FREE-STREAM TURBULENCE AND ITS INFLUENCE ON BOUNDARY-LAYER TRANSITION

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ABSTRACT

Free-stream turbulence (FST) gives, undoubtedly, rise to the most complicated boundary-layer transition-to-turbulence scenario. The reason for the complexity is that the boundary layer thickness grows with the downstream distance at the same time as the turbulence intensity (Tu) of the FST decays and the FST characteristic length scales grow. The FST is present everywhere in the free stream, but changes characteristics with the downstream distance. This implies that the actual forcing by the FST on the boundary layer changes gradually, which makes it an intricate receptivity problem. Today, we cannot honestly say that we are capable to *accurately* predict the transition location subject to FST in the simplest boundary layer flow, namely the one developing over a flat plat under zero-pressure gradient condition.

Based on a set of original experimental data, consisting of 42 unique FST conditions, we here report on a semi-empirical transition prediction model, which takes into account both the integral length scale and the turbulence velocity fluctuation at the leading edge. We show that the Tu, used in *all* existing models, is not the leading variable. Instead, our data show that the necessary ingredients in a successful transition prediction model includes, firstly, a FST Reynolds number (Re_{fst}) as leading variable, secondly, an FST parameter being the integral length scale Reynolds number (Re_Λ) which further accounts for the effect of different length scales and, thirdly, a scale-matching model between the FST and the boundary layer. However, the importance of Tu can still be realized, since it constitutes the quotient of the two Reynolds numbers, namely $Tu \equiv Re_{\text{fst}}/Re_\Lambda$, even though Tu does not explicitly appear in the model.

INTRODUCTION

When a boundary layer flow undergoes laminar-to-turbulence transition under the presence of free-stream turbulence (FST) the transition scenario is different from the condition with a low background disturbance level. Under the latter condition, the initial phase of velocity disturbance growth is characterized by small amplitude exponentially growing Tollmien–Schlichting (TS) waves, while the former condition is characterized by algebraically growing unsteady streamwise velocity streaks. The designation *bypass transition* is still often used when referring to FST induced boundary layer transition, although somewhat incorrectly. The term *bypass* was coined by Morkovin (1969), and was introduced to denote any transition process that bypassed common knowledge, which at the time was limited to the TS wave transition scenario (Tollmien, 1929; Schlichting, 1933; Schubauer & Skramstad, 1947). Originally it was referred to surface roughness induced transition but became a common notation for FST induced transition.

The streamwise turbulence intensity, defined as the ratio between the root-mean-square value of the velocity (u_{rms}) and the free-stream velocity (U_∞), i.e. $Tu = u_{\text{rms}}/U_\infty$, is a simple measure to quantify the level of FST. As Tu is increased there is a gradual shift from the TS to the FST transitoin scenario. Arnal & Juillen (1978) noted that for $Tu > 1\%$, the FST transition scenario with unsteady streamwise streaks dominate the transition process over the TS wave scenario. The early experimental studies on FST induced transitoin originates from around 1940 (see e.g. Hall & Hislop, 1938; Taylor, 1939; Hislop, 1940), but were all carried out using pitot tube measurements and, hence, only mean velocity profiles and transition locations could be reported. Since then, a significant amount of work, both experimentally and numerically, has been performed dealing with FST induced transition in boundary layer flows. For reviews and progress on the subject the interested reader is referred to e.g. Matsubara & Alfredsson (2001); Jacobs & Durbin (2001); Saric *et al.* (2002); Brandt *et al.* (2004); Fransson *et al.* (2005).

