# LARGE-SCALE INTERMITTENCY IN TURBULENT POISEUILLE FLOWS AT LOW REYNOLDS NUMBER UNDER STABLE DENSITY STRATIFICATION

**Koji Fukudome**
Department of Mechanical Engineering
Ritsumeikan University
1-1-1 Noji-higashi, Kusatsu, Shiga, 525-8577, Japan
fukudome@cfd.ritsumei.ac.jp

**Yoshifumi Ogami**
Department of Mechanical Engineering
Ritsumeikan University
1-1-1 Noji-higashi, Kusatsu, Shiga, 525-8577, Japan
y_ogami@cfd.ritsumei.ac.jp

## ABSTRACT

Direct numerical simulations using a spectral method were performed in order to study relaminarizing turbulent structures in Poiseuille flow under stable density stratification. In the case of a small computational domain, the turbulence attenuated with increasing effect of stable stratification. Relaminarization occurred on one wall side at *Ri* > 0.30, while the turbulence remained on the other wall. *Ri* represents the bulk Richardson number, defined by the bulk mean velocity. In contrast, in the case of a large computational domain, relaminarization on one wall side was not observed, while a large-scale intermittent flow structure appeared at *Ri* > 0.17. Turbulent stripes were also observed at *Ri* = 0.56. These results showed that the flow dynamics in small and large computational domains differed from each other. The flow structure of the turbulent stripe was similar to the ones in the turbulent Poiseuille flow in transitional regions in the absence of external forces. Besides, intermittent flow structure appear even in the stable region estimated by liner instability theory. Moreover, one-wall-sided turbulence appear in SB under strong stratification through the state with large temporal oscillation of turbulent intensities.

## INTRODUCTION

Turbulence under stable density stratification is observed in the atmosphere at night, in the oceans, and in industrial systems. For this reason, the study of stably stratified turbulence is important in the fields of physics and engineering. The relationship between stratification and the shear effects are focused on this study. Garg et al. (2000) and Iida et al. (2002) carried out direct numerical simulation (DNS) with a small computational domain and confirmed that the turbulence in horizontal Poiseuille flows is attenuated by stable density stratification. Iida et al. (2002) also confirmed that an internal gravity wave becomes dominant in the channel center and strong stratification causes relaminarization on one wall side, while turbulence remains on the other wall. Hereafter, we call this state, "one-wall-sided turbulence."

In a different case, García-Villalba et al. (2011) carried out DNSs in a large computational domain. They found a large-scale intermittent flow structure consisting of turbulent and spotty quasi-laminar regions in stable stratified turbulent Poiseuille flow. However, the flow structures under stable stratification have not been studied systematically with a change in stratification. Even so, problems still exist, if the flow is similar to the one observed by Iida et al. (2002).

Brethouwer et al. (2012) carried out direct numerical simulations of open channel flow at moderate Reynolds numbers with stratification using a large computational domain. Open-channel flow is the flow bounded by parallel walls consisting of no- and free-slip walls. For this reason, it is considered to be similar to that of the half in the Poiseuille flow. It was also revealed that the turbulent stripe appeared in the open-channel flow under stable stratification. From the flow geometry, there are interactions between the turbulent structures near the upper and lower wall sides in Poiseuille flow. Hence, we cannot expect the flow structure with the interactions in Poiseuille flow using the result from the open-channel flow.

Fukudome et al. (2015) performed DNSs of stably stratified Poiseuille flow and showed that the turbulent stripe, consisting of turbulent and quasi-laminar bands, appeared in a large computational domain. This turbulent stripe was also found in turbulent channel flows consisting of open channel flows (He and Basu, 2015), Poiseuille flow (Fukudome and Iida, 2012), and Couette flow (ex. Manneville, 2015) with moderate stratification.

In this study, we performed DNSs in order to examine the turbulent structure in Poiseuille flows with stable density stratification. The flow structure was further clarified in detail in detail to compare the flow structure in two computational domains.

