COMPUTATION OF WALL PRESSURE SPECTRA FROM CFD RANS SOLUTIONS INDUCED BY INCOMPRESSIBLE TURBULENT BOUNDARY LAYERS

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ABSTRACT

A method is developed to compute the wall pressure spectrum under a turbulent boundary layer based on RANS solutions. High-Reynolds number flows, such as the ones encountered in naval architecture studies, are considered. The model solves a specific equation for the wall pressure, based on the integral solution of a Poisson equation. A new model is proposed, the Extended Anisotropic Model (EAM), for the space-time velocity correlations which are necessary in order to close the equation. The method is applied to a turbulent boundary layer flow over a flat plate and satisfying results are obtained for the frequency pressure spectrum.

INTRODUCTION

Noise and vibrations due to turbulent boundary layer (TBL) flows constitute recurrent issues for many industrial applications, particularly for military naval architecture. In order to analyze the stochastic response of a structure to wall pressure excitations, the pressure spectrum is required. Usually, empirical models of wall pressure spectra (e.g. Chase (1987), Smol'yakov (2006), Goody (2004)) are used. However, these empirical models were developed for zero pressure gradient turbulent boundary layers with ideal conditions, which may be far from conditions of interest. The aim of this work is to replace empirical models, introduced as boundary conditions of noise and vibrations studies, with the numerical solution of a specific Poisson equation for pressure fluctuations according to Peltier & Hambric (2007) and Monté (2013). The method involves mean flow fields and turbulence quantities which can be obtained by computational fluid dynamics (CFD) solutions. This allows us to introduce local flow conditions in the model. Accurate predictions of the pressure spectrum may be obtained using direct numerical simulations (DNS) or large-eddy simulations (LES) but their computation times are still too large for complex industrial problems where numerous simulations have to be done. Instead, Reynolds-averaged Navier-Stokes (RANS) solutions are used and terms which can not be obtained from these solutions are modeled.

At this stage of development, the method is applied to a turbulent boundary layer flow over a flat plate with zero pressure gradient. The results obtained for the frequency spectrum are compared to Goody (2004) empirical model.

METHOD

The Poisson equation for the pressure is obtained by taking the divergence of the incompressible Navier-Stokes equations. By integrating this equation using an appropriate Green function, a solution for the wall pressure can be written. The expression for the wall pressure fluctuations is obtained by applying the Reynolds decomposition to separate the mean and fluctuating wall pressures and then subtracting the mean pressure. Finally, the space-time correlations of the wall pressure fluctuations (see Monté (2013), Peltier & Hambric (2007) for details) take the form:

$$\frac{\overline{p(\vec{x}_s,t)p(\vec{y}_s,\tau)}}{\rho^2} = \int_{\Omega_x}\int_{\Omega_y}\left[4\frac{\partial U_i}{\partial x_j}\frac{\partial V_k}{\partial y_l}\overline{u_j v_l}\frac{\partial G(\vec{x},\vec{x}_s)}{\partial x_i}\frac{\partial G(\vec{y},\vec{y}_s)}{\partial y_k} + 2\,\overline{u_i v_k}\;\overline{u_j v_l}\frac{\partial^2 G(\vec{x},\vec{x}_s)}{\partial x_i \partial x_j}\frac{\partial^2 G(\vec{y},\vec{y}_s)}{\partial y_k \partial y_l}\right]d\vec{y}d\vec{x} \qquad (1)$$

with $U_i(\vec{x})$ the mean velocity at point $\vec{x}$; $V_k(\vec{y})$ the mean velocity at point $\vec{y}$; $u_j(\vec{x},t)$ the fluctuating velocity at point $\vec{x}$ and time t; $v_l(\vec{y},\tau)$ the fluctuating velocity at point $\vec{y}$ and time τ; $\vec{x}_s$, $\vec{y}_s$ surface points. The symbol $\overline{\;.\;}$ denotes ensemble averaging.

Two types of terms can be identified: the turbulence-mean shear (TMS) interactions source terms (first group on the right-hand side) and the turbulence-turbulence (TT) interactions source terms (second group on the right-hand side).

