Improvement of SGS model using analysis of SGS force and SGS energy transfer

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ABSTRACT

The subgrid-scale (SGS) model is improved based on analysis of SGS force and energy transfer around an elliptic Burgers vortex. Abe (2013) proposed an anisotropy-resolving SGS model where the Bardina term of similarity model is mixed to the eddy viscosity model. The Bardina term gives no effect to the SGS energy transfer but affects the SGS force. Using this concept, we newly propose the similarity model with the Clark term. The SGS energy transfer is determined by not the eddy viscosity term but the similarity term, whereas the SGS force is improved by using the SGS kinetic energy as well as Abe (2013). We found that the SGS force and energy transfer with the Clark term yield high correlation with the true distributions around the elliptic vortex better than the Bardina term. The SGS model based on the Clark term gives good performance for turbulent channel flows in $Re_\tau = 180$ and 590 even in extremely coarse grid resolutions. Especially, the SGS model with the SGS kinetic energy fairly improves the mean streamwise velocity profile.

INTRODUCTION

As the advancement of computer power, large-eddy simulation (LES) has been the promising computation method of turbulent flows. The eddy viscosity model like the Smagorinsky model (Smagorinsky, 1963) gives adequate energy dissipation. However, the correlation of the subgrid-scale (SGS) energy transfer distribution is not high compared with the true SGS energy transfer distribution obtained from the filtered direct numerical simulation (DNS). In order to improve the correlation, the terms similar to the SGS stress tensor are proposed by Clark *et al.* (1979) and Bardina (1980), here we call them Clark term and Bardina term, respectively. Those similarity terms give low energy dissipation, but high correlations of the SGS energy transfer and stress tensor with the filtered DNS results.

Recently, Abe (2013) proposed an anisotropy-resolving SGS model where the Bardina model is used for the Reynolds term. That feature is that (1) eddy viscosity is determined by one equation for the SGS kinetic energy proposed by Inagaki (2011) and (2) the Bardina term is mixed in the SGS stress tensor but does not affect the SGS energy transfer. Thus, the SGS energy transfer is given only by the eddy viscosity and the mixed Bardina term affects only the SGS force in Navier-Stokes equations.

The Abe model shows perfect predictions of streamwise velocity in turbulent channel flows in $Re_\tau = 395 \sim 2000$ even for extremely coarse grid resolutions. However, it is unclear why Abe model gives good performance by using the Bardina model. Real turbulence is considerably complicated, so that it is difficult to understand how modeled terms behave in the complex turbulence.

In order to understand that, Kobayashi (2015) showed the SGS energy transfer around an elliptic Burgers vortex which is seen a lot in DNS. It is found that the SGS energy transfer distribution by Bardina model shows a negative correlation with that by the filtered DNS. As is well known, the SGS energy transfer distribution by the eddy viscosity has low spatial correlation with that by the filtered DNS.

In the present study, we improve the SGS model based on (a) the results of the SGS force and SGS energy transfer around the elliptic Burgers vortex and (b) the concept used in the Abe model, i.e., no contribution of non-linear term to the energy transfer. The proposed SGS models are examined for the turbulence statistics in the turbulent channel flows of $Re_\tau = 180$ and 590.

SGS MODEL AND NUMERICAL METHODS

We use the asymptotic solution of the elliptic Burgers vortex presented by Moffatt *et al.* (1994) for the background straining flow $U = (\alpha x, \beta y, \gamma z)$ where $\alpha + \beta + \gamma = 0$, $\alpha < 0 < \gamma$, $\beta > \alpha$, the strain parameter $\lambda = (\alpha - \beta)/(\alpha + \beta) = 3.63$, $\alpha = -2.31$, $\beta = 1.31$ as shown in Kobayashi (2015). The axis of the vortex aligns in z direction. For the obtained velocity distribution around the elliptic vortex, we take a differential filtering operation with the second-order finite difference method.