## NUMERICAL PROCEDURE

The objective flow is a Poiseuille flow as shown in Figure 1, which is bounded by parallel walls consisting of an upper hot wall and a lower cold wall, driven by a constant pressure gradient. The numerical method used in this study was the same as that used by Iida et al. (2002). The governing equations were the incompressible Navier–Stokes equation with an added buoyancy term using a Boussinesq approximation, and continuity and energy equations. DNSs of the equations were carried out through use of the spectral method, employing the Fourier series in the streamwise $x$ (= $x_1$) and spanwise $z$ (= $x_3$) directions and a Chebyshev polynomial expansion in the wall-normal direction $y$ (= $x_2$). The boundary conditions were periodic for the $x$ and $z$ directions and non-slip at the walls. For dealiasing, the 3/2 rule was adapted for both spatial discretization. Time advancement was carried out using the Crank–Nicolson method for the viscous terms and the second–order Adams–Bashforth method for the nonlinear and buoyancy terms. All calculations were carried out at a Reynolds number of $Re_e = u_e\delta/\nu = 150$ and Prandtl number of $Pr = \nu/a = 0.71$. Here, $u_e$, $\delta$, $\nu$ and $\alpha$ represent

Table 1. Configuration of Computational domain and flow geometry for SB (a) and LB (b).

| $Re_e$ | $Gr$ | SB |  |  | LB |  |  |
|---|---|---|---|---|---|---|---|
|  |  | $Re_m$ | $C_f$ | $Ri$ | $Re_m$ | $C_f$ | $Ri$ |
| 150 | 0.00 | 4579 | $8.58\times10^{-3}$ | 0.00 | 4522 | $8.70\times10^{-3}$ | 0.00 |
|  | $13\times10^5$ | 4771 | $7.91\times10^{-3}$ | 0.06 | 4715 | $8.06\times10^{-3}$ | 0.06 |
|  | $44\times10^5$ | 5129 | $6.84\times10^{-3}$ | 0.17 | 5100 | $6.93\times10^{-3}$ | 0.17 |
|  | $72\times10^5$ | 5466 | $6.02\times10^{-3}$ | 0.24 | 5408 | $6.15\times10^{-3}$ | 0.25 |
|  | $100\times10^5$ | 5766 | $5.41\times10^{-3}$ | 0.30 | 5605 | $5.76\times10^{-3}$ | 0.32 |
|  | $150\times10^5$ | 6532 | $4.22\times10^{-3}$ | 0.35 | 5849 | $5.27\times10^{-3}$ | 0.44 |
|  | $175\times10^5$ | 6599 | $4.13\times10^{-3}$ | 0.40 | 5927 | $5.13\times10^{-3}$ | 0.50 |
|  | $200\times10^5$ | 6751 | $3.95\times10^{-3}$ | 0.44 | 5976 | $5.06\times10^{-3}$ | 0.56 |

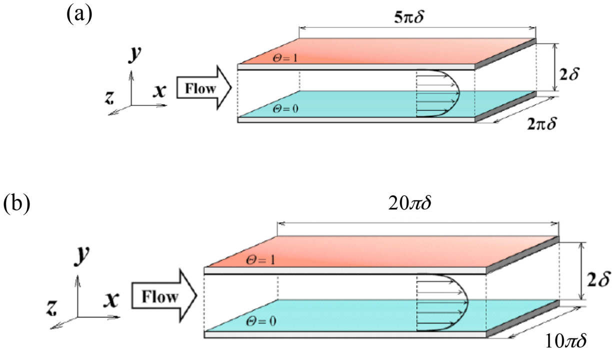

Figure 1. Configuration of Computational domain and flow geometry for SB (a) and LB (b).

the characteristic velocity at the mean pressure gradient, the channel half width, the kinetic viscosity, and the thermal diffusivity, respectively. The Reynolds number $Re_e$ is same as friction Reynolds number $Re_\tau$. The computational domains in the *x*-, *y*-, and *z*-directions are $5\pi\delta\times2\delta\times2\pi\delta$ (SB) as in [2], of a $128\times129\times128$ grid system and $20\pi\delta\times2\delta\times10\pi\delta$ (LB) of a $512\times129\times512$ grid system. As an initial condition, fully developed flows at $Re_e=150$ were calculated in advance. The Grashof number, $Gr=g\beta\Delta\Theta(2\delta)^3/\nu^2$, was then systematically increased as shown in Table 1. Here, $g$, $\beta$, and $\Delta\Theta$ are the gravitational acceleration, the volumetric expansion coefficient, and the temperature difference between the hot and cold walls, respectively. Hereinafter, all parameters are normalized by $u_e$ and $\nu$, and normalization is indicated by the superscript $+$.

