RE-LAMINARIZATION AND RE-TRANSITION OF A TURBULENT BOUNDARY LAYER FROM A VORTICITY POINT OF VIEW

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ABSTRACT

The presentation at TSFP 9 by Brown, Lee and Moser (2015) on the near wall transfer of viscous stress to Reynolds stress, based on a vorticity transport perspective, was motivated by the connection between the transport of vorticity and the Reynolds stress, i.e.

$$\frac{d}{dy}\left(-\overline{u'v'}\right) = \overline{v'\omega'_z} - \overline{w'\omega'_y} \ ,$$

(for plane, parallel turbulent flow) to which Taylor (1915) first drew attention. That work has now been expanded to include Couette flow and is being submitted for publication (BLM (2017)). We consider here the role played by these vorticity transport terms in an understanding of the mechanics of a highly accelerating, re-laminarizing, turbulent boundary layer. An initial, zero pressure gradient, turbulent boundary layer is accelerated by a very favorable pressure gradient, which provides an increase by a factor of three in free stream velocity. It is followed by a relaxation towards zero pressure gradient at this much higher free stream velocity. A recent experimental and numerical investigation (Patwardhan (2015), Patwardhan and Ramesh (2017)) shows a dramatic fall in local skin friction coefficient and, particularly, a local fall in the actual wall friction due to this acceleration. Since an acceleration and fall in static pressure is a source of spanwise vorticity at the wall this decrease in wall vorticity demands a mechanistic explanation. A 're-laminarization' of the near wall flow, as the flow accelerates, is found, which is then followed by a re-transition to turbulence in the subsequent, approximately zero pressure gradient, flow.

This relaminarizing flow attracted early experimental and theoretical attention by Sreenivasan (1972), and Narasimha and Sreenivasan (1973, 1979)) who provided a mechanistic description, largely based on momentum considerations. They developed a two layer model with the

idea of a 'laminar sub boundary layer' near the wall and a 'rapid distortion' model for the outer flow, but it was not possible at that time to measure the components of vorticity near the wall. DNS computions now offer detailed and complementary insights. In particular these computations provide results for the vorticity field and for the vorticity transport terms in the above equation.

As pointed out in (BLM) (2015), in a channel flow the two vorticity transport terms are equal at the location y_m where the Reynolds stress is a maximum. Nearer the wall, $\overline{w'\omega'_y}$ dominates and in the outer flow, ($y \gg y_m$), $\overline{v'\omega'_z}$ has the larger magnitude. Importantly, $\overline{v'\omega'_z}$, acts to transport the mean spanwise vorticity in the same direction as laminar diffusion, whereas near the wall $\overline{w'\omega'_y}$ acts to transport the spanwise vorticity against the mean vorticity gradient! (Both effects cancel at $y = y_m$ and the vorticity is transported there only by the viscous diffusion, as discussed by BLM (2015, 2017). These two vorticity transport terms have now been calculated from the Direct Numerical Simulation database of the re-laminarizing turbulent boundary layer flow (initial $R_\tau = 461$) of Patwardhan (2015).

RESULTS AND DISCUSSION

Fig. 1(a) shows the free stream velocity, (b) the corresponding development of the boundary layer thickness, (c) the friction coefficient and (d) the wall shear stress as functions of streamwise distance. Fig. 2 provides a contour plot of the respective magnitudes of these two vorticity transport terms, (spanwise and time-averaged). These contour plots provide an overall view of the effect of the pressure gradient. In particular, they show the