In order to estimate turbulent pressure correlations from Eq. (1) we need: an appropriate Green function, the mean velocity field at two different points and two-point and two-time velocity correlations $\overline{u_i(\vec{x},t)v_k(\vec{y},\tau)}$. The velocity field can be obtained by RANS solutions but the two-point and two-time velocity correlations (or space-time velocity correlations) have to be modeled. These velocity correlations can be written as a function of Reynolds-stresses (i.e. one-point velocity correlations which can be obtained from RANS solutions) and a space-time correlation coefficient:

$$\overline{u_i(\vec{x},t)v_k(\vec{y},\tau)} = \sqrt{\overline{u_i u_i}(\vec{x})}\sqrt{\overline{v_k v_k}(\vec{y})}\;C_{ik}(\vec{x},\vec{y},t,\tau) \qquad (2)$$

For a TBL flow over a flat plate, four correlation coefficients are necessary: C_{11}, C_{22}, C_{33} et C_{12}.

A few models exist for the correlation coefficient C_{ik} such as the Simplified Anisotropic Model (SAM) developed by Gavin (2002) :

$$C_{ik}(\vec{\xi}) = \frac{\xi_i \xi_k}{{r^*}^2}\left[f(r^*) - g(r^*)\right] + \delta_{ik} g(r^*) \tag{3}$$

with :

$$\vec{\xi} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 + U_c \Delta t \\ r_2 \\ r_3 \end{bmatrix} \tag{4}$$

$$r^*(\vec{\xi}) = \sqrt{\left(\frac{\xi_1}{\gamma_1}\right)^2 + \left(\frac{\xi_2}{\gamma_2}\right)^2 + \left(\frac{\xi_3}{\gamma_3}\right)^2} \tag{5}$$

$$f(r^*) = \exp\left(-\frac{r^*}{L}\right) \tag{6}$$

$$g(r^*) = f(r^*)\left(1 - \frac{r^*}{2L}\right) \tag{7}$$

where $\vec{r}$ denotes the spatial separation vector between the points $\vec{x}$ and $\vec{y}$; Δt the time separation between the times t and τ; U_c the convection velocity; θ the inclination angle of the velocity correlations to the wall; f, g the longitudinal and transverse velocity correlation functions; $L = 0.35\delta$ a turbulence correlation length (based on the boundary layer thickness δ); γ_i the stretching coefficients which allow to introduce anisotropy in the velocity correlation coefficients. The values of the different parameters are presented in Table 1. Taylor's frozen flow hypothesis is used to relate space correlation coefficients to space-time correlation coefficients.

According to Peltier & Hambric (2007), the SAM was developed for the outer part of the boundary layer. Nevertheless, they applied it to compute the wall pressure for a channel flow, by using another definition for the turbulence correlation length: $L \equiv 0.54k^{3/2}/\varepsilon$ (where k denotes the turbulent kinetic energy and ε its dissipation rate), which is a function of the wall distance. Since no fit was provided for the coefficient C_{12} with the SAM, Peltier & Hambric (2007) used the relation : $C_{12} = \sqrt{C_{11}C_{22}}$.

Table 1. Parameters of the SAM.

C_{ik}	θ	γ_1	γ_2	γ_3
C_{11}	20°	1.000	0.700	0.520
C_{22}	90°	0.500	0.525	0.350
C_{33}	35°	0.800	0.220	0.400

In this work, we propose a new model for the space-time velocity correlation coefficients (including C_{12}), the Extended Anisotropic Model (EAM), based on Gavin (2002) model:

$$C_{ik}(\tilde{r}) = \frac{\tilde{r}_i \tilde{r}_k}{\tilde{r}^2}\left[f(\tilde{r}) - g(\tilde{r})\right] + g(\tilde{r}) \tag{8}$$

with :

$$\tilde{r}_i(\xi,\, x_2\,) = \frac{\xi_i}{L_i(x_2)} \tag{9}$$

$$f(\tilde{r}) = \begin{cases} \exp(-\tilde{r}), & \text{for } C_{22}, C_{33}, C_{12} \\ \alpha \exp(-\tilde{r}) + (1-\alpha)\exp(-\beta\tilde{r}), & \text{for } C_{11} \end{cases} \tag{10}$$