## RESULT AND DISCUSSION

Table 1 shows the flow statistics of our results, consisting of flow rate (bulk Reynolds number) $Re_m=2U_m\delta/\nu$, friction coefficient, $C_f=2\tau_w/\rho U_m^2$, and bulk Richardson number, $Ri=Gr/Re_m^2$, as a function of $Gr$ in both cases of SB and LB. Here, $U_m$, $\tau_w$, and $\rho$ are the mean velocity in the entire channel, the wall shear stress, and the density, respectively. Figure 2 represents the relationship between $Ri$ and $Re_m$ in both SB and LB cases. Current results show good agreement with the results of Iida et al. (2002) using a fine grid. The flows were relaminarized by the stratification, so that the increase of $Re_m$ and decrease of $C_f$ were raised. It is noted that the deviation of the flow rate $Re_m$ between SB and LB becomes significant in the case of $Ri$ > 0.3, where the one-wall-side turbulence which was observed in SB, as reported by Iida et al. (2002), was shown in Figure 3(e) and (f). This deviation of bulk Reynolds number shows the gap of statistics between SB and LB. Figure 3 represents the instantaneous flow structure in the case of SB for $Ri$ = 0 (Fig. 3(a)), 0.17 (Fig. 3(b)), 0.30 (Fig. 3(c, d)), and 0.35 (Fig. 3(e, f)). The flow becomes spatially intermittent at $Ri$ = 0.17, compared to the non-stratified case. At $Ri$ = 0.30, the turbulent state was observed in both wall sides as shown in Fig. 1(c) and (d). However, one-wall-sided turbulence was obtained at $Ri$ > 0.35, as shown in Figure 1(e) and (f).

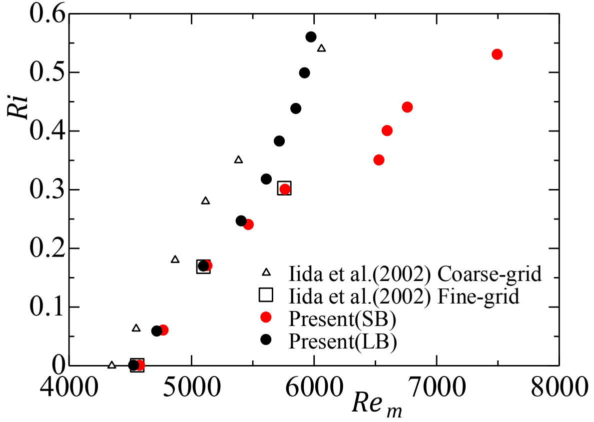

Figure 2. Distribution of the bulk Richardson number $Ri$ as a function of bulk Reynolds number $Re_m$. Red symbols represents the result in SB as same as Iida et al. (2002).

Figure 4 illustrates the instantaneous flow structure in the case of LB for $Ri$ = 0.17 and $Ri$ = 0.56. At $Ri$ = 0.17, the intermittent flow structure is clearly observed as reported by García-Villalba et al. (2011). The weak turbulent regions tended to incline from the streamwise direction. This intermittent flow structure was not observed at this Reynolds number flow without the stratification. At $Ri$ = 0.56, one-wall-sided turbulence was obtained in the case of SB. High- and low-speed fluids exist upstream and downstream of the turbulent region where many vortical structures were clustered. The turbulent regions formed a stripe pattern, which is called as a turbulent stripe. In this case, we got a V-shaped turbulent stripe. In addition, we confirmed that