collapse of $\overline{w'\omega'_y}$ near the wall during re-laminarization and its re-establishment during the following re-transition.
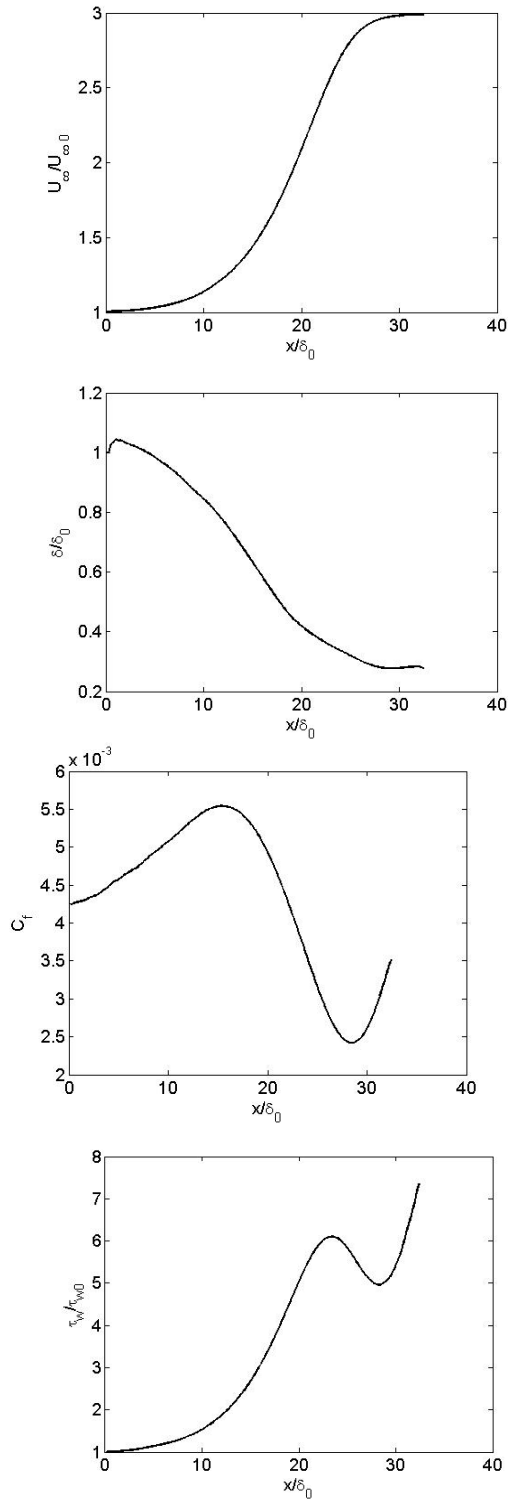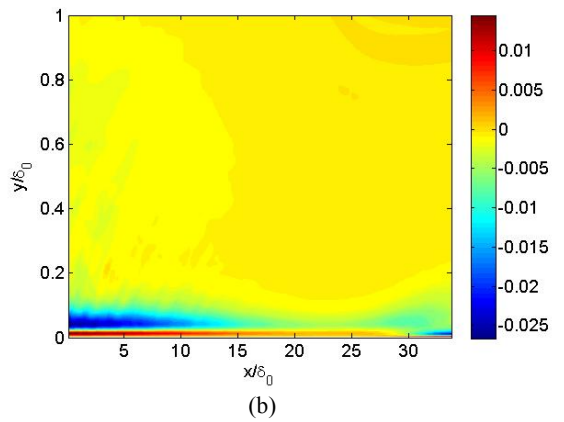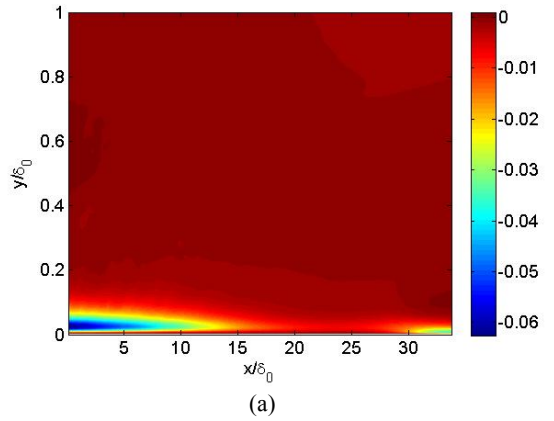

(a)

(b)