$$g(\tilde{r}) = f(\tilde{r})\left(1 - \frac{\tilde{r}}{2}\right) \tag{11}$$

where $\beta = 10$ and α is a function of the wall distance x_2. Contrary to Gavin (2002), we choose to use an anisotropic form for the correlation coefficient tensor, leading to the appearance of the last term of the right-hand side of Eq. (8) which should be zero for C_{12} in the pure isotropic case. Moreover, instead of the stretching coefficients, we use three correlation lengths, denoted L_i, to take into account the anisotropy of the flow. These correlation lengths are also functions of x_2. We also propose another expression for the velocity correlation functions f and g. The parameters α, θ and L_i are fitted to the DNS results of Sillero *et al.* (2014) for TBL flows at several positions across the boundary layer height. Thus, the EAM is adapted for both inner and outer parts of the boundary layer.

For both the SAM used by Peltier & Hambric (2007) and the EAM, the correlation coefficients tend to one for $\vec{r} = 0$ and $\Delta t = 0$. Since the two-point and two-time velocity correlations should tend to one-point correlation for $\vec{x} = \vec{y}$ and $t = \tau$, the coefficient C_{12} has to be multiplied by $\overline{u_1 u_2}(\vec{x}) / \sqrt{\overline{u_1 u_1}(\vec{x})}\sqrt{\overline{u_2 u_2}(\vec{x})}$.

Examples of the correlation coefficients calculated with the SAM (with $L \equiv 0.54k^{3/2}/\varepsilon$) and the EAM are presented in Figure 1 where they are compared to the DNS results of Sillero *et al.* (2014). Even though the results obtained with the EAM do not fit exactly the DNS results in all areas of the boundary layer, the EAM appears to be more appropriate than the SAM, particularly for the coefficient C_{11}.

Finally, to solve Eq. (1), a double volume integral must be evaluated numerically. Moreover, the Green function and its derivatives are singular in the case of a flow over a flat plate. We use a discrete Gauss-Legendre integration method combined with an adaptive Quadtree grid generation to take into account singularities. The RANS data used in this work are generated with the open-source CFD software *Code_Saturne* developed by EDF R&D (Archambeau *et al.*, 2004). The elliptic blending Reynolds Stress model (EBRSM) (Manceau & Hanjalić, 2002) is used for the turbulence closure.

RESULTS

We assume that pressure fluctuation correlations can be written as a function of the spatial separation $\vec{r_s} = \vec{y_s} - \vec{x_s}$ and the time separation $\Delta t = \tau - t$:

$$\overline{p(\vec{x_s}, t)\, p(\vec{y_s}, \tau)} = R_{pp}(\vec{r_s}, \Delta t) \tag{12}$$

The pressure spectrum is obtained by the Fourier transform of the pressure correlations. In this study we only calculate the frequency spectrum which is the Fourier transform of $R_{pp}(\vec{0}, \Delta t)$. The results presented are obtained by solving Eq. (1) for a zero pressure gradient TBL flow over a flat plate with both the TMS and TT source terms.

The time correlations and the frequency spectra obtained with the SAM and the EAM for C_{ik} are displayed in Figures 2 and 3. Frequency spectra are compared to the empirical frequency spectrum of Goody (2004) and time correlations are compared to the