$$\overline{u}_i = u_i + \frac{\overline{\Delta}_k^2}{24} \frac{\partial^2 u_i}{\partial x_k^2} \tag{1}$$

where the overline (-) shows filtered variables for a resolved scale, and this filtering operation is commonly used to obtain the test filtered velocity in dynamic SGS models as shown in Kobayashi (2005). By using this filtering operation, we obtain the SGS energy transfer and SGS force around the elliptic vortex. The SGS energy transfer term in the SGS kinetic energy equation is described as

$$-\tau_{ij}\overline{S}_{ij}, \quad \tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j, \quad \overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_j}{\partial x_i} + \frac{\partial \overline{u}_i}{\partial x_j}\right) \tag{2}$$

where τ_{ij} is the SGS stress tensor and $\overline{S}_{ij}$ is the velocity strain tensor. If $-\tau_{ij}\overline{S}_{ij}$ is positive, it means forward scatter, i.e., energy transfer from large scale to small scale. The SGS force is defined as

$$-\frac{\partial \tau_{ij}}{\partial x_j} \tag{3}$$

in the Navier-Stokes equations.

Abe (2013) proposed the anisotropy-resolving SGS model with the Bardina term as a scale-similarity model.

$$\tau_{ij}^* = -2\nu_{SGS}\overline{S}_{ij} + \frac{2k_{SGS}}{\tau'_{kk}} \left\{ \tau'^*_{ij} - \left(-2\nu' \overline{S}_{ij}\right) \right\} \tag{4}$$

$$\tau'_{ij} = \left\{ \left(\overline{u}_i - \widehat{\overline{u}}_i\right)\left(\overline{u}_j - \widehat{\overline{u}}_j\right) \right\}, \quad \nu' = -\frac{\tau'^*_{ab}\overline{S}_{ab}}{2\overline{S}_{ab}\overline{S}_{ab}} \tag{5}$$

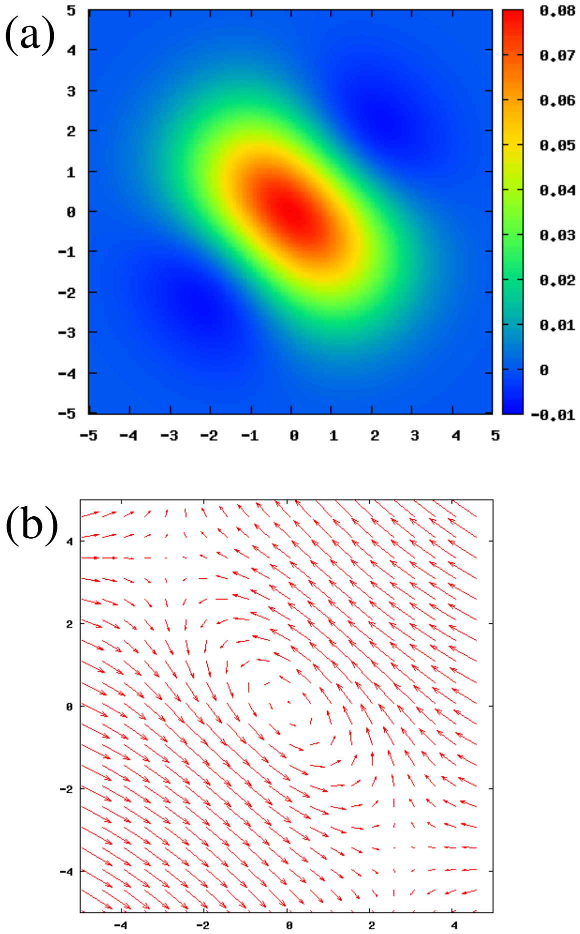

Figure 1. Distributions of vorticity and velocity vectors for an elliptic Burgers vortex.