Figure 2 (a) contour plot of the magnitude of $\overline{w'\omega'_y}$ (b) contour plot of the magnitude of $\overline{v'\omega'_z}$ (both normalized by local friction velocity and inner length scale)

More detailed profiles of $\overline{w'\omega'_y}$, are shown in Fig 2(c); they confirm the reduction near the wall in this term by an order of magnitude. The explanation is discussed below.
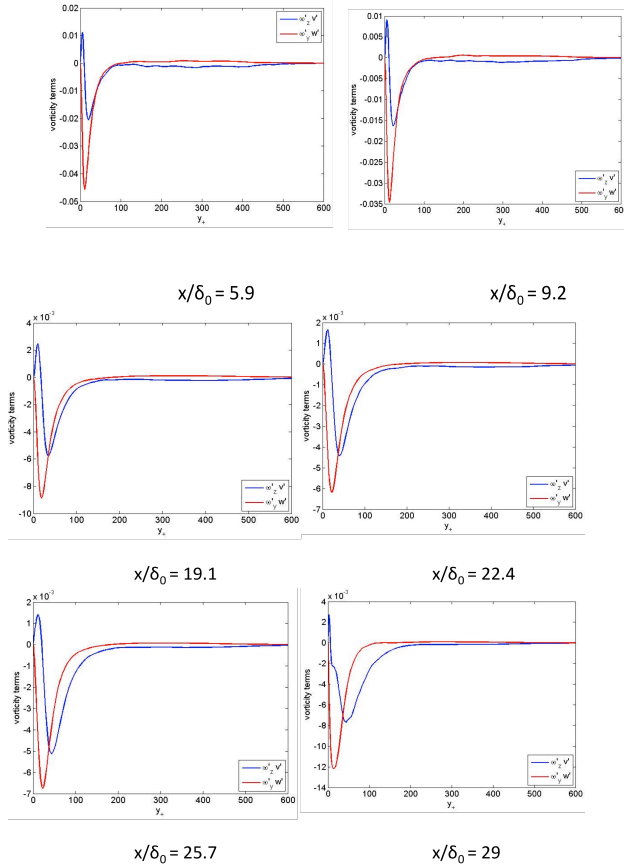

x/δ₀ = 0

x/δ₀ = 2.63

Figure 1 (a) free stream velocity, (b) the development of the boundary layer thickness, (c) the friction coefficient, (d) the wall shear stress. (top to bottom)

$x/\delta_0 = 5.9$ $x/\delta_0 = 9.2$

$x/\delta_0 = 19.1$ $x/\delta_0 = 22.4$

$x/\delta_0 = 25.7$ $x/\delta_0 = 29$
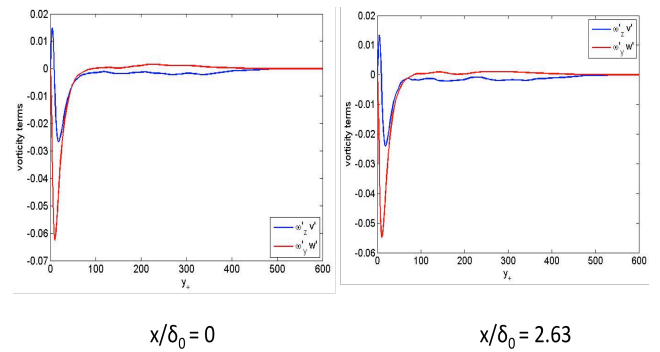
Fig 2(c) Note the decrease near the wall by an order of magnitude in $\overline{w'\omega'_y}$ (i.e approx. from -.06 initially to -.006 at $x/\delta_0 = 19.1$) with downstream distance and then the beginning of the increase with the subsequent reduction in pressure gradient and re-transition. (y_+ is scaled using the local friction velocity).
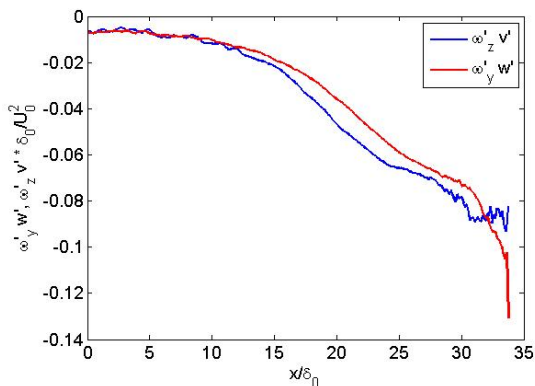

Fig (3) The two vorticity transport terms on the streamline passing through $y/\delta = 0.2$, upstream in the zero pressure gradient boundary layer, vs. distance downstream.