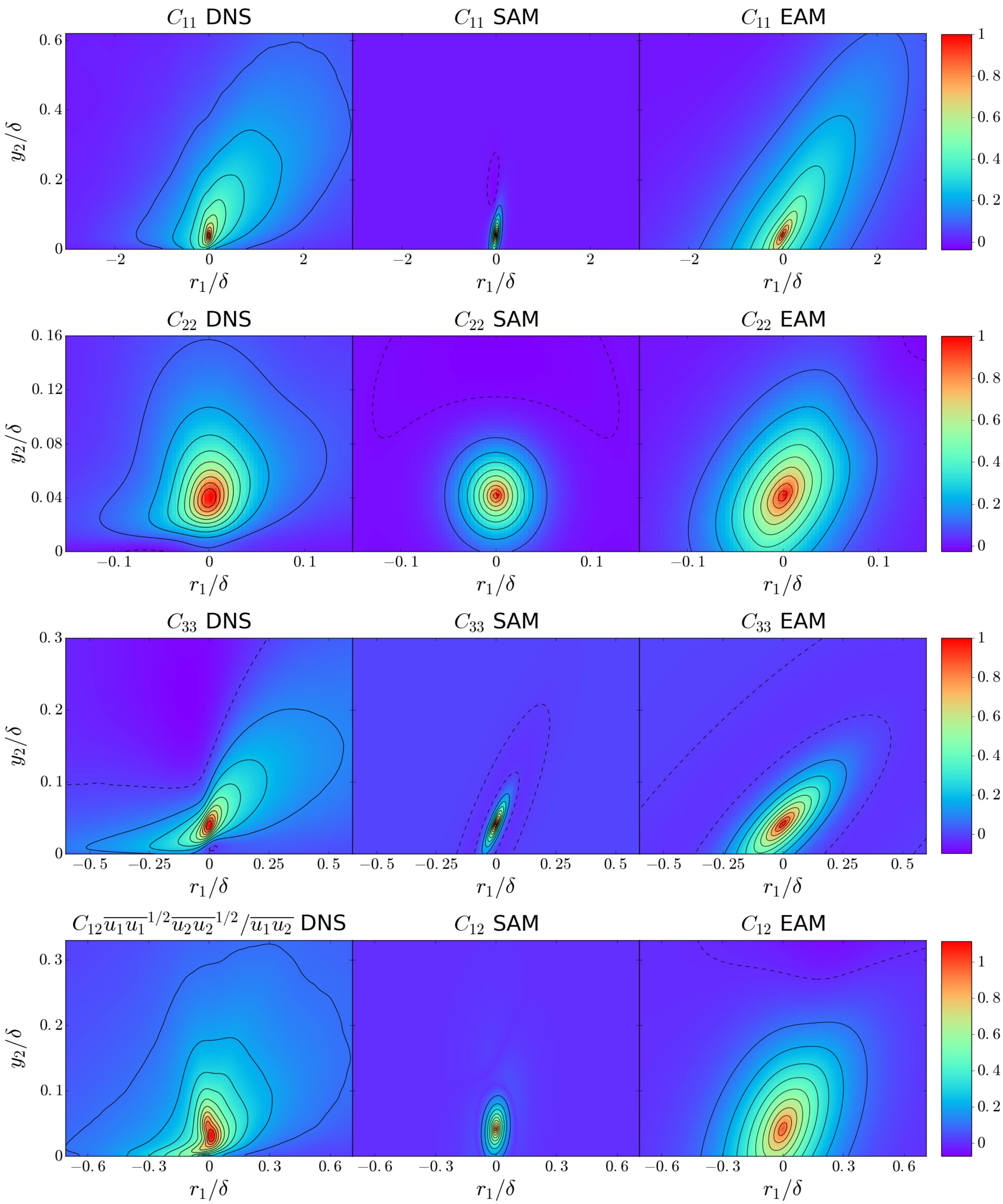

Figure 1. Streamwise (r_1, y_2) sections of the correlation coefficients C_{11}, C_{22}, C_{33} and C_{12} at $x_2/\delta = 0.04$ and $Re_\theta \approx 4860$, obtained with DNS and calculated with the SAM and the EAM. On each panel, positive contours (—–) are from 0.1 to 0.9 with increments of 0.1, the negative one (- - -) is -0.01.

inverse Fourier transform of Goody (2004) model. We choose to use Goody (2004) model as a reference since it provides satisfactory predictions of the frequency spectrum for a TBL flow over a flat plate (Hwang *et al.*, 2009). Differences up to ± 5 dB with this model are acceptable for our applications. The discrepancies between the spectra obtained with the SAM and the EAM show the impact of the correlation coefficients on the results. The wall pressure fluctuations estimated with the EAM, which takes into account the evolution of the anisotropy parameters with the wall distance, are in much better agreement with Goody (2004) model.

