On the scaling of turbulent asymptotic suction boundary layers

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ABSTRACT

An analysis of turbulent suction boundary layers is carried out on the basis of new experimental data. The streamwise extent of the suction region of the present experimental apparatus is significantly longer than previous studies, allowing us to better investigate the development of boundary layers with wall suction. We show that it is possible to experimentally realize a turbulent asymptotic state where the boundary layer becomes independent of the streamwise direction and of the initial condition, so that the suction rate constitutes the only control parameter. Turbulent asymptotic suction boundary layers appear to be characterized by a mean velocity with a long logarithmic region, with a slope independent of the suction rate if outer scaling is adopted. In addition to the mean-velocity scaling of turbulent asymptotic suction boundary layers, the suction rate threshold for self-sustained turbulence is also investigated.

INTRODUCTION

Wall-normal suction and blowing is a rather simple but effective technique to modify the behavior of a boundary layer. In particular, the application of uniform suction at the wall can lead to a state for which the momentum loss due to wall-friction is exactly compensated by the entrainment of fluid due to the suction, hence the boundary layer thickness remains constant in the streamwise direction. This condition is known as the Asymptotic Suction Boundary Layer (ASBL). For a laminar ASBL an analytical solution of the Navier-Stokes equations can readily be derived, resulting in an exponential velocity profile as first obtained by Griffith & Meredith (1936) and experimentally verified by Kay (1948) and Fransson & Alfredsson (2003). Despite the numerous experimental and numerical investigations, there is no general agreement on the description of the turbulent asymptotic suction boundary layer (TASBL): contradictory results can be found in the literature regarding the evolution toward the asymptotic state, the threshold suction rate for self-sustained turbulence and the scaling of the mean velocity profile.

The very existence of an asymptotic state for any value of the suction ratio $\Gamma = -V_0/U_\infty$ (where $V_0 < 0$ is the suction velocity and U_∞ is the free stream velocity) has been questioned: Dutton (1958) concluded, based on experimental results, that there is just one suction rate for which an asymptotic state can be obtained, while the experiments by Tennekes (1965) suggested that no asymptotic state can be observed for suction rates lower than a certain threshold. In a recent large-eddy-simulations study by Bobke *et al.* (2016) it was concluded that it is impossible to obtain a turbulent asymptotic state in a practically realizable facility, due to the very long streamwise suction length required. It should be noticed, however, that the initial condition of the simulations was the laminar ASBL, while the common approach in the experimental studies is to start the suction downstream of an initial impermeable entry length where a turbulent boundary layer has been allowed to grow. Even in this case the evolution toward the asymptotic state appears to be slow, nevertheless the approach to the asymptotic state can be hastened if the boundary layer thickness at the beginning of the suction is chosen to be close to the asymptotic one (Dutton, 1958; Black & Sarnecki, 1958; Tennekes, 1964).

Already in the first studies on turbulent suction boundary layers it was noted that an initially turbulent boundary layer would relaminarize if the suction rate is large enough, and the laminar ASBL would eventually be reached. Dutton (1958) and Tennekes (1965) suggested a critical suction rate above which a turbulent state could not be maintained of $\Gamma_{\text{crit}} \approx 0.1$. In recent numerical simulations Khapko *et al.* (2016) obtained the much lower threshold value of $\Gamma_{\text{crit}} = 0.0037$.

Different scalings of the mean velocity profile have been proposed for the turbulent boundary layer with suction. As any other turbulent boundary layer flow, the turbulent suction boundary layer can be divided in a viscous sublayer where the viscous stresses are prevalent and a turbulent layer where the Reynolds stresses dominate. The asymptotic description of the viscous sublayer can be derived as:

$$U^+ = \frac{1}{V_0^+}\left(e^{y^+V_0^+} - 1\right) \tag{1}$$

where the superscript "+" indicates normalization in viscous units. For the turbulent layer, instead, two different scalings have been proposed. A bi-logarithmic law where the streamwise velocity is proportional to the squared logarithm of the wall-normal coordinate has been derived from Prandtl's momentum transfer theory by a number of authors (Black & Sarnecki, 1958; Clarke *et al.*, 1955; Mickley & Davis, 1957; Simpson, 1970; Stevenson, 1963) and more recently via analytical methods (Vigdorovich & Oberlack, 2008).

