Unsteady turbulence cascades and their effects on the T/NT interface

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ABSTRACT

We identify three different types of turbulence cascade for constant density incompressible 3D turbulence all of which imply a particular turbulence dissipation scaling: the well-known Kolmogorov equilibrium cascade and two different types of non-equilibrium cascade with two different turbulence dissipation scalings. Turbulence dissipation scalings are closely related to the scalings of the local entrainment velocity u_e of the turbulent/non-turbulent (T/NT) interface in an axisymmetric and self-similar turbulent wake. The turbulence dissipation scaling implied by the Kolmogorov equilibrium cascade is consistent with a Kolmogorov scaling of u_e whereas the different non-equilibrium dissipation scaling present in the case of one of the two types of non-equilibrium cascade is consistent with a different scaling of u_e.

INTRODUCTION

The equilibrium Richardson-Kolmogorov cascade applies in cases where the incompressible Navier-Stokes equation is forced in a way which keeps the turbulence steady without significant large-scale oscillations. This situation can be realised in Direct Numerical Simulations (DNS) of periodic turbulence forced to achieve such particular statistical stationarity but it is not known how widely it can be found in nature and engineering. On the other hand it is now known that Kolmogorov equilibrium is absent in various grid-generated turbulent flows, turbulent shear flows, and periodic turbulence which is either decaying or with significant large-scale oscillations (Vassilicos, 2015; Goto & Vassilicos, 2015; Dairay et al., 2015; Obligado et al., 2016). In these unsteady turbulent flows the turbulence dissipation scaling can be different over a very significant extent in time or space (according to case) from the well-known $\varepsilon = C_\varepsilon K^{3/2}/L$ scaling where ε is the turbulence dissipation rate per unit mass, K is the turbulent kinetic energy, L is an integral length-scale and C_ε is a dimensionless number which is constant in the Kolmogorov equilibrium phenomenology. Even though these unsteady flows can be quite different from each other, the same non-equilibrium dissipation scaling can be found in all of them, namely $C_\varepsilon \sim Re_0/Re_L$ where $Re_0 = U_\infty L_b/\nu$ (in terms of an inlet/initial velocity U_∞ and length L_b and the kinematic viscosity ν) and $Re_L = \sqrt{K}L/\nu$, i.e. $\varepsilon \sim U_\infty L_b K/L^2$ and equivalently $C_\varepsilon \sim \sqrt{Re_0}/Re_\lambda$ (where Re_λ is the Taylor length-based Reynolds number).

In the next section we study the non-equilibrium cascade in periodic decaying turbulence. Both dissipation scalings mentioned in the previous paragraph are present during periodic turbulence decay in different time ranges and we show how they are related to two different types of non-equilibrium (unsteady) turbulence cascades. Whilst Kolmogorov equilibrium implies $\varepsilon \sim K^{3/2}/L$, the inverse is not true and this scaling appears in the long-time regime after the $\varepsilon \sim U_\infty L_b K/L^2$ scaling regime even though the cascade is not in equilibrium at any time during decay.

In the section after next we demonstrate that the turbulence dissipation scaling $\varepsilon \sim K^{3/2}/L$ is consistent with a Kolmogorov scaling of u_e whereas the non-equilibrium dissipation scaling $\varepsilon \sim U_\infty L_b K/L^2$ is consistent with a different scaling of u_e. We also present results from a DNS of a spatially developing axisymmetric and self-similar turbulent wake which supports this conclusion and the assumptions that this conclusion has been based on.

UNSTEADY TURBULENCE CASCADES

To study the turbulence cascade in homogeneous or periodic turbulence one often starts from the scale-by-scale energy balance

$$\frac{\partial K^{>}(k,t)}{\partial t} = \Pi(k,t) - \varepsilon^{>}(k,t) \tag{1}$$

where the notation used is defined in terms of the energy spectrum $E(k,t)$: $K^{>}(k,t) \equiv \int_k^\infty E(k,t)dk$ and $\varepsilon^{>}(k,t) \equiv 2\nu \int_k^\infty k^2 E(k,t)dk$ are, respectively, the turbulent kinetic energy and the turbulence dissipation in wavenumbers larger than k and $\Pi(k,t)$ is the inter-scale flux of turbulent kinetic energy from wavenumbers smaller to wavenumbers larger than k (we omit "per unit mass" for brevity). Kolmogorov stationarity/equilibrium is a situation where

$$\left| \frac{\partial K^{>}(k,t)}{\partial t} \right| \ll \varepsilon^{>}(k,t) \tag{2}$$

and therefore

$$\Pi(k,t)/\varepsilon^{>}(k,t) \approx 1. \tag{3}$$

The balance $\Pi(k,t) \approx \varepsilon^{>}(k,t)$ is a cornerstone property of the Kolmogorov equilibrium cascade in the range of intermediate wavenumbers where this cascade is present when it exists. Given the scalings