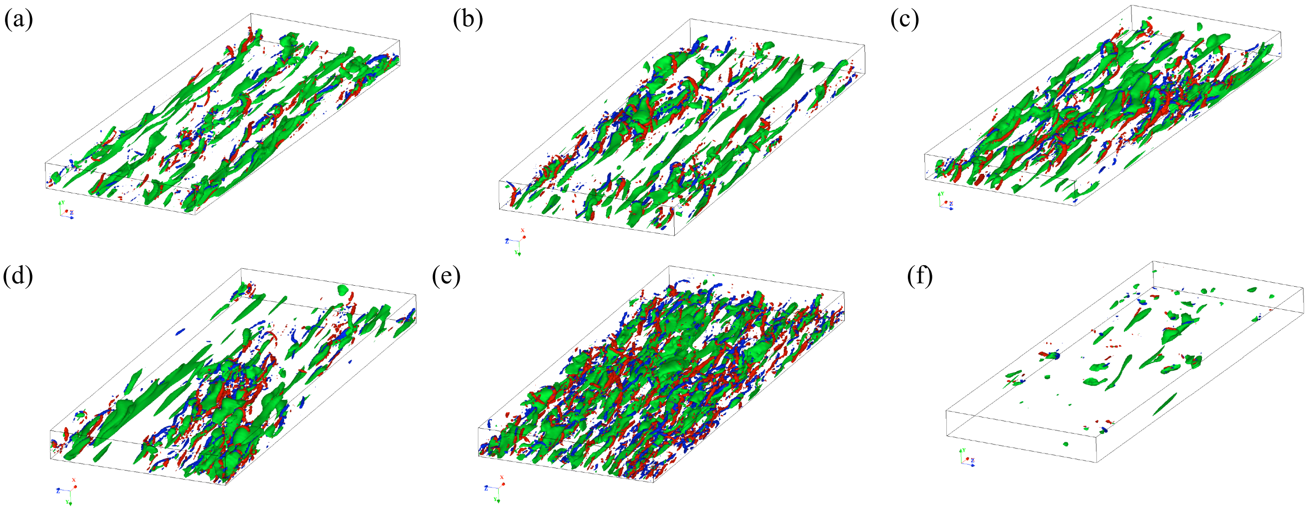

Figure 3. Instantaneous distributions of low-speed streaks and the vortical structure identified by the second invariant of the velocity gradient tensor $II^+ = -0.5u'^+_{i,j}u'^+_{j,i} = 0.25$ over channel half width. The red and blue iso-surfaces correspond to $II^+ = 0.05$. The green contours correspond to $u'^+ = -3.0$. (a) $Ri = 0.00$. (b) $Ri = 0.17$. (c) $Ri = 0.30$ at upper wall side. (d) $Ri = 0.30$ at lower wall side. (e) $Ri = 0.35$ at upper wall side. (f) $Ri = 0.35$ at lower wall side.

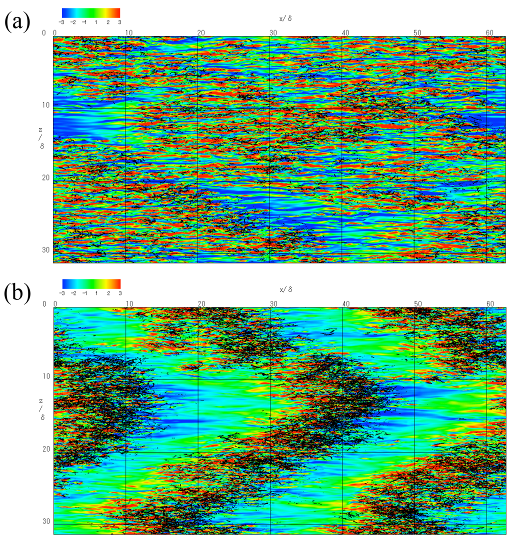

Figure 4. Instantaneous distributions of high- and low-speed fluids in the near-wall region at $y^+$=10 and the vortical structure identified by the second invariant of the velocity gradient tensor $II^+$ over the entire channel in LB. The black iso-surfaces correspond to $II^+ = 0.05$. The red to blue contours correspond to $u'^+ = -3$ to 3. (a) $Ri = 0.17$. (b) $Ri = 0.56$.

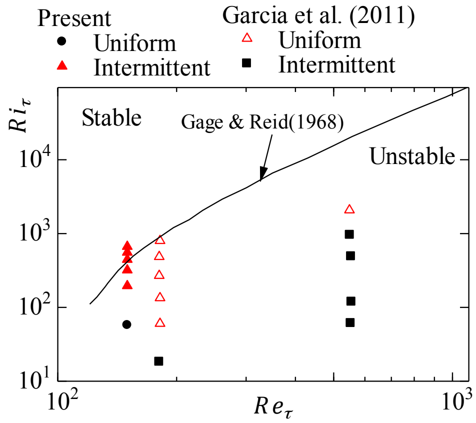

Figure 5. Distribution of the friction Richardson number $Ri_\tau$ as a function of friction Reynolds number $Re_\tau$. Red and black symbols represent intermittent and uniform flow structures, respectively. Line is a neutral curve obtained by Gage and Reid (1968).

the flow structures are nearly the same between upper and lower wall sides. In this instance, the interaction between the turbulent structure in the upper and lower wall sides becomes non-negligible. The flow structure of the turbulent stripe was similar to the ones in the turbulent Poiseuille flow in transitional regions without any external forces (Fukudome et al. 2012, Manneville, 2015). As a result, the turbulent stripe appears at a higher Reynolds number to impose the stable density stratification in Poiseuille flows. Moreover, one-wall-sided turbulence was not observed when the turbulent stripe appeared in LB. It was concluded that, the flow dynamics were not similar between SB and LB for $Ri > 0.3$.

Figure 5 represents neutral curve obtained by Gage and Reid (1968) with linear stability analysis. Here, $Ri_\tau$ is friction Richardson number defined by $Gr / Re_\tau^2$. Garcia et al. (2011) showed that the intermittent flow structure appeared at unstable region at the moderate Reynolds number flow. However, it is noted that the intermittent flow structure appear in not only unstable region but also stable region in the present case.