The bi-logarithmic law can be expressed in the form (Stevenson, 1963)

$$\frac{2}{V_0^+}\left(\sqrt{1+U^+V_0^+}-1\right)=\frac{1}{\kappa}\ln y^+ + B \tag{2}$$

where the L.H.S. is sometimes referred to as pseudo-velocity. There is no consensus on the numerical values of the parameters κ and B, which in general should be considered function of the suction velocity. Nevertheless, a common choice among the supporters of the bi-logarithmic scaling is to set κ to the value of the non-transpired case.

Other authors (Dutton, 1958; Tennekes, 1965; Andersen *et al.*, 1972; Bobke *et al.*, 2016) have instead proposed a logarithmic dependency of the streamwise velocity on the wall-normal coordinate, analogously to what is found for non-transpired boundary layers

$$U^+ = A\ln y^+ + B \tag{3}$$

with the slope A and the intercept B of the line dependent on the suction level. Among these, particularly original is the approach by Tennekes (1965), who derived a law of the wall and a velocity defect law for turbulent suction boundary layers in the modified set of variables $V_0^+U^+ = f(y^+V_0^+)$ and $V_0^+\left(U^+ - U_\infty^+\right) = G(y/\delta)$. Tennekes concluded that in the range $0.04 < -V_0^+ < 0.1$, the logarithmic part of the profile shows the constant slope

$$-V_0^+U^+ \propto 0.06\,\ln\left(-y^+V_0^+\right) \tag{4}$$

For turbulent asymptotic boundary layers, though, $-V_0^+U^+ = U/U_\infty$, hence eq. (4) can be rewritten in outer scaling as

$$U/U_\infty \propto 0.06\,\ln\eta \tag{5}$$

where η is the nondimensional outer wall-normal distance.

In order to settle the controversy on the mean velocity scaling and to explore the scaling of the higher order statistics, for which the available experimental data is rather scanty, a larger database on turbulent suction boundary layer is required. A new experimental apparatus for wall-transpired boundary layers has recently been built and brought into operation at the MTL wind-tunnel at the Odqvist Laboratory of KTH, Stockholm. In this paper we will report the new findings, with main focus on the mean velocity profiles.

EXPERIMENTAL SETUP AND DATA REDUCTION

The experimental setup consists in a 6.62 m long and 1.2 m wide flat-plate, with a top surface made of titanium sheets with 64 µm laser-drilled holes with centre-to-centre distance of 0.75 mm. The flat plate starts with a 122 mm long impermeable elliptical leading edge followed by 8 individual plate elements, each of them equipped with 18 suction hoses arranged such that the suction uniformity is ensured. Each set of 18 hoses is connected to a manifold, and a pipe is driven from each of the 8 manifold to a suction chamber. The chamber is then connected to a fan with a pipe equipped with a flowmeter measuring the total volume flowrate withdrawn through the surface of the plate. The plate is installed in the test section such that the test surface constitutes the wind-tunnel bottom floor. A bleed slot between the wind-tunnel contraction and the plate leading edge allows the development of a fresh boundary layer with a definite origin. Adjustment of the bleed-slot opening and of the ceiling allowed to maintain zero pressure gradient condition, with a max-min variation of $U_\infty < \pm 0.8\%$ in the whole measurement domain for every suction rate considered. A series of V-shaped embossing tapes glued on the leading-edge section act as tripping device, and a turbulent boundary layer grows on a impermeable surface for a certain downstream distance. After this initial length, uniform wall suction is applied along the surface in the downstream direction on the whole spanwise width of the plate. The streamwise component of velocity has been measured with single hot-wire probes with an expected accuracy of $\pm 1\%$. The variation of suction velocity between different plates is less than 2% around the mean value, ensured measuring the pressure drop across the sheets in combination with a permeability measurement. The expected accuracy on the suction rate is $\pm 4\%$. For all the suction profiles shown in this proceeding the friction velocity u_τ has been obtained from Von-Kármán momentum integral equation modified for mass-transfer