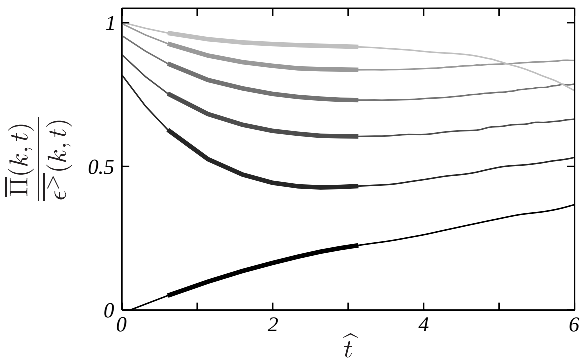

Figure 1. Interscale energy flux to wavenumbers larger than k divided by the turbulence dissipation in wavenumbers larger than k as a function of number of turnover times $\hat{t}$. The overbar signifies an average over ten 1024^3 DNS runs; curves from black to grey and from the lower to the upper parts of the plot correspond to $k = 4, 8, 16, 32, 64, 128$.

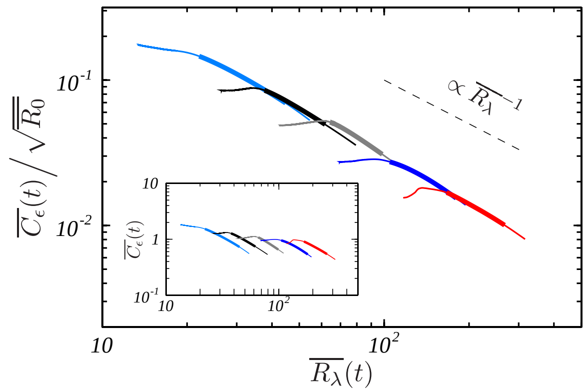

Figure 2. Log-log plot of $\overline{C_\varepsilon}(t)/\sqrt{\overline{R_0}}$ versus $\overline{R_\lambda}(t)$. The dotted line in the plot represents $\overline{R_\lambda}^{-1}$ and the overbar is an average over many DNS runs. Red 2048^3, blue 1024^3, grey 512^3, black 256^3 and light blue 128^3 DNS. The insert is a log-log plot of $\overline{C_\varepsilon}(t)$ versus $\overline{R_\lambda}(t)$. The thick part of the lines marks the same time range as the thick part of the lines in figure 1.

of the turbulent kinetic energy input rate at the large scales, it implies the well-known dissipation scaling $\varepsilon = C_\varepsilon K^{3/2}/L$ where C_ε is a constant dimensionless number.

We have run a total of 311 DNS of decaying three-dimensional Navier-Stokes turbulence in a periodic box with values of the Taylor length-based Reynolds number up to about 300 and an energy spectrum with a wide wavenumber range of close to $-5/3$ power-law dependence at the higher Reynolds numbers. On the basis of these runs we have found a critical time t_c when the ratio of interscale energy flux to high-pass filtered turbulence dissipation, i.e. $\Pi(k,t)/\varepsilon^{>}(k,t)$, changes from decreasing to very slowly increasing in the inertial range (see figure 1). This ratio's departure from 1 is a measure of how unsteady the cascade is at wavenumber k. Clearly there are two types of unsteady cascade, one for times before t_c when the cascade is increasingly unsteady as time progresses and one for times larger than t_c when the cascade remains unsteady and only very slowly becomes slightly less so as times progresses.

The time t_c is also the time when the scaling of the turbulence dissipation changes from $C_\varepsilon \sim \sqrt{Re_0}/Re_\lambda$ to $C_\varepsilon \approx Const$ (see figure 2). Even though the customary theoretical basis for $C_\varepsilon = Const$ is a statistically stationary cascade where large scale energy flux balances dissipation, this is not the case thoughout the entire time-range of integration in all our DNS runs. The turbulence cascade is unsteady in different ways in the two time ranges demarcated by t_c and the turbulence dissipation scalings are different in these two time ranges accordingly. A theoretical framework for making sense of these two non-equilibrium turbulence dissipation scalings has been worked out and will be included in the conference presentation if time allows.