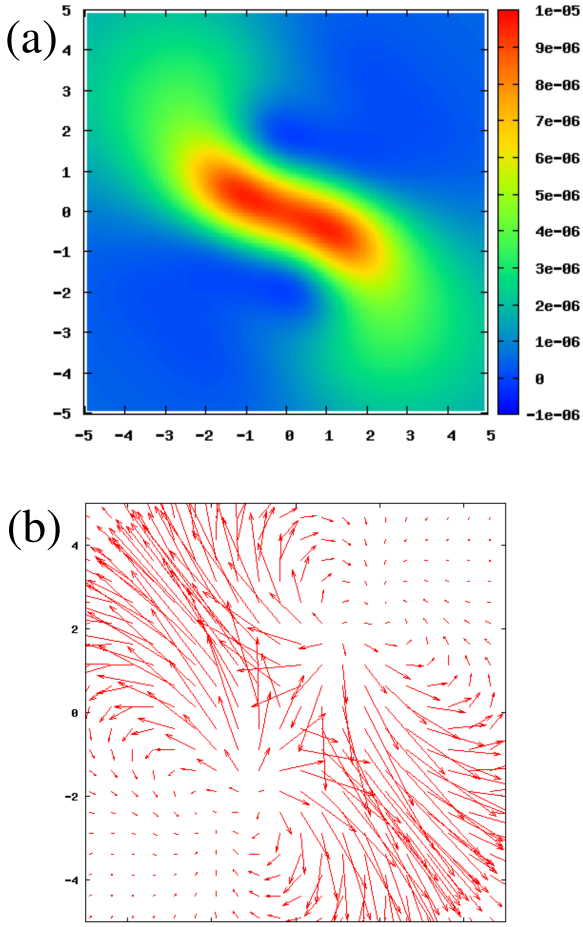

Figure 2. Distributions of SGS energy transfer and SGS force vector for Clark term.

where $()^*$ indicates the traceless tensor, ν_{SGS} is the SGS eddy viscosity, k_{SGS} is the SGS kinetic energy, τ'_{ij} is the Bardina term, and $\widehat{()}$ shows a test filter with $\widehat{\overline{\Delta}} = 2\overline{\Delta}$. In Eq. (4), the first term yields the eddy viscosity model, whereas the second term has no contribution to the SGS energy transfer but affects the SGS force as the similarity term.

The SGS stress tensor is newly modeled as follows.

$$\tau^*_{ij} = \frac{-L_{ab}\overline{S}_{ab} + |-L_{ab}\overline{S}_{ab}|}{-L_{ab}\overline{S}_{ab}} L^*_{ij} + \frac{2k_{SGS}}{L_{kk}} \left(L^*_{ij} + \frac{-L^*_{ab}\overline{S}_{ab}}{\overline{S}_{ab}\overline{S}_{ab}} \overline{S}_{ij} \right) \tag{6}$$

$$L_{ij} = \frac{\overline{\Delta}_k^2}{12} \frac{\partial \overline{u}_i}{\partial x_k} \frac{\partial \overline{u}_j}{\partial x_k} \tag{7}$$

where L_{ij} is the Clark term and is the first term of Taylor expansion of modified Loenard term. In Eq. (6), the first term gives forward scatter and is a similarity term. The second term is modeled based on the concept proposed by Abe (2013), i.e., it does not contribute to the SGS energy transfer but improves the SGS force. The k_{SGS} in Eq. (6) is determined by one equation model proposed by Inagaki (2011), but that equation is simplified as below. We use only the turbulence production and SGS energy dissipation rate. As a result, we use a single model parameter $C_\varepsilon = 0.835$.

$$\frac{\partial k_{SGS}}{\partial t} = -\tau_{ij}\overline{S}_{ij} - \varepsilon_{SGS} \tag{8}$$

$$\varepsilon_{SGS} = C_\varepsilon \frac{2k_{SGS}^{3/2}}{\overline{\Delta}} \tag{9}$$

In this paper, we call the model LS-1eq. When we do not solve the one equation for k_{SGS}, we call the model LS, namely, $k_{SGS} = 0$ in Eq. (6).

Table 1. Numerical condition and grid resolution.

Re_τ	$L_x \times L_y \times L_z$	$N_x \times N_y \times N_z$	Δx^+	Δz^+
180	$4\pi\delta \times 2\delta \times \frac{4}{3}\pi\delta$	$16 \times 64 \times 16$	141	47
590	$2\pi\delta \times 2\delta \times \pi\delta$	$64 \times 64 \times 64$	58	29
590	$2\pi\delta \times 2\delta \times \pi\delta$	$32 \times 64 \times 32$	116	58

The central finite difference method is used with fourth-order accuracy in streamwise (x) and spanwise (z) directions and the second-order accuracy in wall-normal (y) direction as described in

