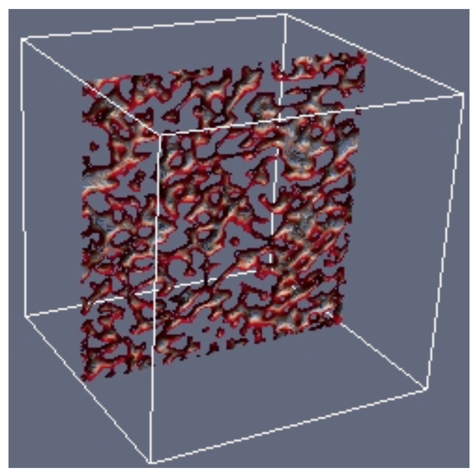
$$\frac{\vec{n}_e^{*} - \vec{n}_e^{n}}{\tau} = -\left(\vec{u}^{n+1}\nabla\right)\vec{n}_e^{*} + \nabla(\nu_T\nabla\vec{n}_e^{*}) + P_e - L_e\vec{n}_e^{n}.$$

Thus, the numerical simulation results in obtaining turbulence characteristics show good agreements with analytical solution of Taylor-Green vortex flow problem.

SIMULATION RESULTS

Navier-Stokes equations for modelling of ionosphere turbulence decay with using finite-difference and spectral methods with high order of accuracy and the efficiently algorithm for parallelization at different Reynolds numbers is developed. For approximation of the convective and diffusion terms of the intermediate velocity field the finite-difference methods in combination with cyclic three-diagonal matrix is used.

The simulation results in Figures 3 – 4 illustrate the contours and dynamics of the kinetic energy with respect to time at different numbers N_0.



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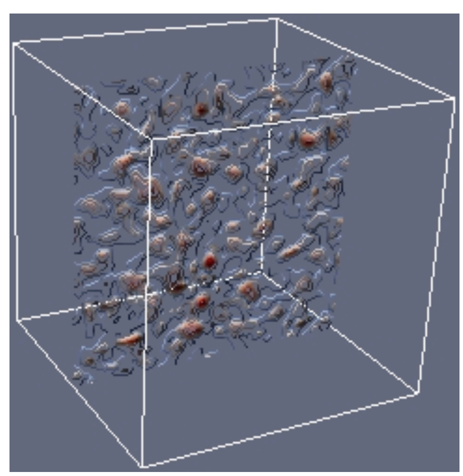

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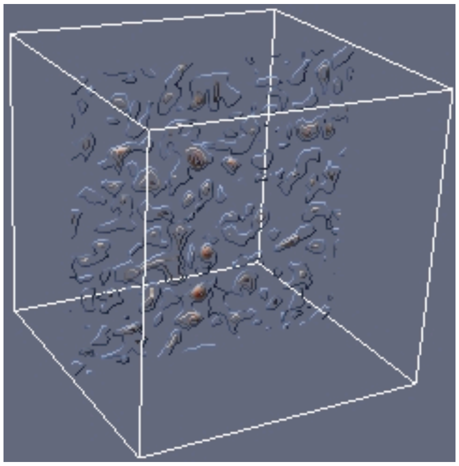