THE LOCAL ENTRAINMENT VELOCITY OF THE T/NT INTERFACE

We now concentrate our attention on spatially developing axisymmetric and self-similalar turbulent wakes. A characteristic local entrainment velocity u_e of the T/NT interface can be defined in such a flow in terms of the time-averaged area A of the fully turbulent region in a plane normal to the flow's axis of symmetry and the time-averaged length $\mathscr{L}$ of the T/NT interface in the same plane. Indeed, it makes sense to write $U_\infty dA/dx = \mathscr{L} u_e$ where U_∞ is the freestream velocity and x is the coordinate in the streamwise direction of the axis of symmetry. If $A(x)$ scales with the wake width $\delta(x)$ of the turbulent wake, i.e. $A(x) \sim \delta^2$, then $U_\infty \delta d\delta/dx \sim \mathscr{L} u_e$. This simple and immediate relation is significant because the modified Townsend-George theory of free turbulent shear flows (see Dairay et al, 2015) implies that the wake width's scaling with x is sensitive to the turbulence dissipation scalings, which means that $\mathscr{L} u_e$ is different for $C_\varepsilon \sim \sqrt{Re_0}/Re_\lambda$ and for $C_\varepsilon \approx Const$. If we also assume that the T/NT interfacial line in a lateral $x = const$ plane has a well-defined fractal dimension D (Sreenivasan et al., 1989) so that $\mathscr{L} \sim \delta(\eta_I/\delta)^{1-D}$ where, in agreement with Corrsin & Kistler (1955), $\eta_I \sim \nu/u_e$, then one can derive scalings for u_e. The dissipation scaling $C_\varepsilon \approx Const$ leads to $u_e \sim u_\eta$ if $D = 4/3$ (as required by the $\nu^{3/4}$ scaling of the Kolmogorov length-scale η_K, see Sreenivasan et al., 1989) and the dissipation scaling $C_\varepsilon \sim \sqrt{Re_0}/Re_\lambda$ leads to $u_e/u_\eta \sim Re_0^{1/4-(D-1)/D}(\frac{x-x_0}{\theta})^{1/8}$ where u_η is the Kolmogorov velocity, L_b in $Re_0 = U_\infty L_b/\nu$ is the size of the wake-generating body, x_0 is a virtual origin and θ is the momentum thickness. In this second case the local entrainment velocity is clearly different from u_η.

We have run a DNS of a spatially developing axisymmetric and self-similar turbulence wake which is identical to the one of Dairay et al. (2015) and we have verified the assumptions and conclusions of the previous paragraph's analysis. Some of the results are given in figures 3, 4 and 5 but more will be presented at the conference. Note that the turbulence dissipation scaling is $C_\varepsilon \sim \sqrt{Re_0}/Re_\lambda$ in the range $10 \le x/L_b < O(100)$ if the Reynolds number is high enough (Dairay et al., 2015; Obligado et al., 2016) and that it can be expected to change to $C_\varepsilon \approx const$ further downstream.

CONCLUSION

It is likely that the most common nonlinear turbulence cascades in nature and engineering are unsteady (non-equilibrium) cascades. We have identified at least two types of unsteady turbulence cascades, each one with a different turbulence dissipation scaling law attached to it. The type of unsteady cascade is therefore an important element in the statistics of the T/NT interface. In the case of axisymmetric self-similar turbulence wakes (and perhaps also in other flows) different types of unsteady turbulence cascade are correlated with different scalings of the local entrainment velocity of the interface. One can predict these scalings if one knows the turbulence dissipation scalings in the region of the flow considered.

More details on the work summarised here can be found in Goto & Vassilicos (2016) and Zhou & Vassilicos (2017).

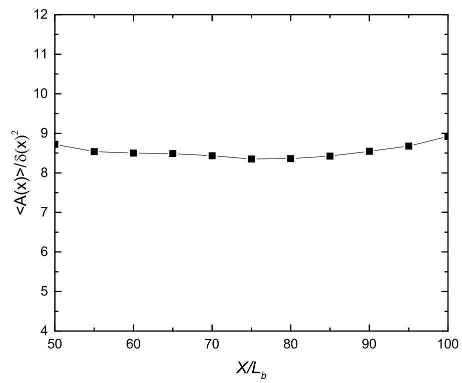

Figure 3. $A(x)/\delta^2(x)$ versus x/L_b.

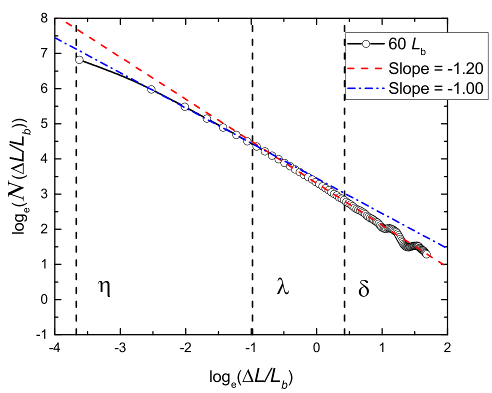

Figure 4. Time-average number $N(\eta_B)$ of squares of side-size η_B needed to cover the T/NT interfacial line in the 2D plane $x/L_b = 60$ versus η_B/L_b. The fractal dimension is well-defined over one decade and found to equal $6/5 \pm 0.02$ in the range $x/L_b = 50$ to $x/L_b = 100$. The values $\eta_B = \eta_K$, λ and δ are indicated for reference.

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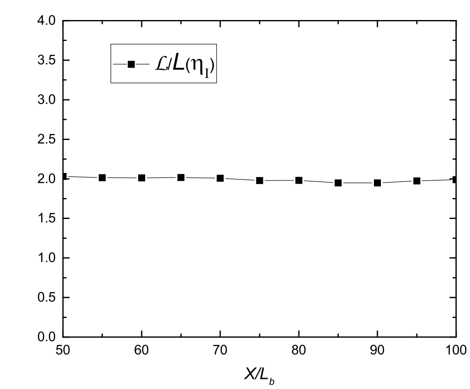

Figure 5. $\mathscr{L}/L(\eta_I)$ versus x/L_b, where $L(\eta_I) \sim \delta(\eta_I/\delta)^{1-D}$ and $\eta_I = \nu/u_e$.