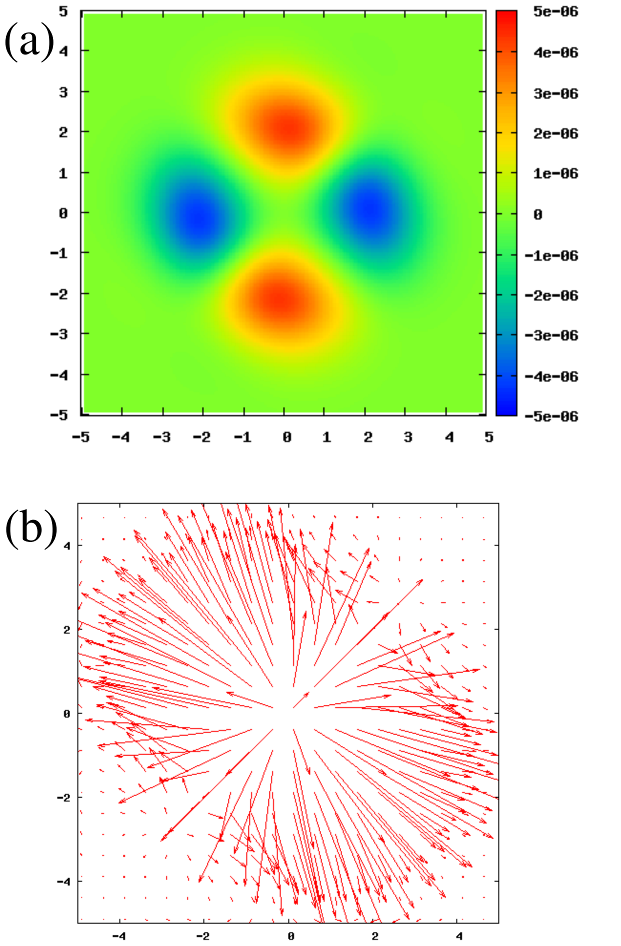

Figure 3. Distributions of SGS energy transfer and SGS force vector for Bardina term.

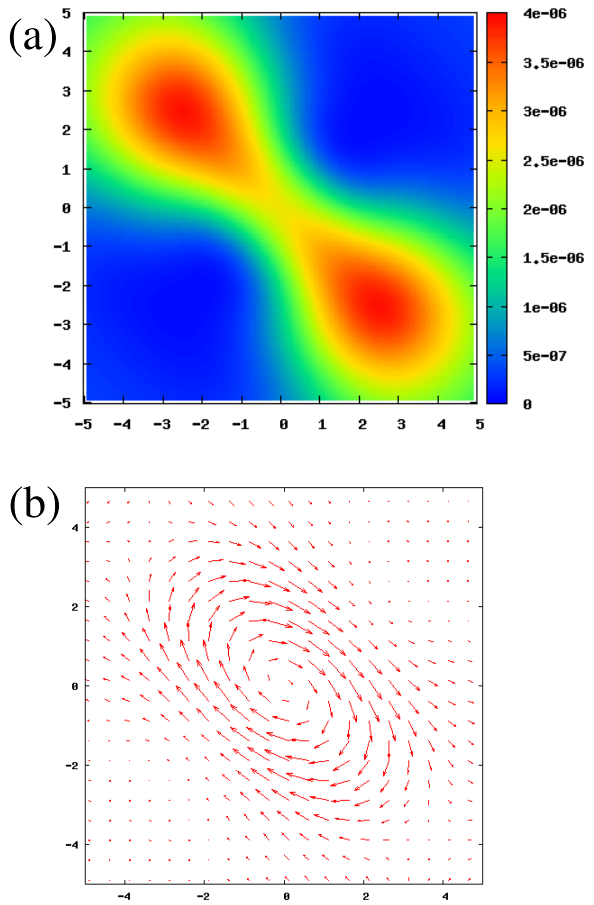

Figure 4. Distributions of SGS energy transfer and SGS force vector for Smagorinsky model.

Kobayashi (2005). The periodic boundary condition is used in x and z directions and no slip condition is adopted in y direction. The MAC scheme is adopted for the coupling of velocity and pressure, and the Poisson equation for pressure is solved by using FFT method. The third order Adams-Bashforth method is used for time marching scheme of Navier-Stokes equations. The linear implicit Euler scheme is adopted for the SGS kinetic energy equation.

Table 1 shows the numerical condition and grid resolution for the turbulent channel flows in $Re_\tau = 180$ and 590, where Re_τ is the Reynolds number based on wall friction velocity, the channel half width δ and molecular kinematic viscosity. The domain size $L_x \times L_y \times L_z$ and grid points $N_x \times N_y \times N_z$ in x, y and z directions are listed in Table 1, and Δx^+ and Δz^+ indicate the grid spaces in the wall unit, respectively. The extremely coarse grid resolution is adopted at $Re_\tau = 180$, whereas fine and coase resolutions are used at $Re_\tau = 590$.