Effect of Integral Length Scale on Transition

Today, the FST transition scenario is fairly well understood even though knowledge about the details on *why* and *how* the FST characteristics can move the transition location back and forth is still lacking. There exists a numerous amount of empirical and semi-empirical relationships between the location of transition onset and the Tu in a flat plate boundary layer. However, already in the doctoral thesis by Hislop (1940) an effect of different FST integral length scales on the laminar-to-turbulence transition location can be pointed out. The transitional Reynolds number for three different turbulence generating grids are tabulated in the thesis of Hislop, and this data is here plotted as a loglog plot in figure 1. Generally, the integral length scale produced by a turbulence generating grid scales with the mesh width of the grid, i.e. the larger the mesh width the larger is the turbulence integral length scale (see e.g. Kurian & Fransson, 2009). From figure 1, using the data from 1940, one may conclude that Hislop was the first one to report that an increase in integral length scale moves the

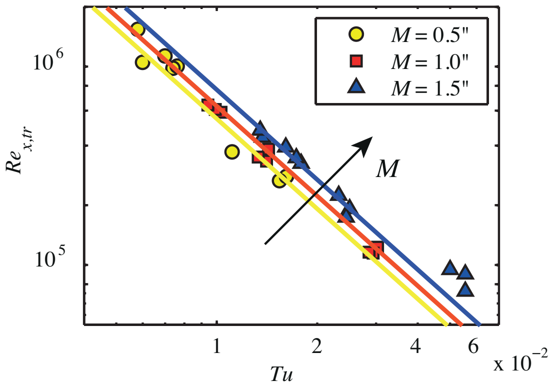

Figure 1. Transitional Reynolds number versus turbulence intensity for three different turbulence generating grids. Experimental data by Hislop (1940).

transition location farther downstream. The trend is modest, but clearly notable with the added straight lines in the loglog plot. More recent investigations have also shown that the level of Tu is not the only dependent variable. An increase in the FST integral length scale Λ_x has shown, both in experiments and numerical simulations (Jonas *et al.*, 2000; Brandt *et al.*, 2004; Ovchinnikov *et al.*, 2004) to advance the transition location. These results contradict the reported results by Hislop since the opposite effect with respect to the movement of the transition location with increasing Λ_x is observed. However, the experimental results by Shahinfar & Fransson (2013), reported in Shahinfar (2013), confirm the existence of both trends, which were observed in the same measurement campaign. Shahinfar & Fransson (2013) report on a critical turbulence intensity level around $Tu_{\rm cr} = 3\%$. For $Tu < Tu_{\rm cr}$ the effect of increasing Λ_x confirm the results by Jonas *et al.* (2000); Brandt *et al.* (2004); Ovchinnikov *et al.* (2004) and for $Tu > Tu_{\rm cr}$ the Λ_x effect by Hislop (1940) is confirmed. The experiments reported in Shahinfar (2013), show on the one hand that for $Tu \approx 2.6\%$, an increase of Λ_x of 12% advances the transition location by 35%. On the other hand, for $Tu \approx 3.9\%$, an increase of Λ_x of 18% the transition location moves downstream by 22%. A physical argument for this change of trend has so far not been suggested. In addition, the results by Shahinfar & Fransson (2013) question the common belief that the FST length scales have a negligible effect on the transition location even though the Tu level is of leading order.

For improved transition prediction models we need: (1) to understand the physical mechanism governing the transition scenario under FST and, (2) to have statistical data, encompassing sufficient cases of different FST conditions, to understand the choice of mother nature when it comes to transition to turbulence. Once there, we can *cherry-pick* the leading variables and boil the complex physics down into a model capable of predicting the transition location with satisfaction. In the present paper a new semi-empirical transition prediction model is proposed based on the experimental results reported in Shahinfar & Fransson (2013), which appears to account for the influence of Λ_x on the transition location in an accurate way.

EXPERIMENTAL DATA

The data used here has been reported in Shahinfar & Fransson (2013) in the doctoral thesis by Shahinfar (2013). A thorough measurement campaign on the FST induced boundary layer transition was carried out in the Minimum-Turbulence-Level wind tunnel at KTH. The experimental setup consisted of a flat plate, and the ceiling of the test section was adjusted such that a zero-pressure gradient boundary layer was developed over the plate. Various passive and active turbulence generating grids were used in order to create different FST conditions. The measurements were carried out using two single hot-wire probes, sufficiently separated from each other in the spanwise direction to not influence each other, and consists of 42 unique FST conditions with thorough measurements throughout the transitional region. Unlike other extensive FST induced transition measurements (e.g. Fransson *et al.*, 2005) the free-stream velocity was here kept constant for all cases, implying that the boundary layer scale is locked up to transition onset. The transition location is determined as the position where the intermittency factor of the velocity signal is 0.5, that is half way through the transition region. An intermittency factor of 0 and 1 correspond to a completely laminar and a fully turbulent flow. The intermittency was calculated using the method proposed in Fransson *et al.* (2005).