Next, we focus on the appearance of one-wall-sided turbulence. Figure 6 represents time evolution of friction coefficient $C_f$ for $Gr = 125\times10^5$ . A flow field of $Gr = 100\times10^5$ is used as an initial flow condition. Large oscillation of $C_f$ arise for long period. Periodic oscillation observed for $t^+ < 20000$ keeping the turbulent state in both sides. For $t^+ > 20000$ , intermittent oscillation arise. However, no oscillation appear for $t^+ > 60000$ , and flow structure become one-wall-sided turbulence.

Figure 7 represents flow structure with intermittent oscillation shown in Figure 6. Figure 7(a) represents time evolution of friction coefficient for entire, upper and lower computational domains, respectively. Large oscillation of $C_f$ occurs with long time in the lower wall side, while it becomes almost constant with high value at upper wall side. Figure 7(b) shows the flow structure in lower wall side of one period of the oscillation. Some lumps of low speed fluid arise in Figure 7(b1). The low speed fluids expand to the streamwise direction (Figure 7(b2)) and form low-speed streaks with generation of streamwise vortices. This turbulence grow in the completely computational domain as shown in Figure 7(b5-b6). Then, the flow structure return to laminar state as shown in Figure 7(b8). However, a few lumps of low speed fluid arise and become the seed of next bursting phenomena.

## CONCLUSIONS

Direct numerical simulations were performed in order to study the turbulent structure in Poiseuille flow under stable stratification. In the case of SB, the turbulence attenuated with increasing effect of stable stratification, and relaminarization occurred on one wall side at $Ri > 0.30$. On the other hand, in the case of LB, relaminarization on one wall side was not observed consistently and large scale intermittent flow structure appeared at $Ri > 0.17$. In addtion, V-shaped turbulent stripes were observed at $Ri = 0.56$. The flow dynamics in SB and LB were different from each other. The flow structure of the turbulent stripe was similar to the ones in turbulent Poiseuille flow in transitional regions without any external forces. Besides, intermittent flow structure appear even in the stable region estimated by liner instability theory. Moreover, one-wall-sided turbulence appear in SB under strong stratification through the state with large temporal oscillation of turbulent intensities.

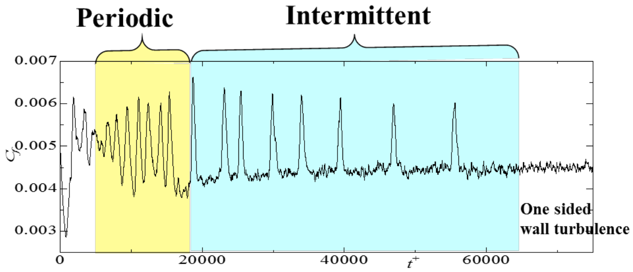

Figure 6. Time evolution of $C_f$ at $Gr = 125\times10^5$ .

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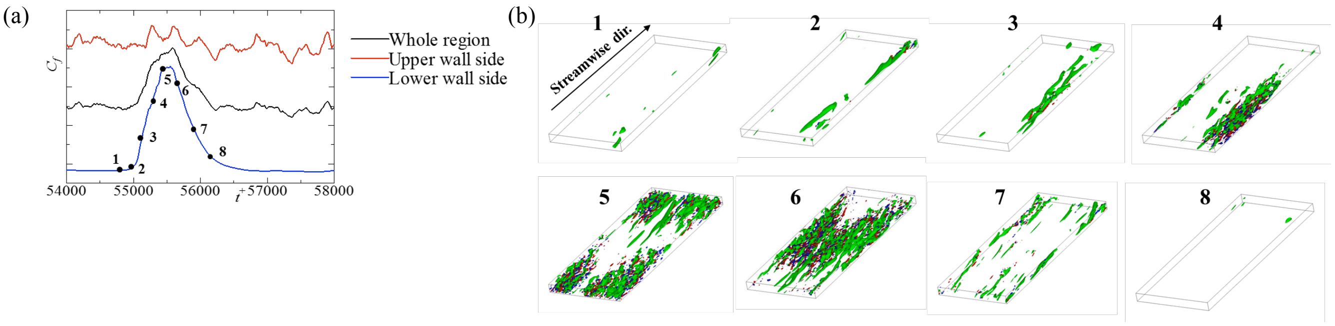

Figure 7. Temporal variation of the flow structure. (a) Temporal evolution of friction coefficients in entire, upper and lower computational domains. (b) Time evolution of the flow structure in lower wall sides. Left upper numbers represents the state indicated in Figure 7(a). Green iso-surfaces correspond to $u'^+ = -3.0$ as low speed fluid. The red and blue iso-surfaces correspond to $II^+ = 0.025$ .

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