RESULTS

Figure 1 shows the distributions of vorticity and velocity vectors for an elliptic Burgers vortex at $Re = 200$ based on circulation and molecular kinematic viscosity. By using filtering operation of Eq. (1) with filter width $1.25r_{max}$ where r_{max} is the radius with the maximum azimuthal velocity of the Burgers vortex (Burgers, 1948), we obtain the SGS energy transfer (red color: forward scatter) described as Eq. (2) and the SGS force vectors of Eq. (3) around the elliptic vortex for the Clark term in Eq. (7) as shown in Fig. 2. Those distributions with Clark term have good correlations of 0.8 with true SGS energy transfer of $-\tau_{ij}\overline{S}_{ij}$ and true SGS force of $-\partial\tau_{ij}/\partial x_j$.

In Fig. 3, shown are the SGS energy transfer and SGS force vectors distributions for the Bardina term in Eq. (5). That SGS energy transfer distribution is different from the true SGS energy transfer one similar to Fig. 2, and it indicates negative correlation with the true SGS energy transfer distribution. In addition, the SGS force is given in the opposite direction of minor axis of the elliptic vortex.

When we use the Smagorinsky model (Smagorinsky, 1963), the SGS energy transfer and SGS force vectors distribute as shown in Fig. 4. The Smagorinsky model as an eddy viscosity model gives a low correlation with the true SGS energy transfer distribution. Abe model results in the SGS energy transfer using the eddy viscosity, so that the correlation of the SGS energy transfer with the filtered DNS would not be so high. The Smagorinsky model as an eddy viscosity model gives the opposed force against the velocity vectors, so that it weakens the vortex. However, the nonlinear terms of the Clark and Bardina terms give rise to radial force. It is considered that these radial force affects the turbulent statistics.

The profiles of streamwise velocity, shear stress and kinetic energies of GS and SGS at $Re_\tau = 180$ with extremely coarse grids are displayed in Fig. 5. The SM (Smagorinsky model with van Driest wall damping function) model shows the overestimation of the mean streamwise velocity profile. The LS model without using one equation for k_{SGS} also overestimates the mean streamwise velocity, but improves the velocity profile better than the SM model. The LS-1eq model with k_{SGS} shows good predictions of mean streamwise velocity in comparison with the LS and SM models. The LS and LS-1eq models give better shear stress profiles than the SM model.