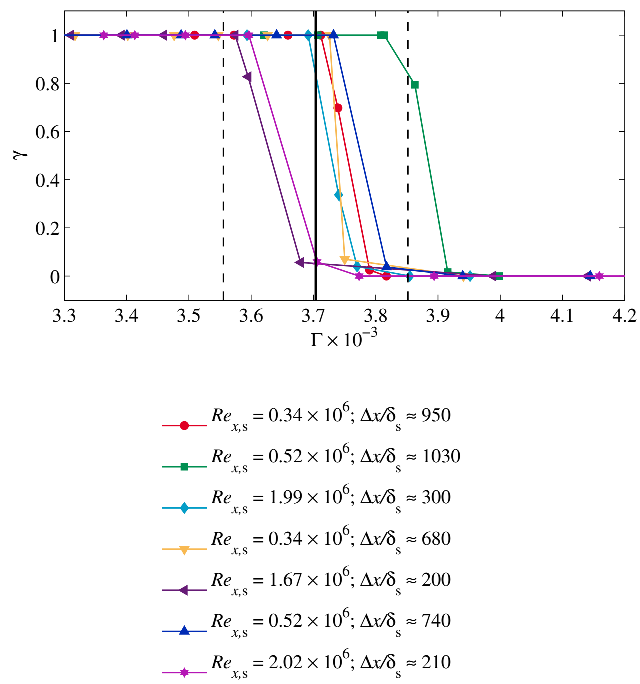

Figure 1. Intermittency γ of the near-wall velocity signal *vs.* the suction rate Γ for different suction start locations Re_{xs} and streamwise evolution lengths $\Delta x/\delta_s$. *Black solid line*: $\Gamma_{\text{crit}} = 3.70\times 10^{-3}$ (Khapko *et al.*, 2016); *Black dashed line*: $\pm 4\%$ deviation on Γ_{crit}.

$$\left(\frac{u_\tau}{U_\infty}\right)^2 = \frac{C_f}{2} = \frac{\mathrm{d}\theta}{\mathrm{d}x} - \frac{V_0}{U_\infty} \tag{6}$$

where $\mathrm{d}\theta/\mathrm{d}x$ was obtained from a fit of the measured momentum thicknesses to an exponential law of the type $Re_\theta = aRe_x^b$. Since for the reported profiles the first term of the R.H.S. of eq. (6) is at least one order of magnitude smaller than the second term, C_f has the same uncertainty as the suction rate.

In the following, the subscript "*s*" indicates the quantities at the streamwise location of the suction start, while the subscript "*as*" indicates the asymptotic conditions. The boundary-layer thickness δ is defined as δ_{99}, i.e. the wall normal location where the velocity reaches 99% of U_∞.