With the collapse of $\overline{w'\omega'_y}$, the mean vorticity diffuses from the wall since it is no longer being kept there by the large counter-gradient diffusive action of $\overline{w'\omega'_y}$. Thus re-laminarization occurs at the wall. [This is not inconsistent with the idea of a laminar sub boundary layer (LSBL), in Narasimha and Sreenivasan (1973 and 1979) where the fall in wall shear stress is accompanied by the growth of the LSBL.]

The behavior far from the wall is the subject of on-going research but one expects that in a two- dimensional acceleration the value of ω'_z following a particle would be weakly affected by the acceleration. In the outer flow it is found that the correlation $\overline{v'\omega'_z}$ actually becomes more negative with downstream distance on a streamline (Fig 3).

This increase in magnitude in the outer flow is not yet understood but the difference between the two vorticity transport terms on a streamline (Fig 3) is shown to increase. This is consistent with the Reynolds stress in the outer flow appearing to be 'frozen' on streamlines, as found in the experiments of Bourassa and Thomas (2009) and Narasimha and Sreenivasan (1973), and also in these numerical simulations (Fig (4)); consequently there is an increase in the gradient in Reynolds stress as the layer thins and therefore in the difference between these fluxes shown in Fig 3 (consistent with the above equation).
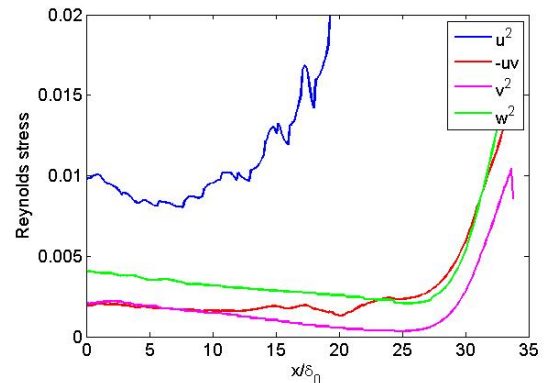

Fig (4) The Reynolds shear stress and the Reynolds normal stresses on the streamline passing through $y/\delta = 0.2$, upstream in the zero pressure gradient boundary layer, vs. distance downstream.

The explanation for the collapse near the wall of $\overline{w'\omega'_y}$ shown from the contours in Fig 2(a) and (b) and in the profiles in Fig 2(c) is found in the development and behavior of the components of the enstrophy near the wall. The equation for the square of each component of the vorticity, derived from the Navier Stokes equations for incompressible fluid, and written in conservation form,

contains a 'production' minus a 'dissipation' term. That is, the equation for $\overline{\omega_x^2}$, as in (BML (2017)), for example, is

$$\frac{d}{dt}\iiint \frac{1}{2}\omega_x^2 dV + \iint \left(u_k \frac{1}{2}\omega_x^2 - \nu \frac{\partial}{\partial x_k}\frac{1}{2}\omega_x^2 \right)n_k dS$$

$$= \iiint (S_x - D_x)dV$$

where $\quad S_x = \omega_x \left(\omega_x \frac{\partial u}{\partial x} + \omega_y \frac{\partial u}{\partial y} + \omega_z \frac{\partial u}{\partial z} \right) \quad$ and

$$D_x = \nu \left[\frac{\partial \omega_x}{\partial x_k} \right]^2 = \nu \left[\left(\frac{\partial \omega_x}{\partial x} \right)^2 + \left(\frac{\partial \omega_x}{\partial y} \right)^2 + \left(\frac{\partial \omega_x}{\partial z} \right)^2 \right]$$