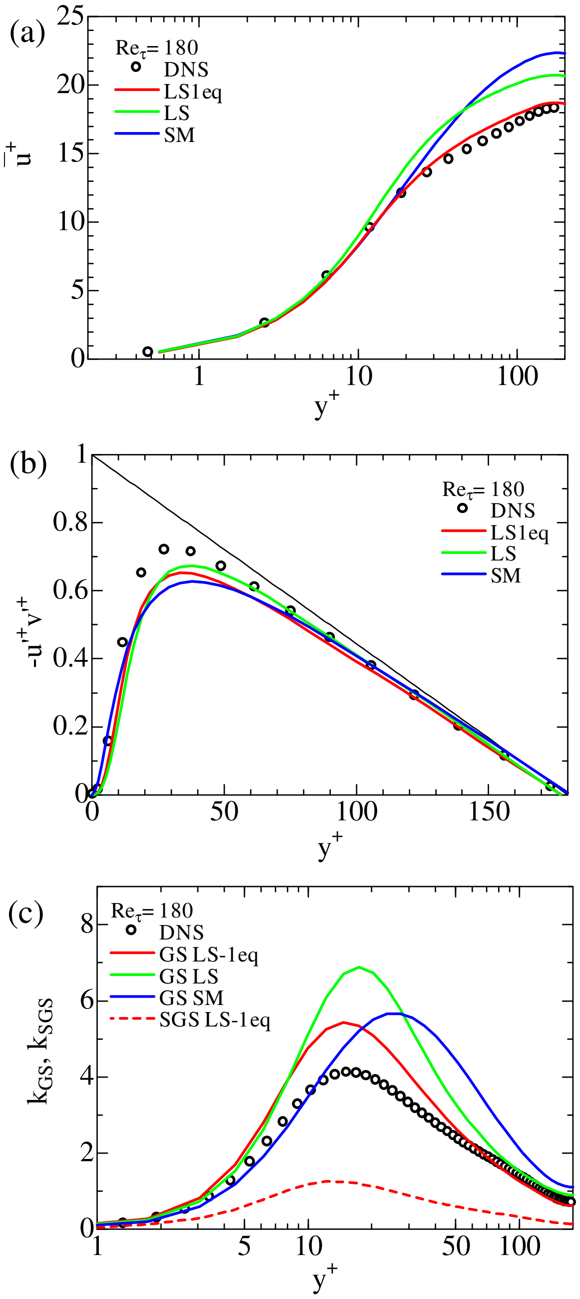

Figure 5. Profiles of mean streamwise velocity, shear stress and kinetic energies of GS and SGS at $Re_\tau = 180$ for extremely coarse grids.

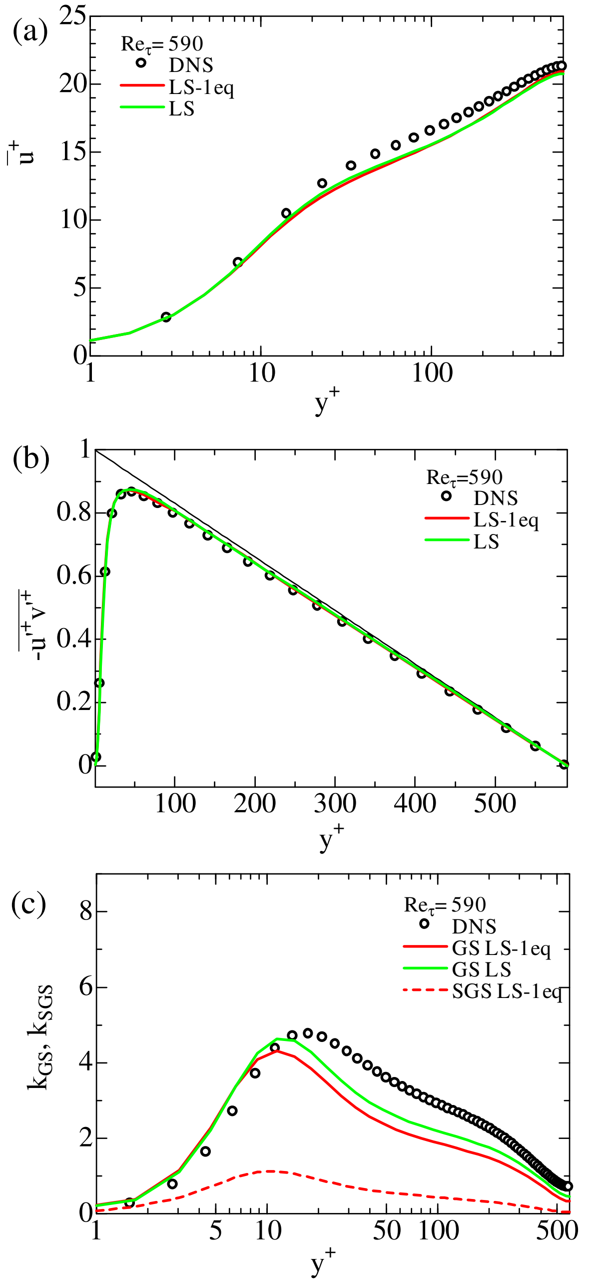

Figure 6. Profiles of mean streamwise velocity, shear stress and kinetic energies of GS and SGS at $Re_\tau = 590$ for fine grids.

The peak locations of the GS kinetic energy by the LS and LS-1eq models are in good agreement with the DNS result (Moser *et al.*, 1998) even in extremely coarse grid resolution. Note that only the LS-1eq model has the profile of the SGS kinetic energy k_{SGS} because it solves one equation of k_{SGS} with Eqs. (8) and (9).

The profiles of mean streamwise velocity, shear stress and kinetic energies of GS and SGS at $Re_\tau = 590$ with fine grid resolution are shown in Fig. 6. The profiles of mean streamwise velocity and shear stress with the LS-1eq model coincide with the DNS results (Moser *et al.*, 1998) as well as the LS model. The GS kinetic energy of the LS-1eq model is slightly diminished in comparison with the LS model, but a part of the kinetic energy is distributed to the SGS kinetic energy.