TRANSITION PREDICTION MODEL

The transition location is defined as $x_{\rm tr}$ and the corresponding boundary layer length scale and transitional Reynolds number as

$$\delta_{\rm tr} = \sqrt{\frac{x_{\rm tr}\,\nu}{U_\infty}} \qquad \text{and} \qquad Re_{x,\rm tr} = \frac{U_\infty\, x_{\rm tr}}{\nu} \tag{1}$$

respectively, where ν denotes the kinematic viscosity. The integral length scale of the FST is defined as

$$\Lambda_x = U_\infty \int_0^\infty f(\tau)\,\mathrm{d}\tau \tag{2}$$

where $f(\tau)$ is the autocorrelation function of the velocity time signal at the position of the leading edge of the flat plate.

Next, we define the FST characteristics at the leading edge of of the plate starting with the turbulence intensity and integral length scale Reynolds numbers as

$$Tu = \frac{u_{\rm rms}}{U_\infty} \qquad \text{and} \qquad Re_\Lambda = \frac{U_\infty\,\Lambda_x}{\nu} \tag{3}$$

respectively. We also define the FST Reynolds number as the product of the two (in eq. 3)

$$Re_{\rm fst} = Tu \cdot Re_\Lambda \equiv \frac{u_{\rm rms}\,\Lambda_x}{\nu} \tag{4}$$

Primary Variable for Predicting Transition

The leading variable in the proposed transition prediction model is $Re_{\rm fst}$ and not Tu as anticipated in *all* previous models. In figure 2(*a*) and (*b*) $Re_{x,\rm tr}$ is plotted versus both variables and at first glance, the choice of Tu seems to be the

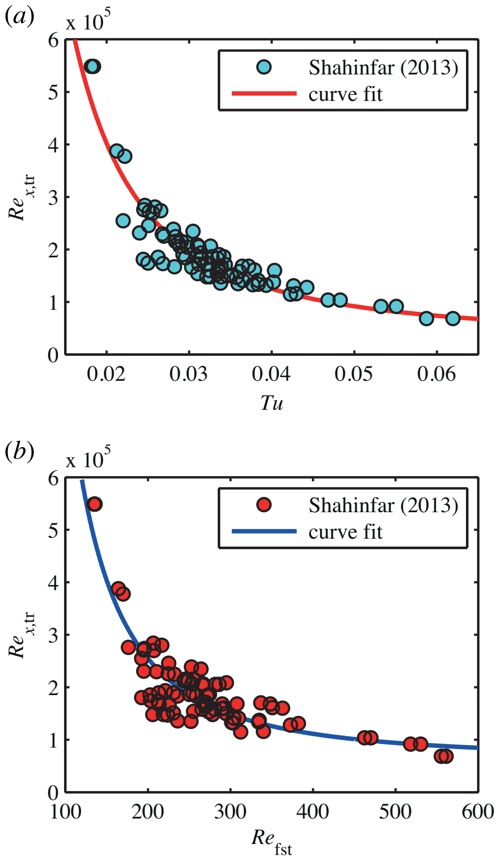

Figure 2. Transitional Reynolds number versus turbulence intensity in (a) and FST Reynolds number in (b). Experimental data from Shahinfar & Fransson (2013).

better option, since it collects the data points closer to the curve fitted line. However, plotting the data versus Re_{fst} reorders the set of data in a favorable way, such that a $\Delta Re_{x,\mathrm{tr}}$ can be added to $Re_{x,\mathrm{tr}}$ and account for the deviation from the curve fit as well be shown.