CONCLUSION

Satisfying results are obtained for the wall pressure fluctuations for a TBL flow over a flat plate without pressure gradient compare to Goody (2004) empirical model. Besides, the results show the influence of the space-time velocity correlation model on the pressure spectrum.

The next step is to compute the wavenumber-frequency spectrum which may validate the numerical approach for a TBL flow without pressure gradient. Then pressure gradients will be gradually taken into account.

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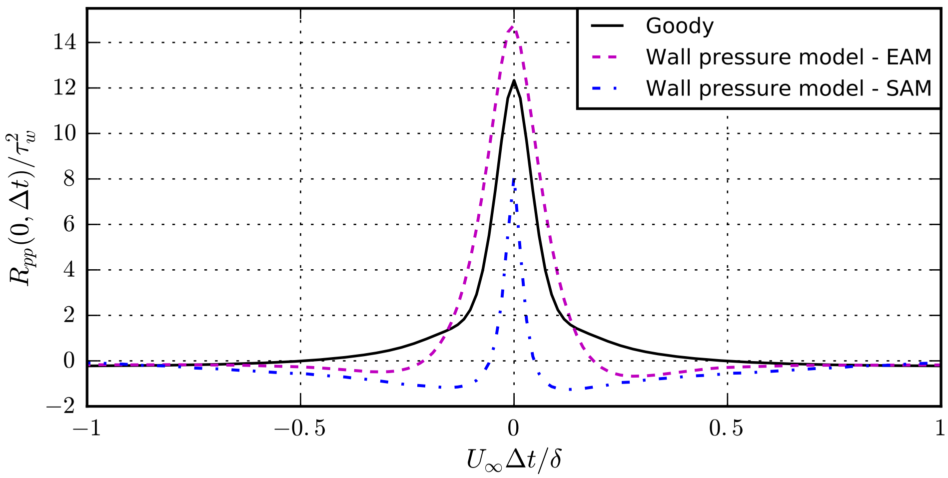

Figure 2. Time correlations of the wall pressure fluctuations obtained at $Re_\theta \approx 4860$.

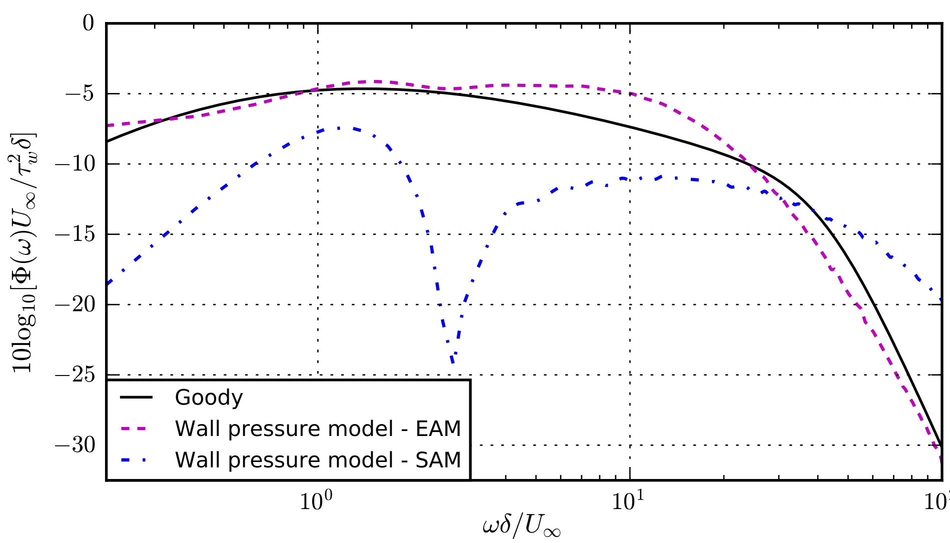

Figure 3. Frequency spectrum of the wall pressure fluctuations obtained at $Re_\theta \approx 4860$.