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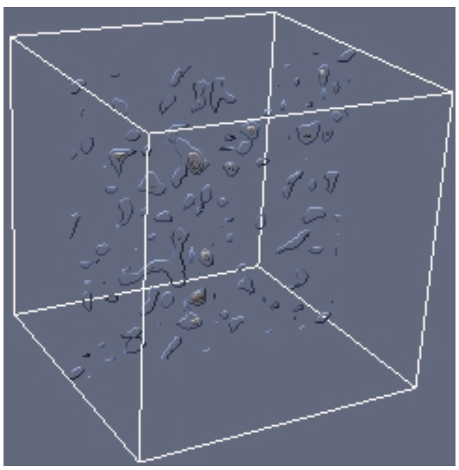
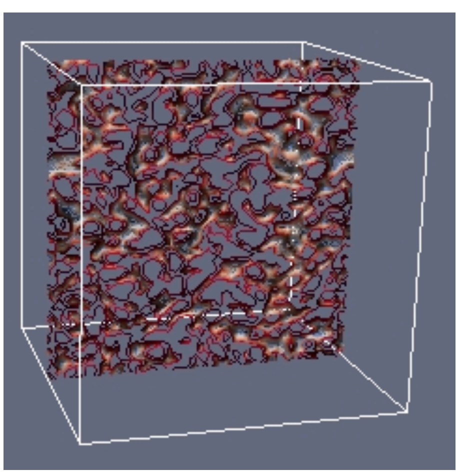
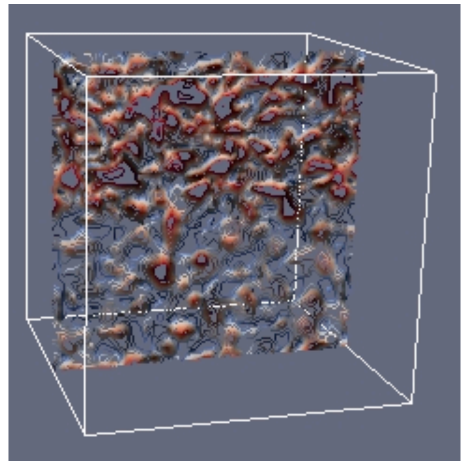


d)

Figure 3. Isoline of turbulent kinetic energy, when $N_0 = 1$ at different moments of time: a) t=0; b) t=0.1; c) t=0.15; d) t=0.2.



a)



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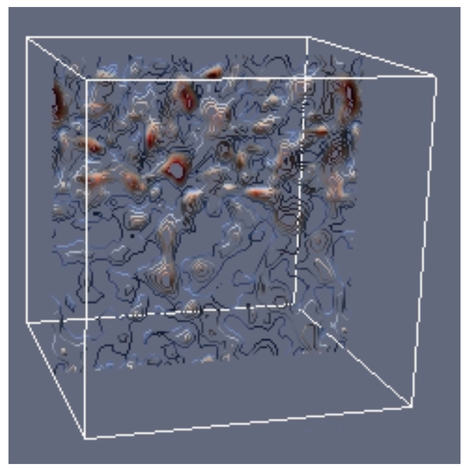

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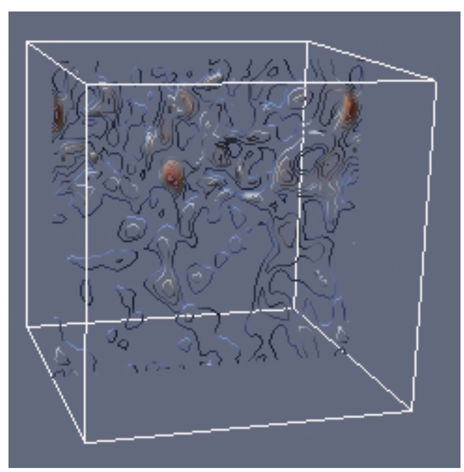

d)

Figure 4. Isoline of turbulent kinetic energy isoline, when $N_0 = 10$ at different moments of time: a) t=0; b) t=0.1; c) t=0.15; d) t=0.2.

As shown in Figures 3 - 4, at the large number N_0, when $N_0 = 10$ the electron density of the medium has influence on the conductivity, and conductivity, in turn, affects to the magnetic viscosity, which determines the dynamics of ionosphere turbulence.

Mathematical model for calculation the evolution of large-scale vortices and the kinetics of ionospheric turbulence, depending on the concentration of electrons in the ionosphere was developed. It was found that, the electron density is directly proportional to the conductivity of the medium, and the medium conductivity coefficient is inversely proportional to the magnetic viscosity of the medium, moreover the coefficient of magnetic viscosity is directly proportional to the rate of dissipation, i.e., with increasing magnetic viscosity, the rate of dissipation increases. With decreasing the magnetic viscosity, large scale vortices gradually increased, and small-scale turbulence structure slowly tends to isotropy.

SAMPLE REFERENCES

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