Using the physical reasoning by Andersson *et al.* (1999) one can argue that the curve in figure 2(a) should have the form

$$(Re_{x,\mathrm{tr}})_{\mathrm{cf}}^{Tu} = \mathscr{A}_1 \cdot Tu^{-2} + \mathscr{A}_2 \qquad (5)$$

where $\mathscr{A}_1$ and $\mathscr{A}_2$ are constants. $\mathscr{A}_2$ has been added here, with the motivation of an existing minimum Reynolds number for self-sustained turbulence. The same physical reasoning, based on input energy, as in Andersson *et al.* (1999), can be used for the variable Re_{fst} and, hence, the curve in figure 2(b) corresponds to

$$(Re_{x,\mathrm{tr}})_{\mathrm{cf}} = \mathscr{B}_1 \cdot Re_{\mathrm{fst}}^{-2} + \mathscr{B}_2 \qquad (6)$$

where $\mathscr{B}_1$ and $\mathscr{B}_2$ are determined in a least-square-fit sense to the data. Equation (6) turns out to be an important relation for the subsequent analysis.

Scale Matching Model

The existing experimental data indicates that there is a change of trend of the influence of Λ_x on the transitional Reynolds number. This information gives a hint about a scale matching between the boundary layer scale and the FST integral length scale. The local integral length scale, which grows with the square root of the downstream distance, is directly proportional to the integral length scale at the leading edge (see e.g. Kurian & Fransson, 2009), namely Λ_x, which suggests that the ratio $\delta_{\mathrm{tr}}/\Lambda_x$ is important for the transition location. Here, the following hypothesis is made:

> *For a given Tu there is an optimal scale ratio $(\delta_{\mathrm{tr}}/\Lambda_x)_{\mathrm{opt}}$ that promotes transition. The transitional Reynolds number versus the scale ratio has a minimum at the optimal scale ratio. A mismatch from the optimal value will give a negative derivative of $Re_{x,\mathrm{tr}}$, with respect to the scale ratio, if the scale ratio is lower than the optimal, and a positive derivate if it is larger.*

Equation for Transitional Reynolds Number

The effect of $\delta_{\mathrm{tr}}/\Lambda_x$ on $Re_{x,\mathrm{tr}}$ is assumed to enter as a correction to eq. (6) and hence, we make the following Ansatz:

$$Re_{x,\mathrm{tr}} = (Re_{x,\mathrm{tr}})_{\mathrm{cf}} + \Delta Re_{x,\mathrm{tr}} \qquad (7)$$

where

$$\Delta Re_{x,\mathrm{tr}} = \Delta Re_{x,\mathrm{tr}}(\Lambda_x/\delta_{\mathrm{tr}}) \qquad (8)$$

If the scale ratio is matched with the one corresponding to the curve of eq. (6), $\Delta Re_{x,\mathrm{tr}}$ will give a zero contribution. On the other hand, the farther the scale ratio deviates from the one corresponding to the curve of eq. (6), the larger becomes the correction. In order to amplify the sensitivity of the scale ratio we work with the square of this ratio, and introduce the correction as

$$\Delta Re_{x,\mathrm{tr}} = \kappa \left[\left(\frac{\Lambda_x}{\delta_{\mathrm{tr}}}\right)^2 - \left(\frac{\Lambda_x}{\delta_{\mathrm{tr}}}\right)^2_{\mathrm{cf}} \right] \qquad (9)$$

where

$$\left(\frac{\Lambda_x}{\delta_{\mathrm{tr}}}\right)^2 \equiv \left(\frac{Re_{\mathrm{fst}}}{Tu}\right)^2 \frac{1}{Re_{x,\mathrm{tr}}} \qquad (10)$$

and $\kappa = \kappa(Re_{\mathrm{fst}})$ is a weighting function. In analogy with eq. (10) we then introduce $(\Lambda_x/\delta_{\mathrm{tr}})^2_{\mathrm{cf}}$ in eq. (9) as

$$\left(\frac{\Lambda_x}{\delta_{\mathrm{tr}}}\right)^2_{\mathrm{cf}} = \left(\frac{Re_{\mathrm{fst}}}{Tu}\right)^2 \frac{1}{(Re_{x,\mathrm{tr}})_{\mathrm{cf}}} \qquad (11)$$