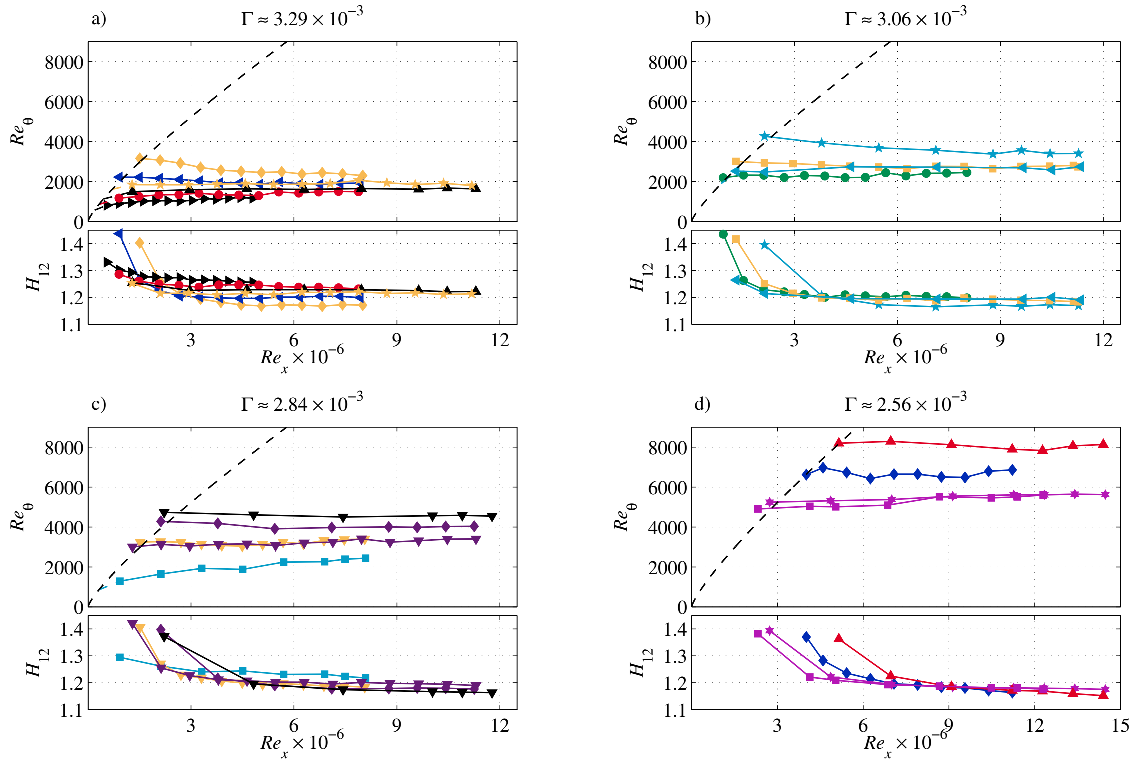

Figure 2. Influence of the boundary-layer thickness at the suction start location on the evolution of the boundary-layer momentum-thickness Reynolds number Re_θ and shape factor H_{12}. *Dashed line*: $Re_\theta = f(Re_x)$ as in Nagib *et al.* (2007)