In Fig. 7, The profiles of mean streamwise velocity, shear stress and kinetic energies of GS and SGS at $Re_\tau = 590$ are shown in coarse grid resolution. In this resolution, the dynamic Smagorinsky model (DSM) (Germano *et al.*, 1991) relatively exhibits the overestimations for the mean streamwise velocity and GS kinetic energy. At the high Reynolds number, the differences of LS and LS-1eq are little. Those models show good performance in comparison with the DNS results (Moser *et al.*, 1998).

In the coarse grid resolutions for $Re_\tau = 180$ and 590, the GS kinetic energies of the LS and LS-1eq models become larger than those of the DNS results. The improvement of this overestimation is left in the future study.

SUMMARY

We proposed the improved SGS model based on the analysis of the SGS force and SGS energy transfer. The model is based on the Clark term as a similarity model. That term yields the distri-

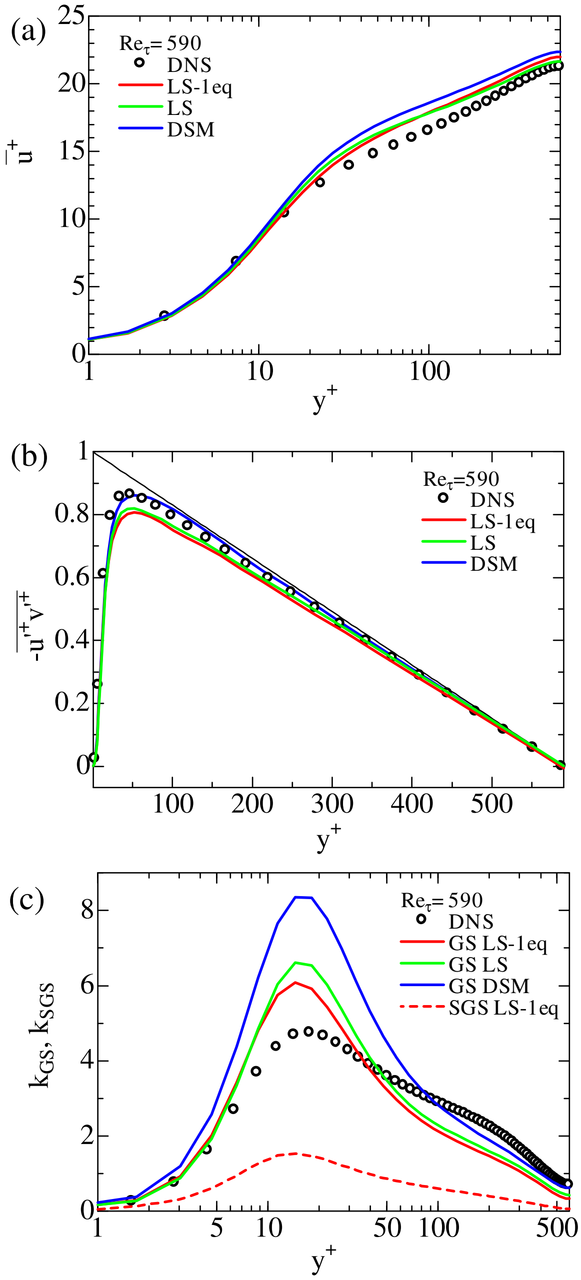

Figure 7. Profiles of mean streamwise velocity, shear stress and kinetic energies of GS and SGS at $Re_\tau = 590$ for coarse grids.

butions of the SGS force and energy transfer with high correlation for the true distributions obtained from the elliptic Burgers vortex. The proposed model consists of the Clark term with only forward scatter and the Clark term mixed with the SGS kinetic energy. The latter term does not affect the SGS energy transfer but contributes to the SGS force. This concept is proposed by Abe (2013). The present model showed good predictions in turbulent channel flows for $Re_\tau = 180$ with extremely coarse grid resolution and $Re_\tau = 590$ with fine and coarse grid resolutions. Especially, in the extremely coarse girds, the SGS model using one equation for the SGS kinetic energy considerably improves the mean streamwise velocity profile.

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