Now, by combining eqs. (11), (10), (9) and (7), and multiplying with $Re_{x,\mathrm{tr}}$ we arrive at the following 2$^{\mathrm{nd}}$ order equation of $Re_{x,\mathrm{tr}}$,

$$Re_{x,\mathrm{tr}}^2 + \underbrace{\left[\kappa\, Re_\Lambda^2 \frac{1}{(Re_{x,\mathrm{tr}})_{\mathrm{cf}}} - (Re_{x,\mathrm{tr}})_{\mathrm{cf}}\right]}_{=\alpha} Re_{x,\mathrm{tr}} - \underbrace{\kappa\, Re_\Lambda^2}_{=\beta} = 0 \qquad (12)$$

with the solution

$$Re_{x,\mathrm{tr}} = -\frac{\alpha}{2} \pm \sqrt{\left(\frac{\alpha}{2}\right)^2 + \beta} \qquad (13)$$

We note that both α and β depend on the parameter Re_Λ and the weighting function κ. While Re_Λ is considered known from the FST condition at the leading edge, κ requires further analysis.

Weighting Function

The weighting function κ is introduced in eq. (9) and corresponds to

$$\kappa = \frac{\Delta Re_{x,\mathrm{tr}}}{\left[(\Lambda_x/\delta_{\mathrm{tr}})^2 - (\Lambda_x/\delta_{\mathrm{tr}})^2_{\mathrm{cf}}\right]} \qquad (14)$$

which may be rewritten using eqs. (10), (11) and (4) to

$$\kappa = -Re_\Lambda^{-2}\, Re_{x,\mathrm{tr}} \cdot (Re_{x,\mathrm{tr}})_{\mathrm{cf}} < 0 \qquad (15)$$

This expression of κ needs to be modeled since it includes $Re_{x,\mathrm{tr}}$ and it renders a trivial solution to eq. (12). In addition, we may note that κ is always negative. For the modeling we plot κ versus Re_{fst} (i.e. eq. 15) in figure 3 using the experimental data and eq. (6). The shape of a curve following the data can be created in many different ways. In this paper, we simply use a 6$^{\mathrm{th}}$ order polynomial fit, which corresponds to the solid curve in figure 3. This model function does a good job in the range of the data, but behaves physically incorrect outside the data range. The model function should have two constraints for an accurate representation, in the limits of $Re_{\mathrm{fst}} \to 0$ and $Re_{\mathrm{fst}} \to \infty$ the function should approach zero. The reasoning behind this is that $\Delta Re_{x,\mathrm{tr}}$ should approach zero in these two limits. For $Re_{\mathrm{fst}} \to \infty$ the transitional Reynolds number will approach the minimum Reynolds number for self-sustained turbulence (cf. $\mathscr{B}_1$ in eq. 6) and the dependence of Λ_x will diminish. For $Re_{\mathrm{fst}} \to 0$ the FST transition scenario will eventually be replaced by the TS wave transition scenario and the effect of Λ_x is expected to disappear. Worth pointing out is, however, that independent of an accurate representation of κ the transition prediction model described by eq. (12) is only valid for the transition scenario dominated by FST.

VALIDATION OF THE MODEL

Figure 4 shows a direct comparison of the transitional Reynolds number as a function of the Reynolds number based on the integral length scale and the turbulence intensity (both taken at the leading edge of the flat plate). The only input to our model, which is based on physical arguments and results in a second order equation (i.e. eq. 12) with analytical solution (see eq. 13), is Re_Λ and Re_{fst}. This parameter and variable, respectively, are directly related to Tu via eq. (4). As shown in figure 4, the comparison between the experimental data in (*a*) with the predicted $Re_{x,tr}$ in (*b*) is very good.