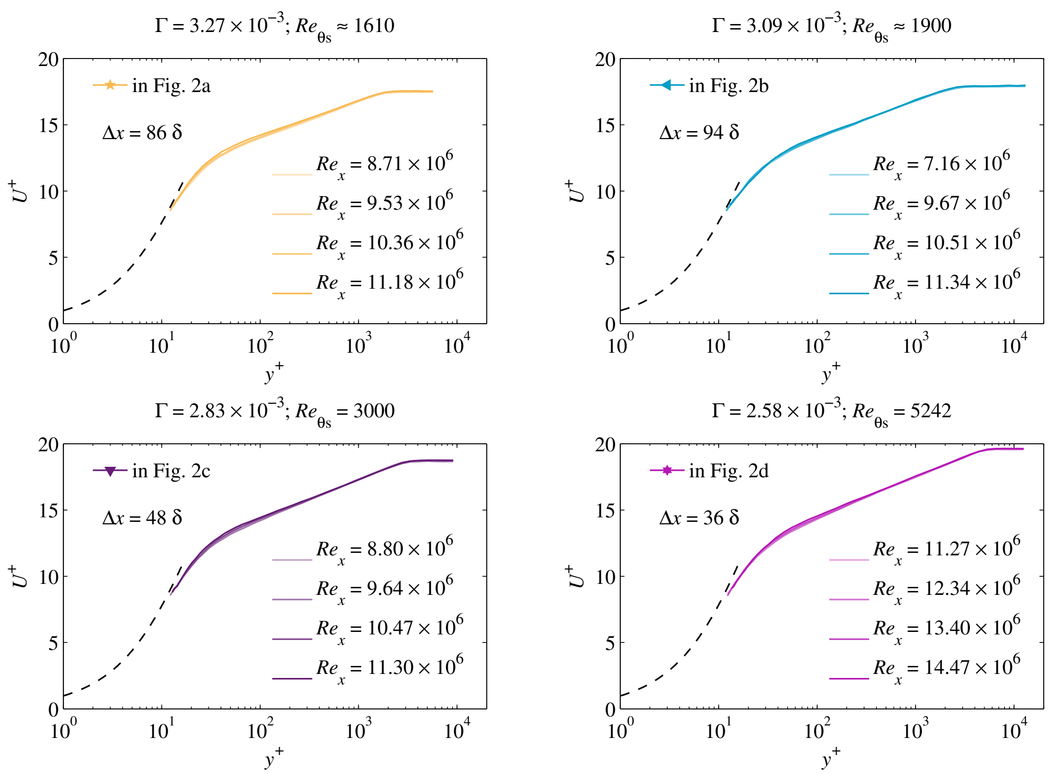

Figure 3. Inner-scaled mean velocity profiles for the four most downstream measurement locations for asymptotic cases in Fig. 2. Δx represents the streamwise distance between the most upstream and the most downstream boundary-layer profile shown in each graph. *Dashed line*: Viscous sublayer as in eq. (1)

RESULTS AND DISCUSSION

Self-Sustained turbulence suction rate threshold

As a first step in this analysis, the threshold suction rate for self-sustained turbulence is considered. While this value is interesting *per se*, its knowledge is also important in order to avoid to include undesired data in the analysis of the scaling of turbulent suction boundary-layers, since profiles which are relaminarizing can bring misleading information.

Suction is applied on an initially turbulent boundary layer, starting at the streamwise Reynolds number Re_x. After the downstream distance Δx, the time series of the velocity is measured in the inner layer of the boundary layer ($9 \lessapprox y^+ \lessapprox 15$) for different values of the suction rate Γ. The intermittency of the velocity signal is then calculated with the method proposed in Fransson *et al.* (2005) to understand if the boundary layer is fully turbulent ($\gamma = 1$), laminar ($\gamma = 0$) or if a relaminarization process is ongoing ($0 < \gamma < 1$). The results are shown in Figure 1. If we define $\Gamma_{\text{crit}} = \max(\Gamma) : \gamma = 1$, i.e. as the maximum value of suction ratio for which a fully turbulent velocity is observed at the measurement location, we notice that for all the initial conditions and evolution length considered $\Gamma_{\text{crit}} = 3.70 \times 10^{-3} \pm 4\%$, in agreement with Khapko *et al.* (2016). It is interesting to notice that all of the boundary-layer reported as turbulent in Dutton (1958), 8 out of 10 of those in Black & Sarnecki (1958) and 7 out of 14 in Tennekes (1964), for instance, were obtained with $\Gamma > \Gamma_{\text{crit}}$, thus were probably undergoing relaminarization. It should be kept in mind, however, that in the aforementioned experiments, Pitot tubes were used as measurement devices, therefore the fluctuating velocity component was unaccessible and the traces of a relaminarization process hard to recognize. The authors opinion is that boundary layers with $\Gamma \gtrapprox 3.7 \times 10^{-3}$ must be excluded from all future analysis on the scaling of turbulent suction boundary layers.