Fig 5 shows a contour plot of the rms value of the streamwise vorticity. The direct connection between streamwise vorticity and the vorticity transport $\overline{w'\omega_y'}$ near the wall, is shown in BML(2015,2017) and thus the collapse in streamwise vorticity shown in Fig 5 is consistent with the collapse in the corresponding vorticity transport near the wall.
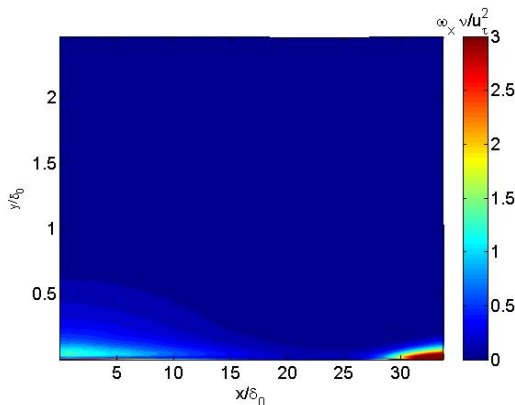

Fig (5) Contour plot of the rms. value of streamwise vorticity.

Thus there is a great reduction in the corresponding transfer of viscous stress to Reynolds stress due to the collapse of the corresponding 'counter-gradient' diffusion of vorticity, and consequently the very high wall vorticity (shear stress) is diffused away and the wall shear stress falls as in (Fig 1(d)), notwithstanding the favorable (but falling) pressure gradient.

The broad explanation for the collapse of the streamwise vorticity is the fact that production minus dissipation (Fig 6) progressively becomes less positive near the wall; this is because the streamwise vorticity is increased by the stretching term (arising from the streamwise acceleration) but the corresponding reduction in scales normal to this vorticity increases the 'dissipation' term and the resulting effect is for the magnitude of streamwise vorticity to decrease, as in Fig 5. Thus the basic mechanism (the root cause of the large counter gradient vorticity transport near the wall), which transfers viscous stress to Reynolds stress,

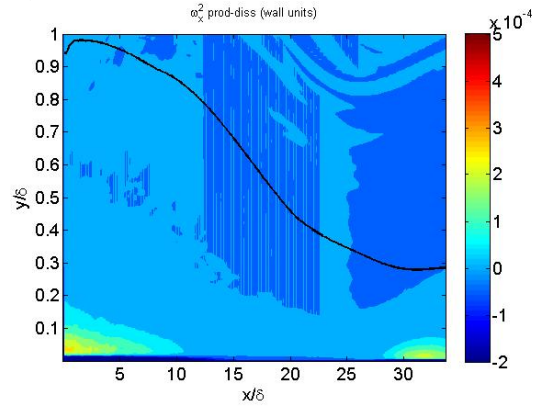collapses and the wall shear stress (i.e the vorticity at the wall) decreases.

Fig (6) Contour plot of 'Production' minus 'Dissipation' for $\frac{1}{2}\omega_x^2$

The results in BLM (2017) show that the counter gradient action of $\overline{w'\omega_y'}$ arises from both the stretching of vortex lines near the wall and the flux of spanwise vorticity fluctuations towards the wall (for $y^+ < 10$, approximately, for channel flow) due to the action of streamwise vorticity (i.e. through its direct connection with $\overline{w'\omega_y'}$).

Results from a more detailed examination of the individual terms in the 'production' of the streamwise component of enstrophy are being obtained and their more detailed connection with the overall mechanics in the collapse and re-establishment of $\overline{w'\omega_y'}$, through the relaminarization and subsequent re-transition regions are being considered. The variation in the Karman constant, that has been found in sink flow with different pressure gradients, is also being considered in terms of the effect of the acceleration on the vorticity transport terms, $\overline{w'\omega_y'}$ and $\overline{v'\omega_z'}$, respectively.

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