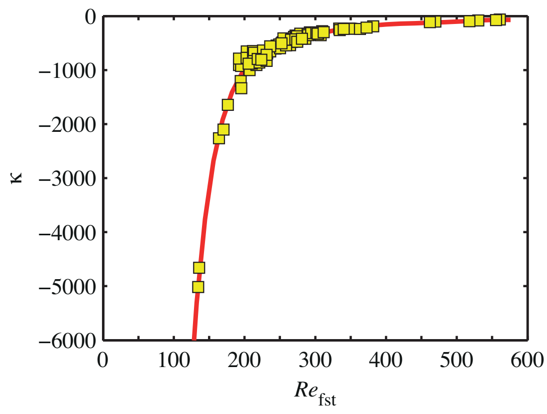

Figure 3. Weighting function (κ) versus FST Reynolds number. Here, the model function, shown as a solid line, corresponds to a 6$^{\mathrm{th}}$ order polynomial.

CONCLUSION

A new transition prediction model has been derived based on a set of original experimental data from a zero-pressure gradient flat plate boundary layer measurement campaign. The semi-empirical model takes into account the integral length scale and the turbulence velocity fluctuation at the leading edge and is capable of predicting the transition location in an accurate way.

All existing models, so far, use Tu as the leading variable, but the present analysis shows that the necessary ingredients in a successful transition prediction model includes, firstly, a FST Reynolds number as leading variable, secondly, an FST parameter being the integral length scale Reynolds number which further accounts for the effect of different length scales and, thirdly, a scale-matching model between the FST and the boundary layer. Even though Tu is related to Re_{fst} and Re_Λ by definition (cf. eq. 4), a similar analysis using Tu as primary variable will not predict the transition location in a satisfactory way, which takes Λ_x into account.

The validity of the model is expected to hold throughout the FST parameter (Re_Λ) and variable (Re_{fst}) ranges that promote the FST dominated transition scenario. Following the criterion by Arnal & Juillen (1978) of $Tu \gtrsim 1\%$ for dominating FST scenario, our derived eq. (12) should be valid for $Re_{\mathrm{fst}} \gtrsim 70$ (using eqs. 5 and 6).

The developed model needs a better description of the weighting function, which behaves physically correct in the limits of Re_{fst} going to zero and infinity. Future work will then address the effect of variable pressure gradient, which would require additional measurement campaigns under controlled wind tunnel settings.

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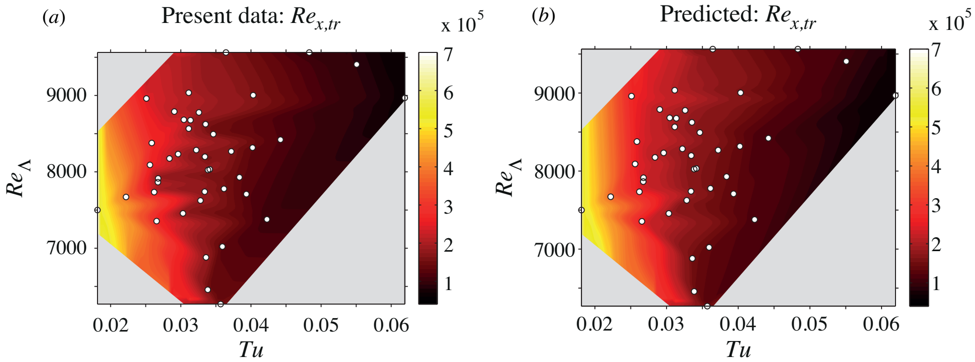

Figure 4. Transitional Reynolds number ($Re_{x,tr}$) as a function of the Reynolds number based on the integral length scale (Re_Λ) and Tu (both taken at the leading edge of the flat plate). (*a*) Experimental data. (*b*) Predicted data based on FST data at the leading edge of the flat plate.

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