The evolution towards the asymptotic state

Figure 2 shows the evolution of the momentum-thickness Reynolds number Re_θ along the streamwise-coordinate Reynolds number Re_x for different suction rates and different boundary layer thickness $Re_{\theta s}$. The variation of $Re_{\theta s}$ was obtained both by regulating the free-stream velocity and by changing the physical location where the suction started. The latter regulation was achieved either by disconnecting the upstream plate elements from the suction system or, when finer adjustment was needed, by covering part of the surface with standard household aluminum foil. For all the suction rates shown in Figure 2, it was possible to experimentally realize a boundary layer with approximately constant boundary layer thickness, moreover the same boundary layer momentum thickness could be obtained with two different $Re_{\theta s}$. This indicates that the turbulent asymptotic suction regime was indeed reached in a strict manner. The boundary layer profiles measured at the four most downstream measurement locations for the cases considered to be asymptotic states are shown in Figure 3. For each of the cases considered the variation of momentum thickness is less than $\pm 3.5\%$ in the last four measurement locations, corresponding to a streamwise distance Δx exceeding 36 times the boundary layer thickness δ. It can be observed that the variation of the mean velocity profile is minimal, supporting the conclusion that the selected profiles represents asymptotic states.

A mean velocity scaling for the asymptotic state

Figure 4 shows the inner-scaled mean velocity profile for the most downstream location of the asymptotic states in Figure 3. The profiles are characterized by a long region where the mean velocity

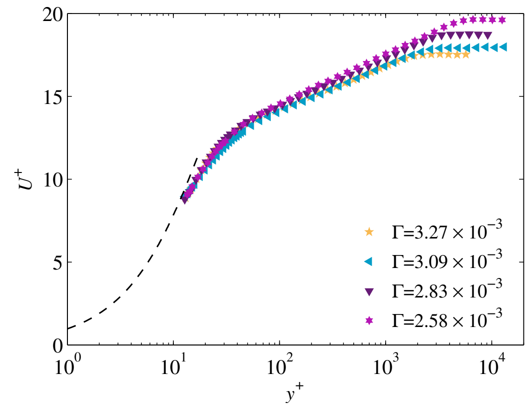

Figure 4. Inner-scaled asymptotic profiles at the most downstream measurement location. Symbols as in Fig. 2. *Dashed line*: Viscous sublayer as in eq. (1)

profile exhibit a logarithmic behaviour, and from the absence of a clearly distinguishable wake region. The disappearance of the wake region appear to be a characteristics of the TASBL, and the presence of a wake region can be considered a symptom that the boundary layer has still not reached its asymptotic state (Simpson, 1970; Bobke *et al.*, 2016). These data support the view according to which the mean velocity profile can be described by a log-law as in eq. (3), with $A = f_1(\Gamma)$ and $B = f_2(\Gamma)$. However, if the profiles are instead plotted in outer scaling, as shown in Figure 5, a good overlap in the logarithmic region between all the asymptotic profiles considered can be observed, independently from the suction rate. In Figure 5 three different choices of outer length scale are shown (with Δ representing the Rotta-Clauser length scale). No significant difference can be noticed, even if slightly better collapse can be observed when $\eta = y/\Delta$. Figure 6 depicts the log-law indicator function based on the outer scaled velocity $\Xi = y\,\mathrm{d}(U/U_\infty)/\mathrm{d}y$, where the derivative was calculated through a cubic spline interpolant of the measured mean velocity profile. A clear plateau of Ξ extending more than a decade of y^+ is observed between $y^+ > 100$ and $y/\delta < 0.6$, with mean value $\overline{\Xi} = 0.0656$. We propose here that the mean velocity profile of the turbulent asymptotic suction boundary layer can be

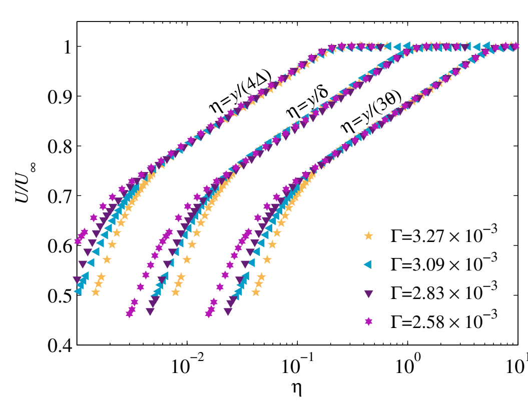

Figure 5. Outer-scaled asymptotic profiles at the most downstream measurement location, for three different choice of the outer length scale. The multiplicative coefficients of the length scales were chosen solely for illustration purposes. Symbols as in Fig. 2.

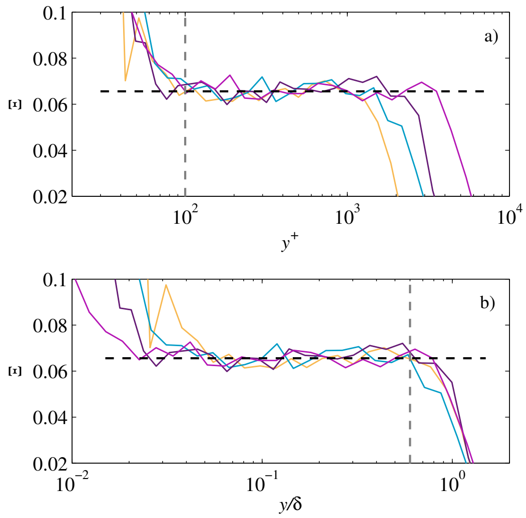

Figure 6. Indicator function Ξ *vs.* the inner scaled (a) and outer-scaled (b) wall-normal coordinate. *Black dashed line*: $\Xi = 0.0656$; *Gray dashed line*: log-law bounds $y^+ = 100$ and $y/\delta = 0.6$

described by the equation

$$\frac{U}{U_\infty} = A \ln \eta + B \tag{7}$$

with $A = 0.0656$ and the value of B, which depends on the choice of outer length scale, equal to $B = 0.811, 0.993, 1.01$ for $\eta = y/\theta, y/\delta, y/\Delta$ respectively. The proposed scaling is in close agreement with the one proposed by Tennekes (1965) (see eq. 5).

Comparison with other experiments and simulation

The asymptotic profiles measured in the current experiment are compared with previous numerical and experimental results in Figure 7. The asymptotic profiles obtained numerically by Khapko *et al.* (2016) and Bobke *et al.* (2016) show outer-scaling similarity for all the suction rates considered, excluding the case representing the reported self-sustain turbulence threshold ($\Gamma = 3.70 \times 10^{-3}$). The slope of the log-law coefficient observed in the simulations with $\Gamma < 3.70$ is however slightly smaller ($A \approx 0.061$) than the one observed in the current experiments. Good agreement on the slope of the logarithmic region is found with the profile measured by Kay (1948) at the suction rate for which he reported that a constant boundary layer thickness was achieved. The profile from Tennekes (1964) deviates considerably from the one measured in the current experiments. This profile, however, represents a case where the boundary-layer momentum thickness was still weakly growing, hence the asymptotic regime was not fully established. As a comparison, in Figure 8 two non-asymptotic states at $\Gamma \approx 2.83 \times 10^{-3}$ are shown together with the asymptotic state at the same suction rate in inner and outer scaling. In the two non asymptotic cases the presence of a small but distinguishable wake region, determine an early departure of the mean velocity profile from the log-law, compromising the validity of the proposed mean velocity profile scaling. In particular, if suction is applied too early ($Re_{\theta s} << Re_{\theta as}$), the wake region appears as an overshoot above the log-law, similarly to the effect of insufficient box-size in the simulations by Bobke *et al.* (2016). If suction is applied too late ($Re_{\theta s} >> Re_{\theta as}$), the departure from the log-law takes the form of an undershoot. These deviations are probably linked to not fully developed outer structures in the

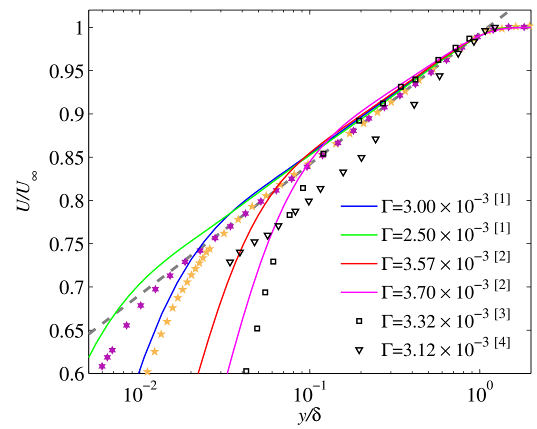

Figure 7. Comparison between present experiments and other experimental and numerical data. *Colored symbols*: current experiments, symbols as in Fig. 5; *Gray dashed line*: log-law as in eq. (7); [1]: LES simulations by Bobke *et al.* (2016); [2]: DNS simulation by Khapko *et al.* (2016); [3] Experiments by Kay (1948); [4]: Experiments by Tennekes (1964) (run 2-312; $x = 878$ mm).

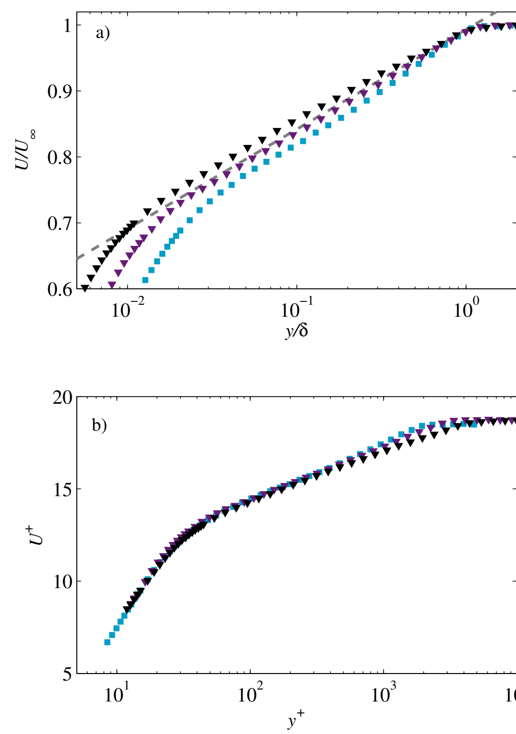

Figure 8. Inner- (a) and outer- (b) scaled mean velocity profiles for the most downstream location of three cases with $\Gamma \approx 2.83 \times 10^{-3}$. Symbols as in Fig. 2c. *Gray dashed line*: log-law as in eq. (7).

case of $Re_{\theta s} << Re_{\theta as}$ or to an excess of low-wavenumber turbulent energy in the case of $Re_{\theta s} >> Re_{\theta as}$ (Coles, 1971; Bobke *et al.*, 2016).

CONCLUSIONS

New experimental results on turbulent suction boundary layer convincingly show that it is possible to experimentally produce a turbulent asymptotic state, provided that the boundary layer thickness at the streamwise location where suction is started is close to the asymptotic one. The largest suction rate for which self-sustained turbulence is observed is $\Gamma \approx 3.7 \times 10^{-3}$, in agreement with Khapko *et al.* (2016). The mean velocity profile of the asymptotic profile is characterized by a long logarithmic region with a slope that appears to be constant in outer variables with a value equal to $A = 0.0656$ independently of the suction rate. To confirm this behavior, data on asymptotic profiles at lower suction rates would be welcomed. However, the larger $Re_{\theta\mathrm{as}}$ expected when Γ is lowered, represents a considerable challenge. From an experimental perspective, lowering Γ would require experiments in which a turbulent boundary layer is allowed to grow for a long downstream distance before suction is applied, in order to have $Re_{\theta\mathrm{s}} \approx Re_{\theta\mathrm{as}}$. Downstream of this location a suction region must be provided, which extends multiple times the (larger) boundary layer thickness. The size of the required facility would hence quickly become limiting. From the numerical perspective, instead, the limiting factor would be the large Re_τ encountered when Γ is lowered (Bobke *et al.*, 2016).

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